	Ministry of Higher Education and Scientific Research - Iraq Northern Technical University Technical Engineering College Kirkuk Department of Fuel and Energy Engineering	

MODULE DESCRIPTION FORM

نموذج وصف المادة الدراسية

Module Information معلومات المادة الدراسية					
Module Title	Mass Transfer			**Module Delivery**	
Module Type	Core			☒ Theory ☒ Lecture ☒ Lab ☒ Tutorial ☐ Practical ☐ Seminar	
Module Code	FEK305				
ECTS Credits	7				
SWL (hr/sem)	175				
Module Level	3		**Semester of Delivery**	1	
Administering Department	FEK		**College**	TCK	
Module Leader	Mohammed Qader Abdulrahman		**e-mail**	Mohammed83@ntu.edu.iq	
Module Leader's Acad. Title	The lecturer		**Module Leader's Qualification**	Ph.D.	
Module Tutor	Name (if available)		**e-mail**		
Peer Reviewer Name	Name		**e-mail**	E-mail	
Scientific Committee Approval Date	01/10/2024		**Version Number**	1.0	

Relation with other Modules العلاقة مع المواد الدراسية الأخرى			
Prerequisite module	None	**Semester**	
Co-requisites module	None	**Semester**	

<table>
<tr><th colspan="2">Module Aims, Learning Outcomes and Indicative Contents
أهداف المادة الدراسية ونتائج التعلم والمحتويات الإرشادية</th></tr>
<tr><td>Module Aims
أهداف المادة الدراسية</td><td>To introduce the basic principles of chemical engineering separation processes and mass transfer and then proceed to study the diffusion of components and also design and operation of separation processes units operation such as distillation, gas-liquid absorption, and stripping, adsorption, liquid-liquid extraction.</td></tr>
<tr><td>Module Learning Outcomes

مخرجات التعلم للمادة الدراسية</td><td>1. Explain The term diffusion (mass transfer), the physical phenomena, theoretical concepts, and design aspects of mass transfer in separation processes, including distillation, gas-liquid absorption, gas-solid adsorption, liquid-liquid extraction.

2. Analyse the important separation processes of distillation, gas absorption, adsorption, liquid-liquid extraction and carry out design calculations appropriately of the above processes.

3. Apply simplifying assumptions to complex problems in order to gain useful design information individually and in a team

4. Communicate (written and verbal) outcome of practical work.</td></tr>
<tr><td>Indicative Contents
المحتويات الإرشادية</td><td>Introduction
The term diffusion (mass transfer) is used to denote the transference of a component in a mixture from a region where its concentration is high to a region where the concentration is lower. Diffusion process can take place in a gas or vapour or in a liquid, and it can result from the random velocities of the molecules (molecular diffusion) or from the circulating or eddy currents present in a turbulent fluid (eddy diffusion).

Distillation (binary and multi-component). Calculation of number of plates, column height and diameter, heat transfer in condenser and reboiler. Crystallisation; application, theory, basic principles, super-saturation effects.
Gas absorption including application, different types of equipment in industry, process design of a column to find the height and diameter of the column, required solvent flow rate. The concepts and procedure for their calculation of the number of theoretical stages, height of theoretical stages, number of theoretical and height of theoretical units will be explained.</td></tr>
</table>

	In absorption (also called gas absorption, gas scrubbing, and gas washing), a gas mixture is contacted with a liquid (the absorbent or solvent) to selectively dissolve one or more components by mass transfer from the gas to the liquid. The components transferred to the liquid are referred to as solute or absorbate. Adsorption is a mass transfer process that is a phenomenon of sorption of gases or solutes by solid or liquid surfaces. The adsorption on the solid surface is that the molecules or atoms on the solid surface have residual surface energy due to unbalanced forces. Liquid-solvent extraction when the phases are immiscible, including application, different types of equipment in industry, solvent selection, process design of a column to find the height and the diameter of the column and the required solvent flow rate. The concepts and procedure for their calculation of number of theoretical stages, height of theoretical stages, number of theoretical and height of theoretical units will be explained. Liquid-solvent extraction when the phases are miscible, including application, different types of equipment in industry, solvent selection, process design of counter-current and cross flow stage wise operations to find the number of stages to meet operation constraints.

Learning and Teaching Strategies استراتيجيات التعلم والتعليم	
Strategies	Learning outcomes will be achieved through interactive lectures, tutorials and laboratory sessions. The lectures will be organized so that the students participate by organizing them in groups in the class and assigning to them points of discussion throughout the lecture. The tutorials will be organized so that the students work in groups discussing the problem at hands and its solution. Each group will be asked to raise and share questions with the rest of the class. All lecture notes and tutorial questions and their solutions will be posted on the VLE. The laboratory sessions will be conducted with students working in groups. During the sessions, the students will be challenged to explain the objectives of the experiments, the operation of the experiment, health safety precautions and error analysis of the data collected and present them in their report. The Learning outcomes covered by the examinations include an understanding of the fundamental principles of mass transfer operations and application of these principles to the design of operations such as distillation, gas absorption, liquid-liquid extraction and crystallisation.

	The assessment will be by formal examination (70%) and course work (30%). The formal exam, worth 70%, is of 2.0hr duration and taken at the end of semester 2. Formative assessment will take the form of frequent class tests on the unit operations of Distillation, Gas Absorption and Solvent Extraction. The coursework consists of a critical report of the group laboratory experiments on Distillation and Gas Absorption. It will include in it design and operation elements as well critical evaluation of the data collected. The coursework is worth 30% and students will have two weeks to complete it. Summative peer evaluation will be taken into account when calculating the individual mark for the coursework.

Student Workload (SWL) الحمل الدراسي للطالب محسوب لـ ١٥ اسبوعا			
Structured SWL (h/sem) الحمل الدراسي المنتظم للطالب خلال الفصل	112	**Structured SWL (h/w)** الحمل الدراسي المنتظم للطالب أسبوعيا	7
Unstructured SWL (h/sem) الحمل الدراسي غير المنتظم للطالب خلال الفصل	88	**Unstructured SWL (h/w)** الحمل الدراسي غير المنتظم للطالب أسبوعيا	6
Total SWL (h/sem) الحمل الدراسي الكلي للطالب خلال الفصل	200		

Module Evaluation تقييم المادة الدراسية					
		Time/Number	**Weight (Marks)**	**Week Due**	**Relevant Learning Outcome**
Formative assessment	**Quizzes**	2	10% (10)	5, 10	LO #1, 2, 10 and 11
	Assignments	2	10% (10)	2, 12	LO # 3, 4, 6 and 7
	Homework.	1	10% (10)	Continuous	All
	Report	1	10% (10)	13	LO # 5, 8 and 10
Summative assessment	**Midterm Exam**	2 hr	10% (10)	7	LO # 1-7
	Final Exam	3hr	50% (50)	16	All
Total assessment			100% (100 Marks)		

Delivery Plan (Weekly Syllabus) المنهاج الاسبوعي النظري	
	Material Covered
Week 1	Diffusion , flick's law, modes of diffusion
Week 2	Diffusivity coefficient in liquid and gas
Week 3	Absorption, equilibrium of gas and liquid
Week 4	Packed tower
Week 5	Tray tower
Week 6	Calculation of tower diameter, stripping
Week 7	Extraction, differential type
Week 8	Completely immiscible
Week 9	Party miscible
Week 10	Distillation , vapor-liquid equilibrium
Week 11	Continuous distillation
Week 12	flash distillation
Week 13	adsorption
Week 14	Physical adsorption and Chemisorption.
Week 15	Exam

Delivery Plan (Weekly Lab. Syllabus) المنهاج الاسبوعي للمختبر	
	Material Covered
Week 1	Lab 1: To determine the Liquid phase mass transfer coefficient in a wetted wall column.
Week 2	Lab 2: To determine the diffusion coefficient of an organic vapour i.e. CCl4 in Air.
Week 3	Lab 3: To study the effect of temperature on the diffusion co-efficient.
Week 4	Lab 4: To determine the Vapour-Liquid Equilibrium (VLE) curve for the CCl4 toluene mixture(Computerized).
Week 5	Lab 5: Continuous Distillation Column
Week 6	Lab 6: Liquid-liquid extraction in packed bed column.

Week 7	Lab 7: Simple batch distillation
Week 8	Lab 8: Absorption in Packed Column with Mass Transfer.

Learning and Teaching Resources مصادر التعلم و التدريس		
	Text	**Available in the Library?**
Required Texts	1. Treybal R.E., Mass Transfer Operations, McGraw Hill 2. McCabe W.L., Smith J.C. & Harriott P., Unit Operations in Chemical Engineering, McGraw Hill.	Yes
Recommended Texts	3. Seader J.D.& Henley E.J., Separation Process Principles. 4. Rousseau R.W., Handbook of Separation Process Technology, John Wiley 5. Foust A.S. et al, Principles of Unit Operations, John Wiley	No
Websites	https://www.usb.ac.ir/FileStaff/6885_2019-4-27-19-27-38.pdf	

Grading Scheme مخطط الدرجات				
Group	**Grade**	التقدير	**Marks (%)**	**Definition**
Success Group (50 - 100)	**A -** Excellent	امتياز	90 - 100	Outstanding Performance
	B - Very Good	جيد جدا	80 - 89	Above average with some errors
	C - Good	جيد	70 - 79	Sound work with notable errors
	D - Satisfactory	متوسط	60 - 69	Fair but with major shortcomings
	E - Sufficient	مقبول	50 - 59	Work meets minimum criteria
Fail Group (0 – 49)	**FX –** Fail	راسب (قيد المعالجة)	(45-49)	More work required but credit awarded
	F – Fail	راسب	(0-44)	Considerable amount of work required

Note: Marks Decimal places above or below 0.5 will be rounded to the higher or lower full mark (for example a mark of 54.5 will be rounded to 55, whereas a mark of 54.4 will be rounded to 54. The University has a policy NOT to condone "near-pass fails" so the only adjustment to marks awarded by the original marker(s) will be the automatic rounding outlined above.

الحقيبة الدراسية

الكلية التقنية الهندسية/كركوك

قسم هندسة تقنيات الوقود والطاقة

انتقال المادة/المرحلة الثالثة

الدكتور محمد قادر عبدالرحمن

Diffusion

Presenter by:

Dr. Mohammed Qader

Fuel and Energy Engineering

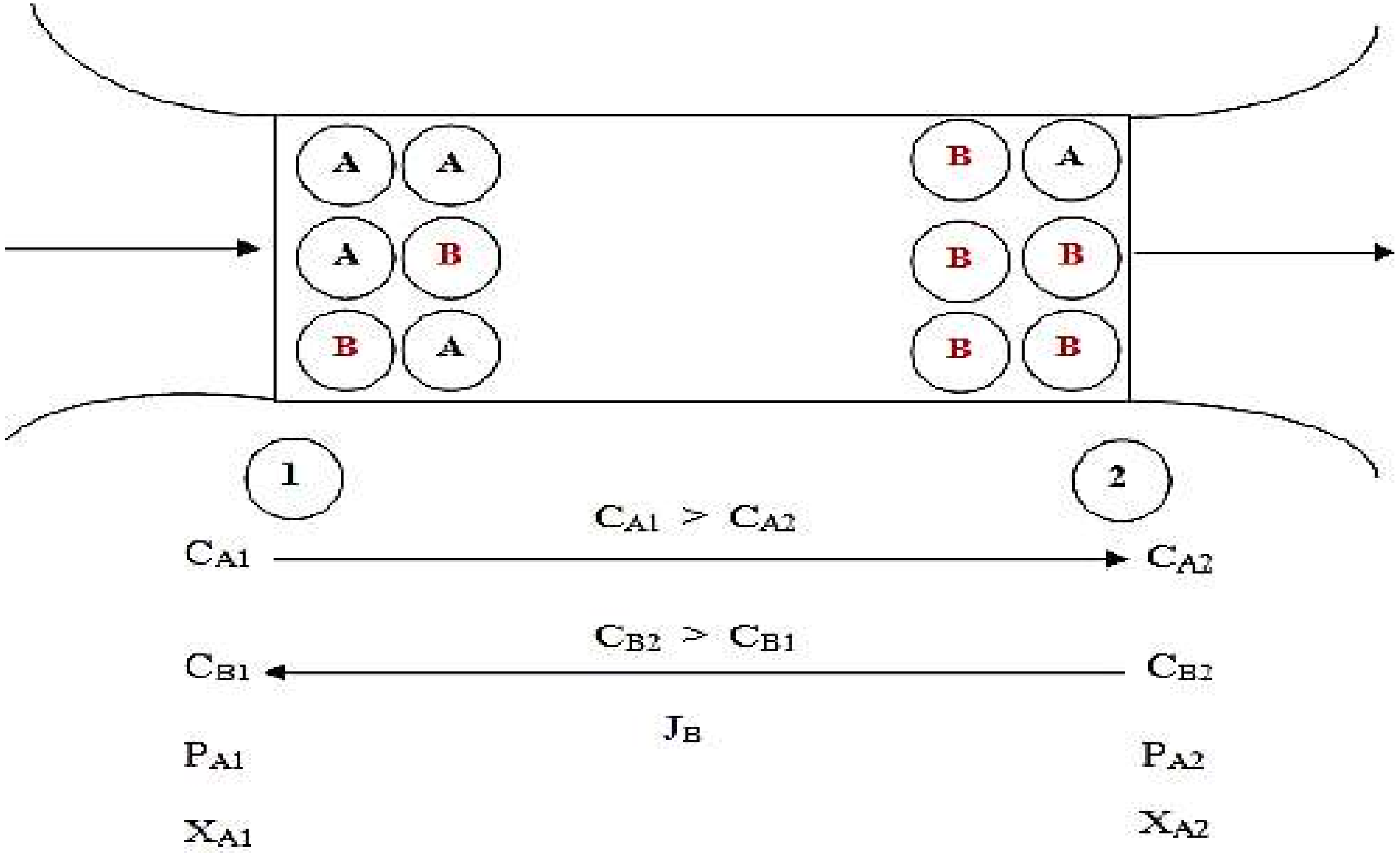
Diffusion

The term diffusion (mass transfer) is used to denote the transference of a component in a mixture from a region where its concentration is high to a region where the concentration is lower. Diffusion process can take place in a gas or vapour or in a liquid, and it can result from the random velocities of the molecules **(molecular diffusion)** or from the circulating or eddy currents present in a turbulent fluid **(eddy diffusion)**.

Diffusion depends on:

1. Driving force (ΔC), moles per unit volume ($kmol/m^3$).
2. The distance in the direction of transfer (Δz), meter (m).
3. Diffusivity coefficient, unit area per unit time (m^2/s).

Diffusion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CA1
CA2
CB2 > CB1
CB1
CB2
JB
PA1
PA2
XA1
XA2

Diffusion

Fick's Law of diffusion:

The rate of diffusion is governed by Fick's Law, first proposed by Fick in 1855 which expresses the mass transfer rate as a linear function of the molar concentration gradient. In a mixture of two gases **A** and **B**, assumed ideal, Fick's Law for steady state diffusion may be written as:

$$J_A \quad \alpha \quad \frac{\Delta C_A}{\Delta z}$$

$$\boxed{J_A = -D_{AB}\frac{dC_A}{dz}} \quad \ldots\ldots\ldots\ldots \qquad \text{Fick's first law of steady state diffusion}$$

Where:

J_A: is the molecular diffusion flux of **A**, (moles per unit area per unit time) $\left[\frac{kmol}{m^2.s}\right]$.

C_A: is the concentration of **A** (moles of A per unit volume) $\left[\frac{kmol}{m^3}\right]$.

D_{AB}: is known as the diffusivity or diffusion coefficient for **A** in **B** (unit area per unit time) $\left[\frac{m^2}{s}\right]$

z: is distance in the direction of transfer (m).

Diffusion

The Fick's first law of diffusion describes the mass transfer from the random movement of molecules of a stationary medium or a fluid in streamline flow. If circulating currents or eddies are present, then the molecular mechanism will be reinforced and the total mass transfer rate may be written as:

Total diffusion = Molecular diffusion + Convection term

Convection term = Eddy diffusion = Molar flux due to convection

Convection term = Concentration * mass transfer velocity = $C_A \cdot V$

Diffusion

Where:

$$\textbf{mass transfer velocity (V)} = \frac{\textbf{mass flux}}{\textbf{concentration}} = \frac{N_A + N_B}{C_T} = \frac{\frac{kmol}{m^2.s}}{\frac{kmol}{m^3}} = \frac{m}{s}$$

$$\textbf{Total diffusion} = N_A = J_A + C_A \cdot V$$

$$N_A = -D_{AB}\frac{dC_A}{dz} + \frac{C_A}{C_T}(N_A + N_B) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Total diffusion equation in the form of concentration (normally used for liquids)

Diffusion

The total diffusion equation can be write in another forms:

a. Partial pressure for gases.
b. Mole fraction for gases and liquids.

a. <u>Total diffusion equation in the partial pressure form:</u>

If **A** and **B** are ideal gases in a mixture, the ideal gas law may be applied to each gas separately and to the mixture:

$$P V = n R T \quad \Longrightarrow \quad P = \frac{n}{V} R T$$

$$P = C R T$$

$$P_A = C_A R T \qquad \text{and} \qquad P_T = C_T R T$$

$$C_A = \frac{P_A}{R T}$$

$$dC_A = \frac{1}{R T} dP_A$$

$$\boxed{N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A + N_B)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Total diffusion equation in the form of partial pressure (normally used for gases)

Diffusion

b. **Total diffusion equation in the mole fraction form:**

$$\bar{y}_A = \frac{P_A}{P_T} \quad \text{or} \quad X_A = \frac{C_A}{C_T}$$

$$P_T\ \bar{y}_A = P_A \quad \text{and} \quad C_T\ X_A = C_A$$

$$P_T\ d\bar{y}_A = dP_A \quad \text{and} \quad C_T\ dX_A = dC_A$$

Then:

$$\mathbf{N_A = \frac{-D_{AB}\ P_T}{RT}\ \frac{d\bar{y}_A}{dz} + \bar{y}_A\ (N_A + N_B)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

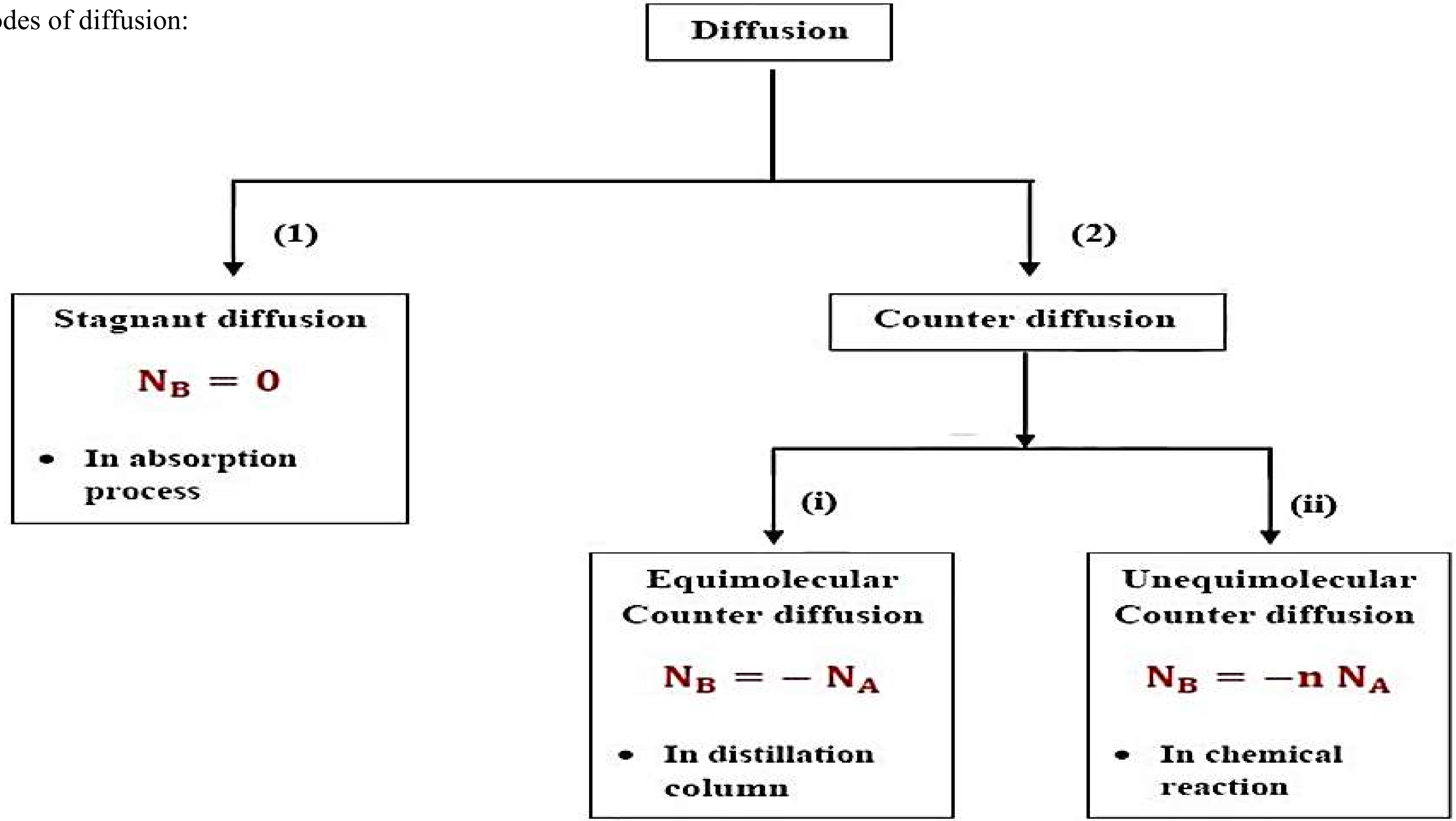
↘

Total diffusion equation in the form of mole fraction (used for gases)

Diffusion

Modes of diffusion

There are two modes of diffusion:



Diffusion

1. Stagnant diffusion (Mass transfer through a stationary second component):

In several important processes, one component in a gaseous mixture will be transported relative to a fixed plane, such as a liquid interface, for example, and the other will undergo no net movement. In gas absorption a soluble gas **A** is transferred to the liquid surface where it dissolves, whereas the insoluble gas **B** undergoes no net movement with respect to the interface. Similarly, in evaporation from a free surface, the vapour moves away from the surface but the air has no net movement. The mass transfer process therefore:

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A + N_B) \qquad \text{……………………………………………}(1)$$

Diffusion

Since stagnant diffusion layer: $N_B = 0$

$$N_A = \frac{-D_{AB}}{RT}\ \frac{dP_A}{dz} + \frac{P_A}{P_T}\ N_A \qquad \dots\dots\dots (2)$$

$$N_A\left(1 - \frac{P_A}{P_T}\right) = \frac{-D_{AB}}{RT}\ \frac{dP_A}{dz} \qquad \dots\dots\dots (3)$$

$$N_A = \frac{-D_{AB}}{RT}\ \frac{1}{dz}\ \frac{dP_A}{\left(1 - \frac{P_A}{P_T}\right)} \qquad \dots\dots\dots (4)$$

$$N_A = \frac{-D_{AB}}{RT}\ \frac{P_T}{dz}\ \frac{dP_A}{(P_T - P_A)} \qquad \dots\dots\dots (5)$$

$$N_A = \frac{D_{AB}}{RT}\ \frac{P_T}{(z_2 - z_1)}\ \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

Diffusion

Example 10.1: Ammonia gas is diffusing at a constant rate through a layer of stagnant air 1 mm thick. Conditions are such that the gas contains 50 percent by volume ammonia at one boundary of the stagnant layer. The ammonia diffusing to the other boundary is quickly absorbed and the concentration is negligible at that plane. The temperature is 295 K and the pressure atmospheric, and under these conditions the diffusivity of ammonia in air is 0.18 cm^2/s. Estimate the rate of diffusion of ammonia through the layer.

Solution:

If the subscripts 1 and 2 refer to the two sides of the stagnant layer and the subscripts **A** and **B** refer to ammonia and air respectively, then the rate of diffusion through a stagnant layer is given by:

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{(z_2 - z_1)} \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

Diffusion

where:

$$P_T = 101.3\ \text{kPa} \quad , \quad P_{A_2} = 0 \quad , \quad P_{A_1} = y_A P_T = 0.5 * 101.3 = 50.65\ \text{kPa}$$

$$\Delta z = z_2 - z_1 = 1\ \text{mm} = 1 * 10^{-3}\ \text{m}$$

$$R = 8.314\ \frac{\text{kJ}}{\text{kmol. K}} \quad , \quad T = 298\ \text{K} \quad \text{and} \quad D_{AB} = 0.18\ \frac{cm^2}{s} = 1.8 * 10^{-5}\ \frac{m^2}{s}$$

$$N_A = \frac{1.8 * 10^{-5}}{8.314 * 295}\ \frac{101.3}{1 * 10^{-3}}\ \ln\left[\frac{101.3 - 0}{101.3 - 50.65}\right] = 5.153 * 10^{-4}\ \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

Diffusion

2. Counter diffusion:

i. Equimolecular counter diffusion:

When the mass transfer rates of the two components are equal and opposite the process is said to be one of equimolecular counter diffusion. Such a process occurs in the case of the box with a movable partition. It occurs also in a distillation column when the molar latent heats of the two components are the same ($\lambda_A = \lambda_B$). At any point in the column a falling stream of liquid is brought into contact with a rising stream of vapour with which it is not in equilibrium. The less volatile component is transferred from the vapour to the liquid and the more volatile component is transferred in the opposite direction. If the molar latent heats of the components are equal, the condensation of a given amount of less volatile component releases exactly the amount of latent heat required to volatilize the same molar quantity of the more volatile component. Thus, at the interface, and consequently throughout the liquid and vapour phases, equimolecular counter diffusion is taking place ($N_B = -N_A$).

Diffusion

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A + N_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

Since equimolecular counter diffusion: $N_B = -N_A$

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A - N_A) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$

$$N_A = \frac{-D_{AB}}{RT}\left(\frac{P_{A_2} - P_{A_1}}{z_2 - z_1}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (5)$$

$$N_A = \frac{D_{AB}}{RT}\left(\frac{P_{A_1} - P_{A_2}}{z_2 - z_1}\right)$$

Diffusion

Drift Factor:

For stagnant diffusion:

$$N_A = \frac{D_{AB}}{RT}\ \frac{P_T}{\Delta z}\ \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

$$N_A = \frac{D_{AB}}{RT}\ \frac{P_T}{\Delta z}\ \left[\frac{(P_T - P_{A_2}) - (P_T - P_{A_1})}{(P_T - P_{A_2}) - (P_T - P_{A_1})}\right]\ \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

From Dalton's Law of partial pressures: $P_T = P_A + P_B$

By definition, P_{Bm}, the logarithmic mean of P_{B1} and P_{B2}, is given by:

Diffusion

$$\frac{(P_T - P_{A_2}) - (P_T - P_{A_1})}{\ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]} = \frac{P_{B_2} - P_{B_1}}{\ln\left[\frac{P_{B_2}}{P_{B_1}}\right]} = P_{Bm}$$

$$N_A = \frac{D_{AB}}{RT} \frac{1}{\Delta z} \left[\frac{P_T}{P_{Bm}}\right] (P_{A_1} - P_{A_2})$$

Where: $\left[\frac{P_T}{P_{Bm}}\right]$ is known as the ***drift factor***.

If the ***drift factor*** = $\left[\frac{P_T}{P_{Bm}}\right] = 1$ (this happen when the concentration of component **A** being transferred is low)

Then,

$$N_A = \frac{D_{AB}}{RT} \left(\frac{P_{A_1} - P_{A_2}}{z_2 - z_1}\right)$$

*** Thus, the bulk flow enhances the mass transfer rate by a factor $\frac{P_T}{P_{Bm}}$, known as the drift factor.**

Diffusion

Example: In an air-carbon dioxide mixture at 298 K and 202.6 kPa, the concentration of CO_2 at two planes (3 mm) apart are 15 vol.% and 25 vol.%. The diffusivity of CO_2 in air at 298 K and 202.6 kPa is $8.2*10^{-6}$ m^2/s. Calculate the rate of transfer of CO_2 across the two planes, assuming:

a. Equimolecular counter diffusion.

b. Diffusion of CO_2 through a stagnant air layer.

Solution:

$P_{A_1} = y_{A_1} . P_T$ = (0.25) (202.6) =50.65 kPa

$P_{A_2} = y_2 . P_T$ = (0.15) (202.6) =30.39 kPa

Diffusion

a. Equimolecular counter diffusion.

$$N_A = \frac{D_{AB}}{RT}\left(\frac{P_{A_1} - P_{A_2}}{z_2 - z_1}\right)$$

$$N_A = \frac{8.2 * 10^{-6}}{(8.314)(298)(3 * 10^{-3})}(50.65 - 30.39) = 2.23 * 10^{-5} \quad \frac{kmol}{m^2.s}$$

b. Stagnant diffusion.

$$N_A = \frac{D_{AB}}{RT}\frac{P_T}{\Delta z}\ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

$$N_A = \frac{8.2 * 10^{-6}}{(8.314)(298)}\frac{202.6}{(3 * 10^{-3})}\ln\left[\frac{202.6 - 30.39}{202.6 - 50.65}\right] = 2.79 * 10^{-5} \quad \frac{kmol}{m^2.s}$$

Diffusion

ii. Unequimolecular counter diffusion:

When the mass transfer rates of the two components are unequal and opposite, the process is said to be the unequimolecular diffusion, such a process occurs in a chemical reaction.

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A + N_B) \qquad \dots (1)$$

Since unequimolecular counter diffusion: $N_B = -n\,N_A$

$$N_A = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A - n\,N_A) \qquad \dots (2)$$

$$N_A\left(1 - \left[\frac{P_A}{P_T}\right](1-n)\right) = \frac{-D_{AB}}{RT}\frac{dP_A}{dz} \qquad \dots (3)$$

$$N_A = \frac{-D_{AB}}{RT}\frac{P_T}{dz}\frac{dP_A}{\left(1 - \left[\frac{P_A}{P_T}\right](1-n)\right)} \qquad \dots (4)$$

$$N_A = \frac{-D_{AB}}{RT}\frac{1}{\Delta z}\frac{dP_A}{(P_T - P_A(1-n))} \qquad \dots (5)$$

Diffusion

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \frac{1}{(1-n)} \ln\left[\frac{P_T - (1-n)\,P_{A_2}}{P_T - (1-n)\,P_{A_1}}\right]$$

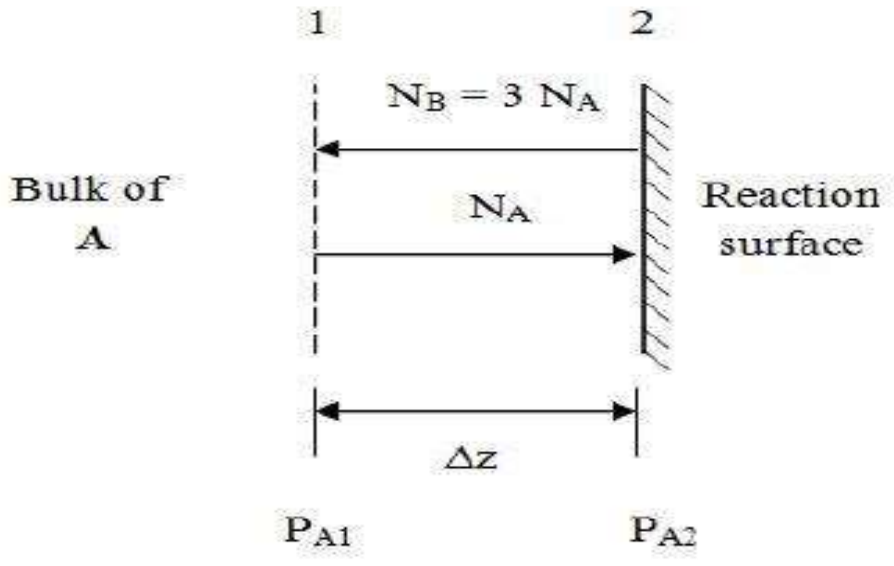
Example: Species **A** in a gaseous mixture diffuses through a (3 mm) thick film and reaches a catalyst surface where the reaction **A** → **3B** takes place. If the partial pressure of **A** in the bulk of the gas is 8.5 kN/m^2 and the diffusivity of **A** is 2*10^{-5} m^2/s. Find the mole flux of **A**, given the pressure and temperature of the system are 101.3 kPa and 297 K, respectively.

Diffusion

Solution:

$$\mathbf{A \rightarrow 3B}$$

$$\mathbf{n = \frac{N_B}{N_A} = \frac{3}{1} = 3}$$



Given:

$D_{AB} = 2 * 10^{-5}\ \frac{m^2}{s}$, $P_T = 101.3\ kPa$

$T = 297\ K$, $P_{A_1} = 8.5\ kPa$

$P_{A_2} = 0$

$$\mathbf{N_A = \frac{D_{AB}}{RT}\ \frac{P_T}{\Delta z}\ \frac{1}{(1-n)}\ ln\left[\frac{P_T - (1-n)\,P_{A_2}}{P_T - (1-n)\,P_{A_1}}\right]}$$

$$\mathbf{N_A} = \frac{2 * 10^{-5}}{8.314 * 297}\ \frac{101.3}{3 * 10^{-3}}\ \frac{1}{(1-3)}\ \ln\left[\frac{101.3 + 2(0)}{101.3 + 2(8.5)}\right] = 2.12 * 10^{-5}\ \frac{kmol}{m^2.s}$$

Diffusion

Maxwell's Law for multicomponent mass transfer

This argument can be applied to the diffusion of a constituent of a multicomponent gas. Considering the transfer of component, **A** through a ***stationary gas*** consisting of components B, C, D, ... etc, if the total partial pressure gradient can be regarded as being made up of a series of terms each representing the contribution of the individual component gases. The mass transfer rate can be calculated from the previous equations using the effective diffusivity of A in the mixture ($\mathbf{D_{Am}}$).

Calculation of the effective diffusivity of (A) in the mixture ($\mathbf{D_{Am}}$):

Let **A** be the diffusing species through *stagnant* mixture of **B**, **C**, **D** ….. etc.

$$N_A = -D_{Am}\, C_T \frac{dX_A}{dz} + X_A\,(N_A + N_B + N_C + N_D) \qquad \text{.......................(1)}$$

where: $\mathbf{D_{Am}}$ is the effective diffusivity of **A** in the mixture.

Since stagnant diffusion layer of the mixture: $\mathbf{N_B = N_C = N_D = 0}$

$$\mathbf{N_A = -D_{Am}\, C_T \frac{dX_A}{dz} + X_A(N_A)} \qquad \text{.............................. (2)}$$

Diffusion

$$\frac{N_A}{C_T}\left(\frac{1-X_A}{D_{Am}}\right) = -\frac{dX_A}{dz} \quad \text{.............................} (3)$$

Now consider binary system for diffusion of **A** in **B**.

$$N_A = -D_{AB}C_T\frac{dX_A}{dz} + X_A(N_A + N_B)$$

Since stagnant diffusion layer: $N_B = 0$

$$N_A = -D_{AB}C_T\frac{dX_A}{dz} + X_A(N_A)$$

$$N_A(1 - X_A) = -D_{AB}C_T\frac{dX_A}{dz}$$

$$\frac{N_A}{C_T}\left(\frac{1 - X_A}{D_{AB}}\right) = -\frac{dX_A}{dz}$$

But: $(1-X_A) = X_B \Longrightarrow -dX_A = dX_B$

$$\frac{N_A}{C_T}\left(\frac{X_B}{D_{AB}}\right) = \frac{dX_B}{dz} \quad \text{........................}(4)$$

Diffusion

Similarly for diffusion of **A** in **C**.

$$N_A = -D_{AC}\ C_T\ \frac{dX_A}{dz} + X_A\ (N_A + N_C)$$

Since stagnant diffusion layer: $N_C = 0$

$$N_A = -D_{AC}\ C_T\ \frac{dX_A}{dz} + X_A\ (N_A)$$

$$N_A(1 - X_A) = -D_{AC}\ C_T\ \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T}\left(\frac{1 - X_A}{D_{AC}}\right) = -\ \frac{dX_A}{dz}$$

But: $(1 - X_A) = X_C \Longrightarrow -dX_A = dX_C$

$$\frac{N_A}{C_T}\left(\frac{X_C}{D_{AC}}\right) = \frac{dX_C}{dz} \quad \text{.................................}(5)$$

Diffusion

Similarly for diffusion of **A** in **D**.

$$\mathbf{N_A = -D_{AD}C_T\frac{dX_A}{dz} + X_A(N_A + N_D)}$$

Since stagnant diffusion layer: $\mathbf{N_D = 0}$

$$N_A = -D_{AD}\ C_T\ \frac{dX_A}{dz} + X_A\ (N_A)$$

$$N_A(1 - X_A) = -D_{AD}\ C_T\ \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T}\left(\frac{1 - X_A}{D_{AD}}\right) = -\ \frac{dX_A}{dz}$$

But: $(1 - X_A) = X_D \Longrightarrow -dX_A = dX_D$

$$\frac{N_A}{C_T}\left(\frac{X_D}{D_{AD}}\right) = \frac{dX_D}{dz} \qquad \text{.. (6)}$$

Diffusion

Now adding Equations (4), (5) and (6):

$$\frac{N_A}{C_T}\left(\frac{X_B}{D_{AB}}+\frac{X_C}{D_{AC}}+\frac{X_D}{D_{AD}}\right)=\frac{d(X_B+X_C+X_D)}{dz}$$

But: $(X_B + X_C + X_D) = 1 - X_A$

$$\frac{d(X_B+X_C+X_D)}{dz}=-\frac{dX_A}{dz}$$

$$\frac{N_A}{C_T}\left(\frac{X_B}{D_{AB}}+\frac{X_C}{D_{AC}}+\frac{X_D}{D_{AD}}\right)=\frac{N_A}{C_T}\frac{(1-X_A)}{D_{Am}}$$

$$\frac{1-X_A}{D_{Am}}=\frac{X_B}{D_{AB}}+\frac{X_C}{D_{AC}}+\frac{X_D}{D_{AD}}$$

For dilute mixture (low concentration of A), $X_A \rightarrow 0$

$$\frac{1}{D_{Am}}=\frac{X_B}{D_{AB}}+\frac{X_C}{D_{AC}}+\frac{X_D}{D_{AD}}$$

Diffusion

Example: Nitrogen is diffusing under steady condition through a mixture of 2% N_2, 20% C_2H_6, 30% C_2H_4 and 48% C_4H_{10} at 298 K and 100 kPa. The partial pressure of nitrogen at two planes (1mm) apart are 13.3 & 6.67 kPa, respectively. Calculate the rate of N_2 across the two planes. The diffusivity of N_2 through C_4H_{10}, C_2H_6 and C_2H_4 may be taken as $9.6*10^{-6}$ m^2/s , $14.8*10^{-6}$ m^2/s and $16.3*10^{-6}$ m^2/s, respectively.

Solution:

Since stagnant diffusion:

$$N_A = \frac{D_{Am}}{RT} \frac{P_T}{\Delta z} \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]$$

$$\frac{1 - y_A}{D_{Am}} = \frac{y_B}{D_{AB}} + \frac{y_C}{D_{AC}} + \frac{y_D}{D_{AD}}$$

$$\frac{1 - 0.02}{D_{Am}} = \frac{0.48}{9.6 * 10^{-6}} + \frac{0.2}{14.8 * 10^{-6}} + \frac{0.3}{16.3 * 10^{-6}}$$

$$D_{Am} = 1.22 * 10^{-5} \ \frac{m^2}{s}$$

$$N_A = \frac{1.22 * 10^{-5}}{8.314 * 298} \ \frac{100}{0.001} \ \ln\left[\frac{100 - 6.67}{100 - 13.3}\right] = 0.0492 \ \frac{kmol}{m^2.s}$$

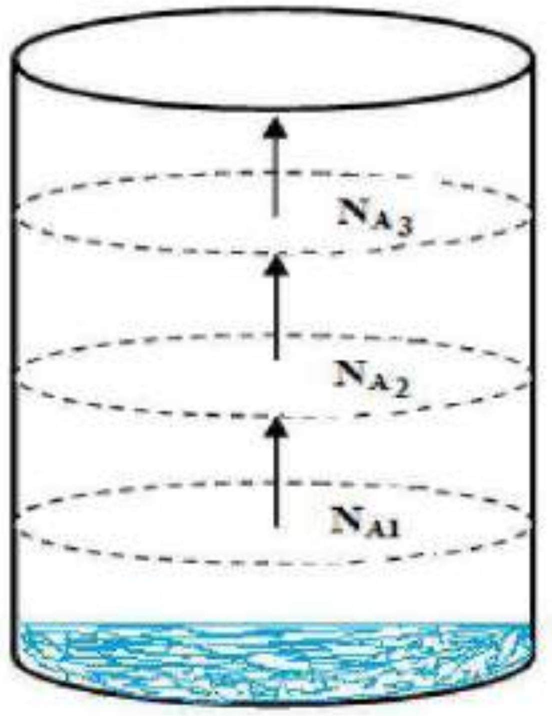
Diffusion

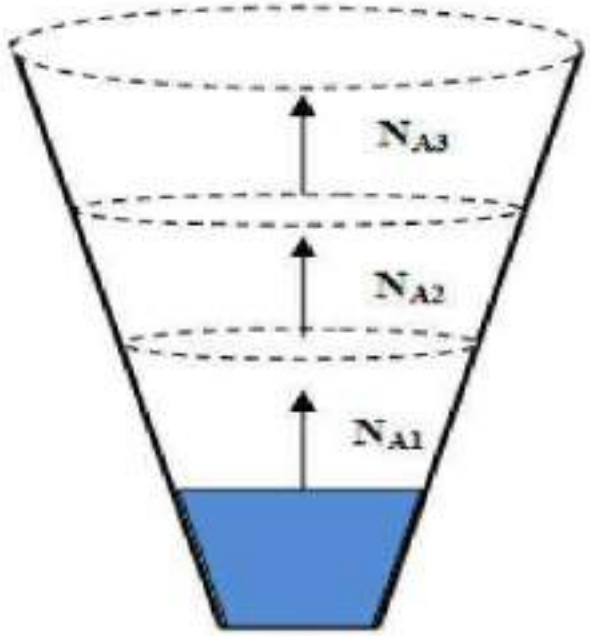
Diffusion through a varying cross-section area

The mole rate ($\bar{N}_A$, $\frac{kmol}{s}$) through a system of a varying cross section area is constant, while the mole flux (N_A, $\frac{kmol}{m^2.s}$) is variable. The mass transfer through a cone and sphere can be consider as a mass transfer through a system of varying cross section area. On the other hand, the transfer through a cylinder can be consider as a mass transfer through a system of constant cross section area.

$$N_A = \frac{\text{mole rate}}{\text{surface area}} = \frac{\bar{N}_A}{A} = \frac{\frac{kmol}{s}}{m^2} = \frac{kmol}{m^2.s}$$

Diffusion

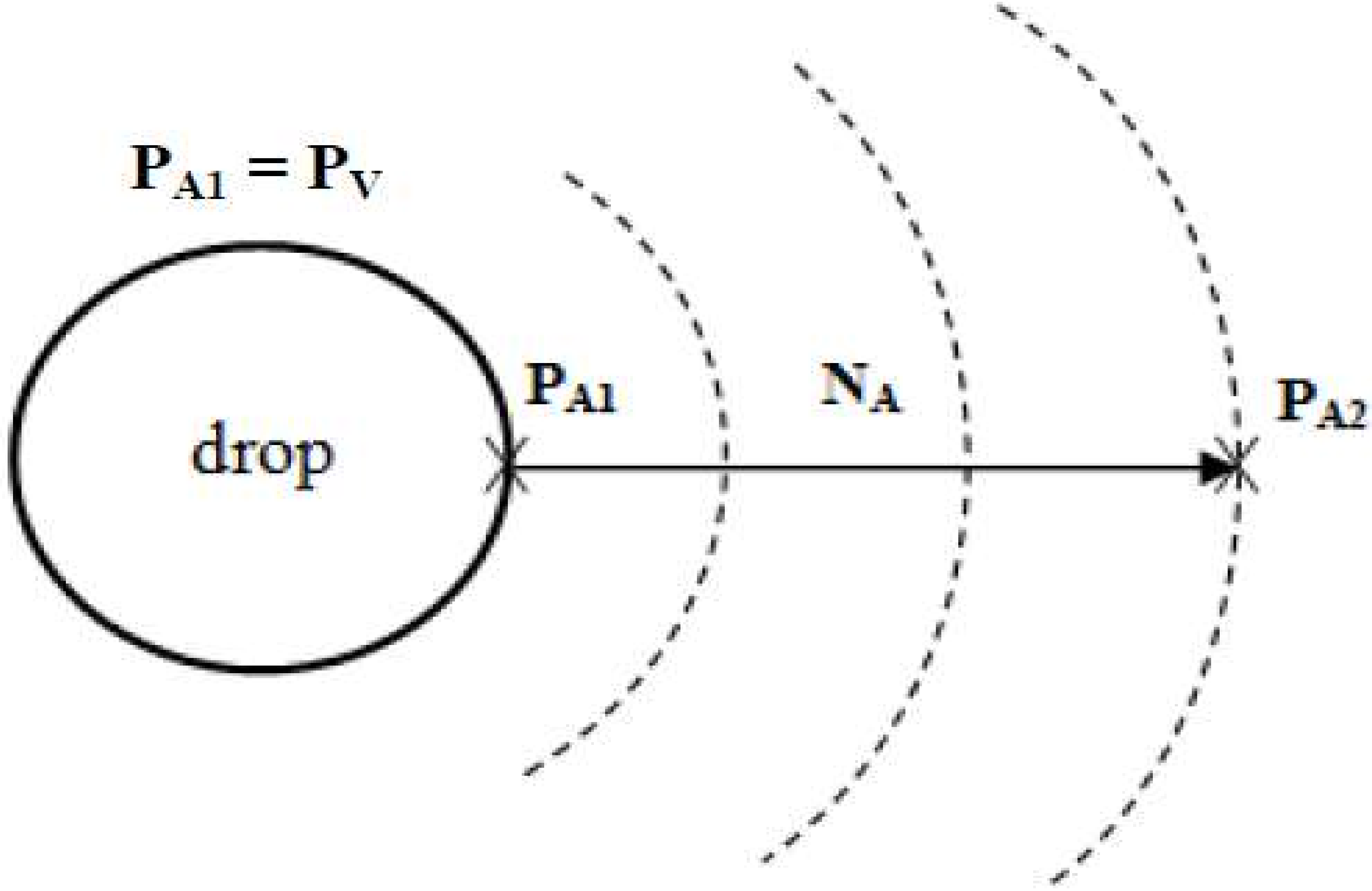




$N_{A1} = N_{A2} = N_{A3}$

$N_{A1} > N_{A2} > N_{A3}$

Diffusion



Diffusion

Diffusion through a spherical body

$$N_A = -D_{AB}\,\frac{dC_A}{dr} + \frac{C_A}{C_T}\,(N_A + N_B) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\bar{N}_A}{A} = -D_{AB}\,\frac{dC_A}{dr} + \frac{C_A}{C_T}\left(\frac{\bar{N}_A}{A} + \frac{\bar{N}_B}{A}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\bar{N}_A = -D_{AB}\;A\;\frac{dC_A}{dr} + \frac{C_A}{C_T}\,(\bar{N}_A + \bar{N}_B) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

But: The surface area of sphere $= A = 4\pi\, r^2$

Diffusion

Case (I): Diffusion through a stagnant layer ($\bar{N}_B = 0$):

$$\bar{N}_A = -\ 4\pi\, r^2\ D_{AB}\ \frac{dC_A}{dr} + \frac{C_A}{C_T}\,(\bar{N}_A + 0)$$

$$\bar{N}_A(C_T - C_A) = -\ 4\pi\, r^2\ D_{AB}\ C_T\ \frac{dC_A}{dr}$$

$$\bar{N}_A \int_{r_0}^{r_1} \frac{dr}{r^2} = 4\pi\ D_{AB}\ C_T \ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]$$

$$\boxed{\bar{N}_A = \frac{4\pi\ D_{AB}\ C_T}{\frac{1}{r_0} - \frac{1}{r_1}}\ \ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

The most important things is to calculate the mass transfer rate for the sphere surface where the surface area is constant ($4\pi\, r_0^2$):

$$N_A . A = \frac{4\pi\ D_{AB}\ C_T}{\frac{1}{r_0} - \frac{1}{r_1}}\ \ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]$$

Diffusion

$$N_A\,(4\pi\, r_0^2) = \frac{4\pi\; D_{AB}\; C_T}{\frac{1}{r_0} - \frac{1}{r_1}}\;\ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]$$

$$\boxed{N_A = \frac{D_{AB}\; C_T}{r_0^2\left(\frac{1}{r_0} - \frac{1}{r_1}\right)}\;\ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]} \quad \dots\dots\dots (2)$$

→ **Mole flux from the sphere surface**

* When the mass transfer from surface to a large distance compare to the sphere surface (r_0):

$r_1 \to \infty$ and $C_{A_2} = 0$

$$N_A = \frac{D_{AB}\; C_T}{r_0^2\left(\frac{1}{r_0} - \frac{1}{\infty}\right)}\;\ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]$$

$$\boxed{N_A = \frac{D_{AB}\; C_T}{r_0}\;\ln\left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}}\right]} \quad \dots\dots\dots (3)$$

Diffusion

In partial pressure form:

$$N_A = \frac{D_{AB}\, P_T}{r_0 \,.\, RT} \ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Example: A sphere of naphthalene having a radius of 2 mm is suspended in a large volume of still air at 318 K and 101.3 kPa. The surface temperature of naphthalene can be assumed to be 318 K and its vapour pressure at this temperature is 0.555 mmHg. The diffusivity of naphthalene in air at 318 K is 6.92 * 10^{-6} m^2/s. Calculate the rate of naphthalene evaporation from surface.

Solution:

The sphere is suspended in a large volume of still air means:

Diffusion

$r_1 \to \infty$ and $P_{A_2} = 0$

$$\mathbf{N_A = \frac{D_{AB}\ P_T}{r_0 \,.\, RT}\ ln\left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}}\right]}$$

$$P_{A_1} = \left(\frac{0.555}{760}\right) * 101.3 = 0.07397\ \text{kPa}$$

$$r_0 = 2 * 10^{-3}$$

$$N_A = \frac{(6.92 * 10^{-6})\,(101.3)}{(2 * 10^{-3})(8.314)(318)}\ \ln\left[\frac{101.3 - 0}{101.3 - 0.07397}\right] = 9.68 * 10^{-8}\ \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

Diffusion

Case (II): Equimolecular Counter Diffusion ($\bar{N}_B = -\bar{N}_A$):

$$N_A = -D_{AB}\,\frac{dC_A}{dr} + \frac{C_A}{C_T}\,(N_A + N_B) \qquad \ldots\ldots\ldots (1)$$

$$\frac{\bar{N}_A}{A} = -D_{AB}\,\frac{dC_A}{dr} + \frac{C_A}{C_T}\left(\frac{\bar{N}_A}{A} - \frac{\bar{N}_A}{A}\right) \qquad \ldots\ldots\ldots (2)$$

$$\bar{N}_A = -\,4\pi\, r^2\, D_{AB}\,\frac{dC_A}{dr} \qquad \ldots\ldots\ldots (3)$$

$$\bar{N}_A \int_{r_0}^{r_1} \frac{dr}{r^2} = -\,4\pi\, D_{AB} \int_{C_{A_1}}^{C_{A_2}} dC_A \qquad \ldots\ldots\ldots (4)$$

$$\bar{N}_A \left[\frac{1}{r_0} - \frac{1}{r_1}\right] = 4\pi\, D_{AB}\,\left(C_{A_1} - C_{A_2}\right) \qquad \ldots\ldots\ldots (5)$$

$$\boxed{\bar{N}_A = \frac{4\pi\, D_{AB}}{\left[\frac{1}{r_0} - \frac{1}{r_1}\right]}\left(C_{A_1} - C_{A_2}\right)}$$

Diffusion

For the mass transfer from surface (A=4π r_0^2):

$$N_A = \frac{D_{AB}}{r_0^2\left[\frac{1}{r_0} - \frac{1}{r_1}\right]}\left(C_{A_1} - C_{A_2}\right)$$

In the case of r_1 is very large $\Longrightarrow \frac{1}{r_1} = 0$

$$N_A = \frac{D_{AB}}{r_0}\left(C_{A_1} - C_{A_2}\right)$$

In the form of partial pressure:

$$N_A = \frac{D_{AB}}{r_0 .\ RT}\left(P_{A_1} - P_{A_2}\right)$$

Diffusion

Case (III): Unequimolecular Counter Diffusion ($\bar{N}_B = -n\,\bar{N}_A$):

$$N_A = \frac{D_{AB}}{RT}\,\frac{P_T}{r_0}\,\frac{1}{(1-n)}\,\ln\left[\frac{P_T-(1-n)\,P_{A_2}}{P_T-(1-n)\,P_{A_1}}\right]$$

Example: Calculate the rate of burning of carbon particle 2.56 cm radius in an atmosphere of pure oxygen at 1000 K and 1 atm. Assuming a very large blanking layer of CO2 has formed a round the particle. At the carbon surface PCO2=1 atm and PO2=0. At very large radius PCO2=0 and PO2=1 atm. Given the diffusivity of oxygen in carbon dioxide = 1.032 cm2/s.

Solution:

$$C + O_2 \rightarrow CO_2$$

The diffusion is equimolecular counter diffusion:

Diffusion

$$N_A = \frac{D_{AB}}{RT \, . \, r_0^2 \left[\frac{1}{r_0} - \frac{1}{r_1}\right]} (P_{A_1} - P_{A_2})$$

In the case of r_1 is very large ($r_1 \rightarrow \infty$) $\Longrightarrow$ $\frac{1}{r_1} = 0$

$$N_A = \frac{D_{AB}}{RT \, . \, r_0} (P_{A_1} - P_{A_2}) = \frac{1.032 * 10^{-4}}{(8.314)(1000)(2.56 * 10^{-2})} (101.3 - 0)$$

$$N_A = 4.95 * 10^{-5} \frac{kmol}{m^2 . s}$$

Diffusion

Mass transfer coefficients

Consider the two-film theory as shown in Figure (1):

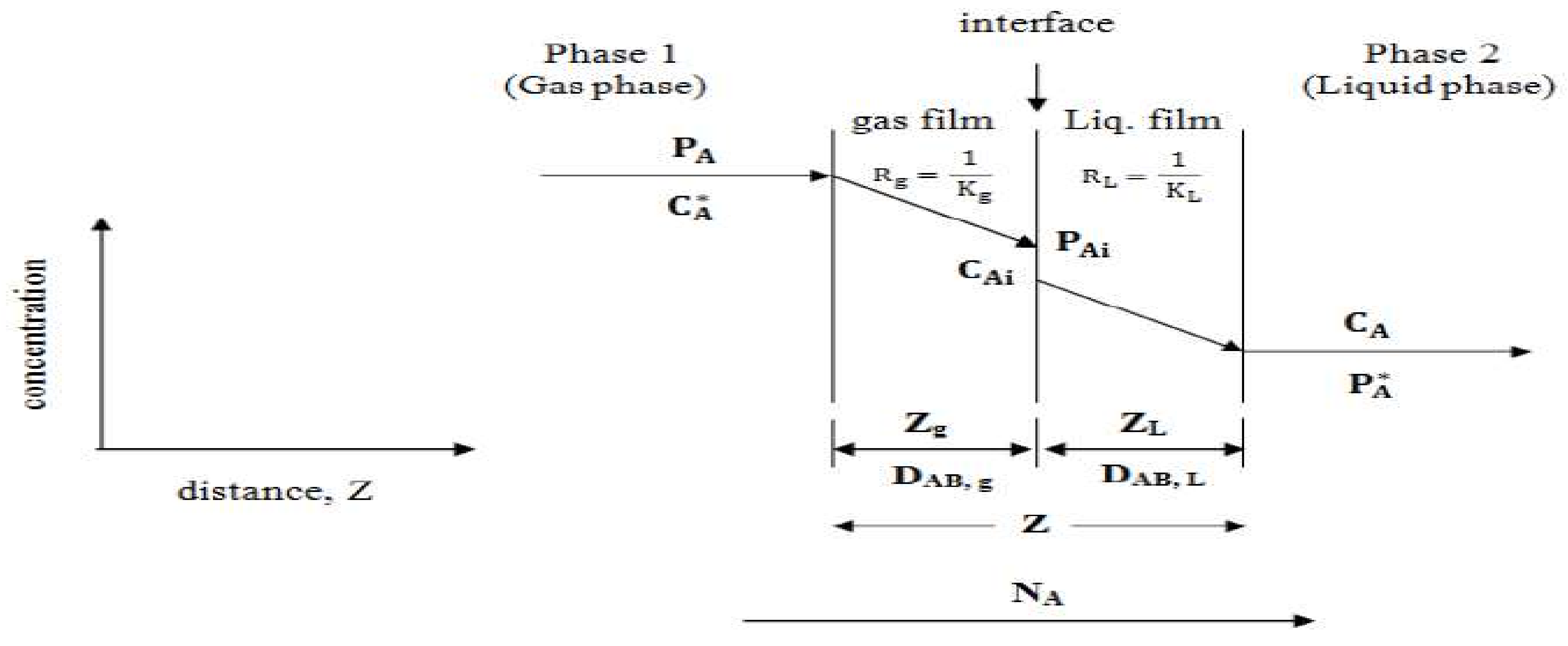


Figure 1: Two - Film Theory

Diffusion

The rate of mass transfer per unit area from the gas film:

$$\mathbf{N_{A_g}} = \frac{(\mathbf{D_{AB}})_\mathbf{g}}{\mathbf{Z_g} \cdot \mathbf{RT}} \; (\mathbf{P_A} - \mathbf{P_{A_i}})$$

The rate of mass transfer per unit area from the liquid film:

$$\mathbf{N_{A_L}} = \frac{(\mathbf{D_{AB}})_\mathbf{L}}{\mathbf{Z_L}} \; (\mathbf{C_{A_i}} - \mathbf{C_A})$$

Where:

$$(\mathbf{D_{AB}})_\mathbf{g} = (\mathbf{D_{AB}})_\mathbf{L}$$

$$\mathbf{N_{A_g}} = \mathbf{N_{A_L}}$$

Diffusion

Since the film thickness $\mathbf{Z_g}$ and $\mathbf{Z_L}$ are difficult to define or estimate, then we rewrite the above equations as follow:

$$\mathbf{N_A = k_g\ (P_A - P_{A_i})}$$

$$\mathbf{N_A = k_L\ (C_{A_i} - C_A)}$$

But: $\mathbf{P_{A_i}}$ and $\mathbf{C_{A_i}}$ are difficult to measure, therefore we define the overall mass transfer coefficient:

$$\mathbf{N_A = K_{OG}\ (P_A - P_A^*)}$$

$$\mathbf{N_A = K_{OL}\ (C_A^* - C_A)}$$

Diffusion

Where:

k_L is the individual liquid film mass transfer coeficient.

k_g is the individual gas film mass transfer coeficient.

K_{OL} is the overall mass transfer coeficient based on liquid phase.

K_{OG} is the overall mass transfer coeficient based on gas phase.

P_{A_i} is the partial pressure of the gas (A) at the interface.

C_{A_i} is the concentration of the liquid (A) at the interface.

P_A^* is the partial pressure of the gas phase which is in equilibrium with the liquid phase C_A.

C_A^* is the concentration of the liquid phase which is in equilibrium with the gas phase P_A.

Diffusion

The Relationships between the various mass transfer coefficients

$$N_A = k_g \left(P_A - P_{A_i}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$N_A = k_L \left(C_{A_i} - C_A\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$N_A = K_{OG} \left(P_A - P_A{}^*\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$N_A = K_{OL} \left(C_A{}^* - C_A\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\frac{1}{K_{OL}} = \frac{1}{H\,k_g} + \frac{1}{k_L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Diffusion

Q: Prove that $\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$

From Eq.(3) above:

$$\frac{1}{K_{OG}} = \frac{P_A - P_A^*}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_A^* + P_{A_i} - P_{A_i}}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{P_{A_i} - P_A^*}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{H\,C_{A_i} - H\,C_A}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{H\,(C_{A_i} - C_A)}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$$

Diffusion

Q: Prove that $\frac{1}{K_{OL}} = \frac{1}{H\,k_g} + \frac{1}{k_L}$

From Eq.(4) above:

$$\frac{1}{K_{OL}} = \frac{C_A{}^* - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{C_A{}^* - C_A + C_{A_i} - C_{A_i}}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{C_A{}^* - C_{A_i}}{N_A} + \frac{C_{A_i} - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{\frac{P_A}{H} - \frac{P_{A_i}}{H}}{N_A} + \frac{C_{A_i} - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{1}{H}\left(\frac{P_A - P_{A_i}}{N_A} + \frac{C_{A_i} - C_A}{N_A}\right)$$

$$\frac{1}{K_{OL}} = \frac{1}{H\,k_g} + \frac{1}{k_L}$$

Diffusion

Notes:

1. The inverse to mass transfer coefficient $\left(\frac{1}{K}\right)$ is termed as a resistance to mass transfer.
2. The term (gas film control) refers to the resistance lie in the gas film.

Thus: $$\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L} \rightarrow 0$$ [when the solute is very soluble in liquid solvent]

3. The term (liquid film control) refers to the resistance lie in the liquid film.

Thus: $$\frac{1}{K_{OL}} = \frac{1}{H\,k_g} \rightarrow 0 + \frac{1}{k_L}$$

4. The units of mass transfer coeficients are as follows

$$N_A = k_L\ (C_{A_1} - C_{A_2}) \quad \rightarrow \quad k_L = \frac{m}{s}$$

$$N_A = \bar{k}_g\ (P_{A_1} - P_{A_2}) \quad \rightarrow \quad \bar{k}_g = \frac{kmol}{m^2.s.kPa}$$

$$N_A = \bar{\bar{k}}_g\ (X_{A_1} - X_{A_2}) \quad \rightarrow \quad \bar{\bar{k}}_g = \frac{kmol}{m^2.s.}$$

Diffusion

$$k_L * \frac{1}{RT} = \bar{k}_g$$

$$k_L * \frac{p_T}{RT} = \bar{\bar{k}}_g$$

$$\bar{k}_g * P_T = \bar{\bar{k}}_g$$

$$kJ = kN.m = kg.\frac{m^2}{s^2} \quad , \quad kPa = \frac{kN}{m^2}$$

$$\frac{m}{s} * \frac{1}{\frac{kJ}{kmol.K} * K} = \frac{m}{s} * \frac{kmol}{kJ} = \frac{m}{s} * \frac{kmol}{kN.m} = \frac{kmol}{S.kN} = \frac{kmol}{S.\frac{kN}{m^2} * m^2}$$

$$= \frac{kmol}{m^2.kPa.S} = \bar{k}_g$$

$$\frac{m}{s} * \frac{kPa}{\frac{kJ}{kmol.K} * K} = \frac{m}{s} * \frac{kPa.kmol}{kJ} = \frac{m}{s} * \frac{kN}{m^2} * \frac{kmol}{kN.m} = \frac{kmol}{m^2.s}$$

$$= \bar{\bar{k}}_g$$

Diffusion

Example: For a system in which component (A) is transfering from the liquid to the gas phase, the equilibrium is given by $\mathbf{y_A}^* = 0.75\,\mathbf{x_A}$. At one point in the appartus the liquid contain 90 mol% of (A) and gas contain 45 mol% of (A). The individual gas film mass transfer coefficient at this point in the appartus of 0.02716 $kmol/m^2.s$, and 70% of the overall resistance to mass transfer is known to be encountered in the gas film: determine:

1. The molar flux of (A).
2. The interficial concentration of (A).
3. The overall mass transfer coefficient for liquid and gas phases.

Diffusion

Solution:

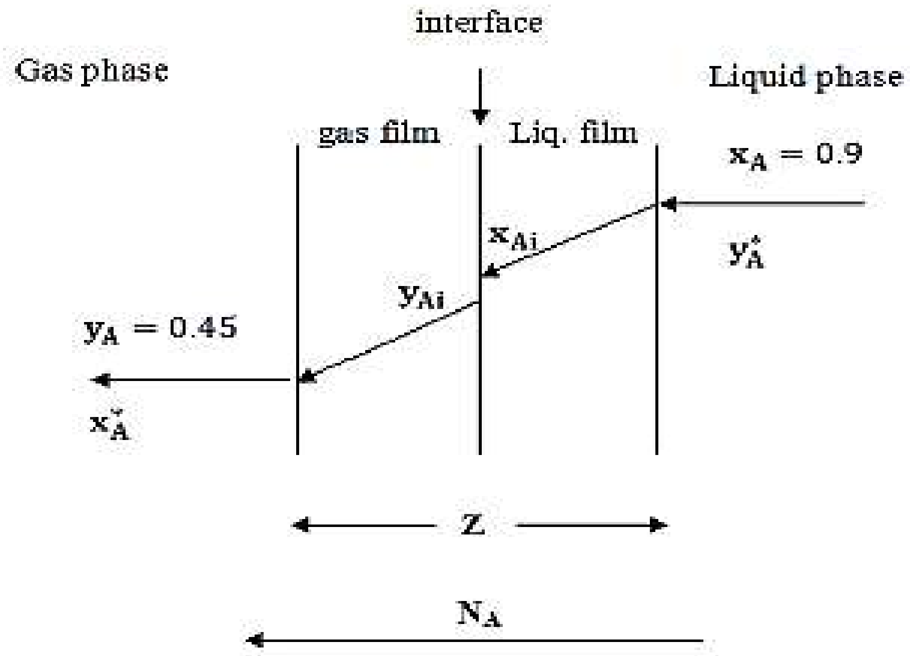
$$\frac{1}{k_g} = 0.7\left(\frac{1}{K_{OG}}\right)$$

$$\frac{1}{0.02716} = 0.7\left(\frac{1}{K_{OG}}\right)$$

$$K_{OG} = 0.019 \quad \frac{kmol}{m^2.s}$$

$$N_A = K_{OG}\ \left(y_A^* - y_A\right)$$

$$y_A^* = 0.75\,x_A = (0.75)(0.9) = 0.675$$



Diffusion

1. $N_A = (0.019)\,(0.675 - 0.45) = 4.274 * 10^{-3}\ \frac{\text{kmol}}{\text{m}^2.\text{s}}$

2. $N_A = k_g\ \left(y_{A_i} - y_A\right)$

$4.274 * 10^{-3} = (0.02716)(y_{A_i} - 0.45)$

$y_{A_i} = 0.607$

3. $\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$

$\frac{1}{0.019} = \frac{1}{0.02716} + \frac{0.75}{k_L}$

$k_L = 0.0476$

$\frac{1}{K_{OL}} = \frac{1}{H\,k_g} + \frac{1}{k_L}$

$\frac{1}{K_{OL}} = \frac{1}{(0.75)(0.02716)} + \frac{1}{0.0476} \quad \rightarrow \quad K_{OL} = 0.0142\ \frac{\text{kmol}}{\text{m}^2.\text{s}}$

Distillation

Presenter by:

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Distillation

The separation of liquid mixtures into their various components is one of the major operations in the process industries, and distillation, the most widely used method of achieving this end, is the key operation in any oil refinery. In processing, the demand for purer products, coupled with the need for greater efficiency, has promoted continued research into the techniques of distillation. In engineering terms, distillation columns have to be designed with a larger range in capacity than any other types of processing equipment, with single columns 0.3–10 m in diameter and 3–75 m in height.

Distillation: is the separation of liquid mixture by partial evaporation. The essential requirement is to have a vapor composition different from liquid.

Distillation

Boiling point: is the temperature at which the $\Sigma P_i = P_T$ and for pure component $P_T = P^o$ (vapor pressure).

- If $P_A^o > P_B^o$

  Volatility of (A) > Volatility of (B)

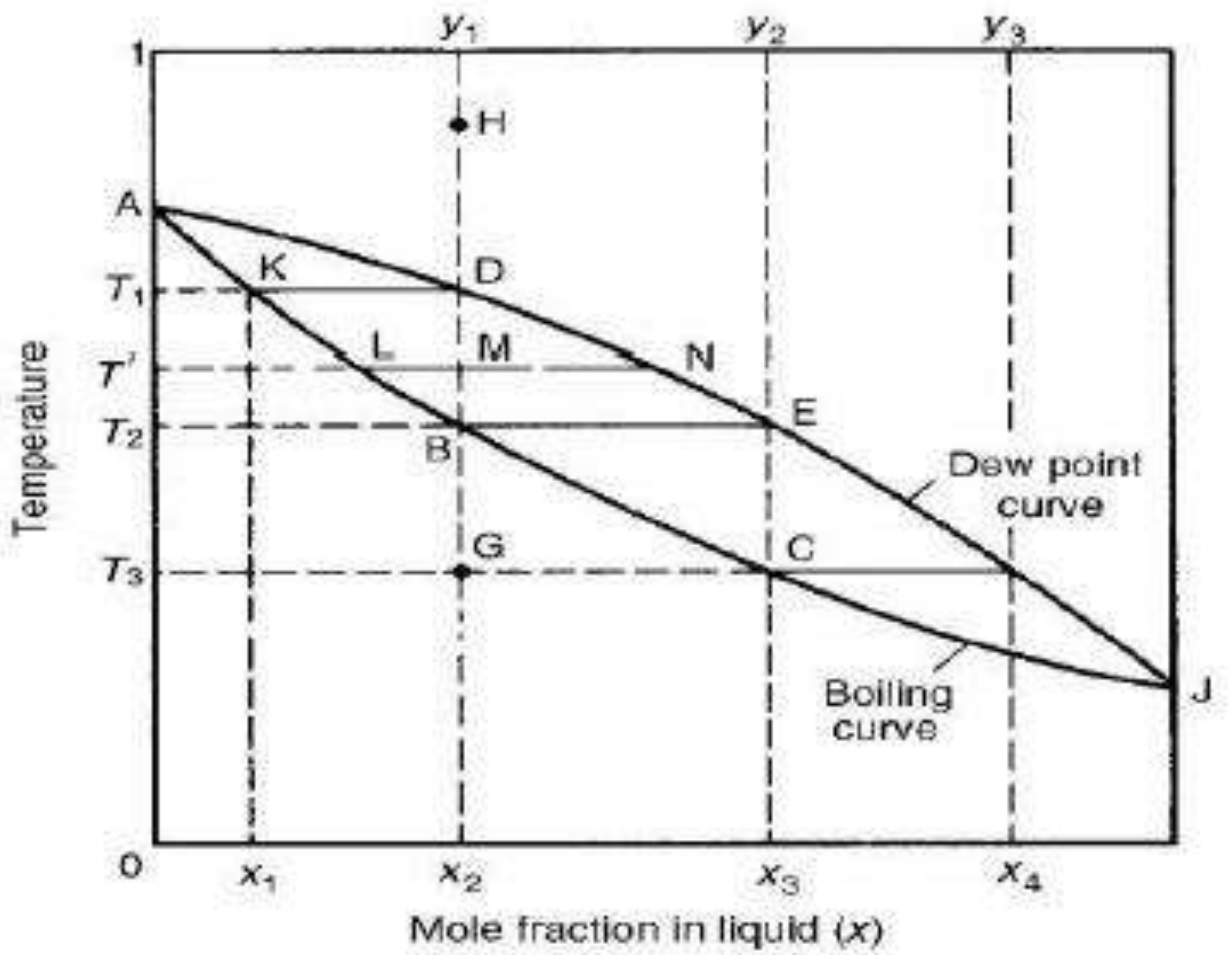
- If $P_B^o > P_A^o$

  Volatility of (B) > Volatility of (A)

Vapour–Liquid Equilibrium

The composition of the vapour in equilibrium with a liquid of given composition is determined experimentally using an equilibrium still. The results are conveniently shown on a temperature–composition diagram as shown in Figure below.

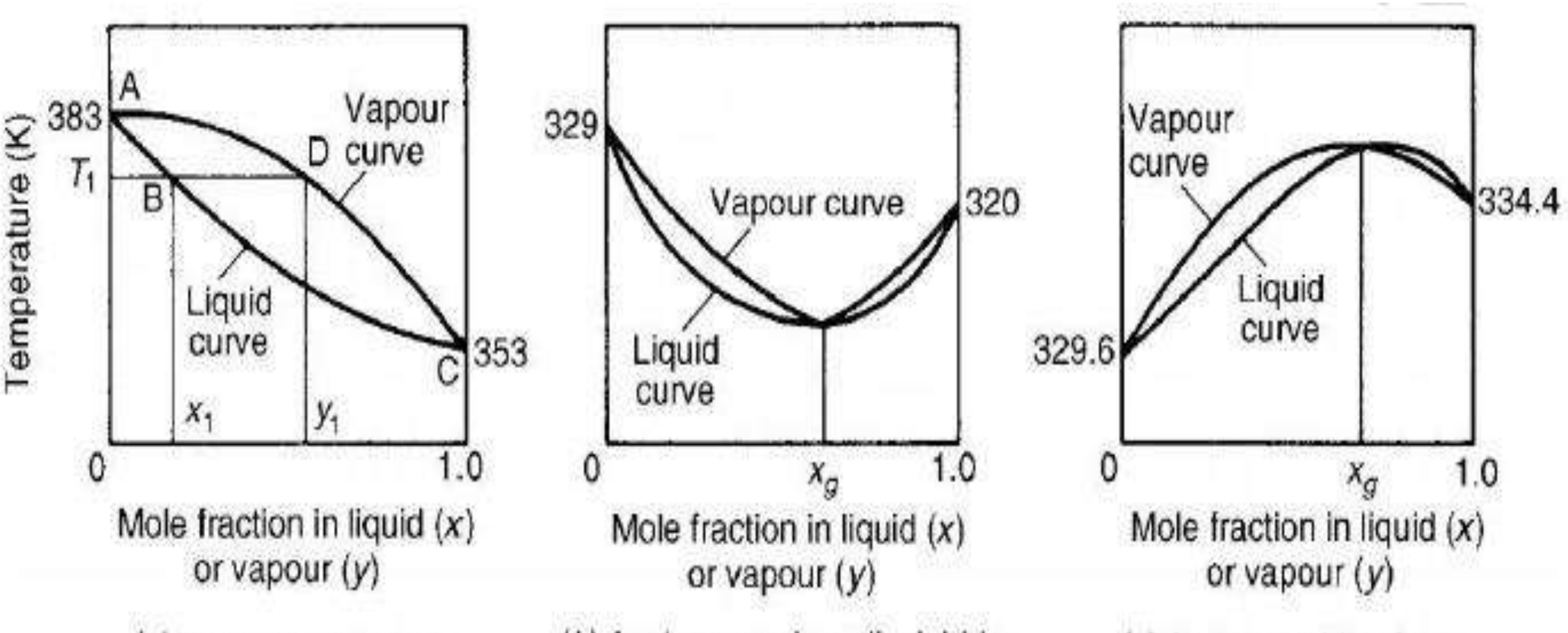
Distillation



Distillation

In the normal case shown in Figure (a), the curve ABC shows the composition of the liquid which boils at any given temperature, and the curve ADE the corresponding composition of the vapour at that temperature. Thus, a liquid of composition x_1 will boil at temperature T_1, and the vapour in equilibrium is indicated by point D of composition y_1. It is seen that for any liquid composition x the vapour formed will be richer in the more volatile component, where **x** is the mole fraction of the more volatile component in the liquid, and **y** in the vapour. Examples of mixtures giving this type of curve are benzene–toluene and n-heptane–toluene.

Distillation



(a) Benzene–toluene (b) Acetone–carbon disulphide (c) Acetone–chloroform

Distillation

In Figures (b) and (c), there is a critical composition xg where the vapour has the same composition as the liquid, so that no change occurs on boiling. Such critical mixtures are called **azeotropes**. For compositions other than xg, the vapour formed has a different composition from that of the liquid. It is important to note that these diagrams are for **constant pressure conditions**, and that the composition of the vapour in equilibrium with a given liquid will change with pressure.

Distillation

For distillation purposes it is more convenient to **plot y against x** at a constant pressure, since the majority of industrial distillations take place at substantially constant pressure.

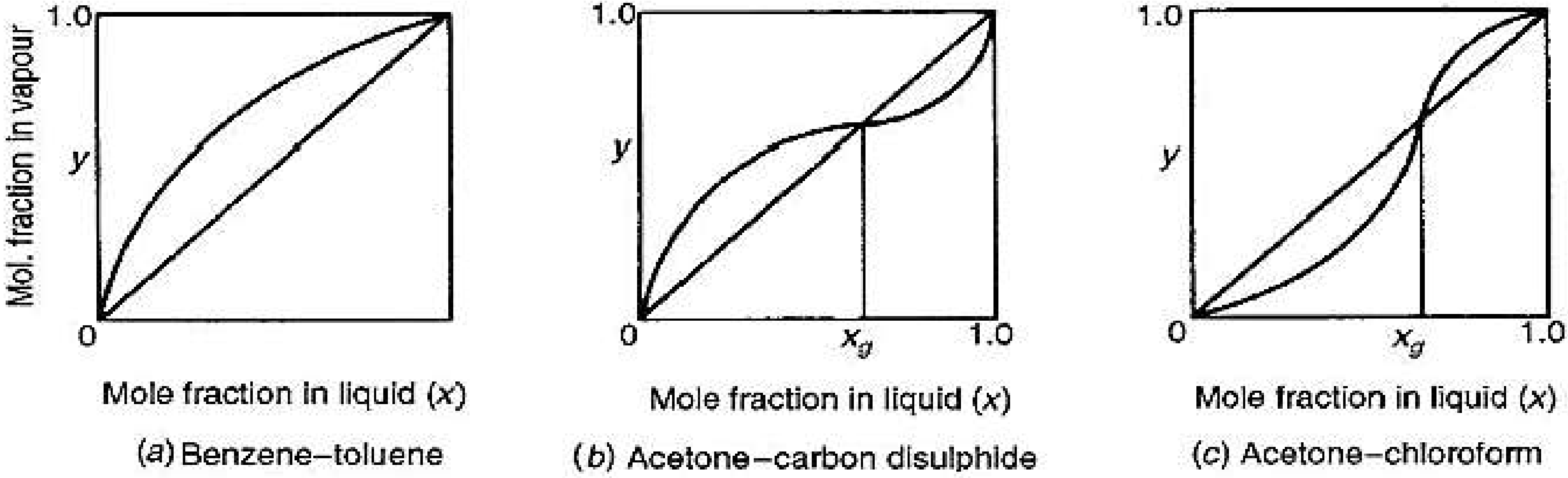


Figure 11.4. Vapour composition as a function of liquid composition at constant pressure

Distillation

The vapour-liquid equilibrium data is calculated from:

1. Raoult's and Dalton's law for ideal system:

For an ideal mixture, the partial pressure is related to the concentration in the liquid phase by **Raoult’s law** which may be written as:

$$P_A = P_A^o \, x_A \quad \text{and} \quad P_B = P_B^o \, x_B \quad \text{or} \quad P_B = P_B^o \, (1 - x_A)$$

Where:
P_A: is the partial pressure of component A.
P_{Ao}: is the vapour pressure of component A.
P_B: is the partial pressure of component B.
P_{Bo}: is the vapour pressure of component B.
x_A: is the mole fraction of component A in liquid phase.
This relation (**Raoult’s law**) is usually found to be true only for high values of x_A, or correspondingly low values of x_B, although mixtures of organic isomers and some hydrocarbons follow the law closely.

Distillation

By **Dalton's law** of partial pressures:

$$P_T = \Sigma P_i$$

Where: $P_A = y_A P_T$

$$P_T = P_A + P_B$$

$$P_T = P_A^o x_A + P_B^o (1 - x_A)$$

$$P_T = P_A^o x_A + P_B^o - P_B^o x_A$$

$$P_T - P_B^o = x_A (P_A^o - P_B^o)$$

$$x_A = \frac{P_T - P_B^o}{P_A^o - P_B^o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$y_A = \frac{P_A^o}{P_T} x_A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Distillation

Temp.	P_A^o	P_B^o	$x_A = \frac{P_T - P_B^o}{P_A^o - P_B^o}$	$y_A = \frac{P_A^o}{P_T} x_A$
-	-	-	Calculated from eq.1	Calculated from eq.2
-	-	-	Calculated from eq.1	Calculated from eq.2

2. Relative volatility (α):

The relationship between the composition of the vapour y_A and of the liquid X_A in equilibrium may also be expressed in a way, which is particularly useful in distillation calculations:

$$\alpha_A = \frac{P_A^o}{P_T} \quad \text{and} \quad \alpha_B = \frac{P_B^o}{P_T}$$

difference for distillation

As the difference increase, the distillation would be easier.

Distillation

Where:

α_A : is the volatility of component A.

α_B : is the volatility of component B.

The relative volatility (α_{AB}):

$$\alpha_{AB} = \frac{\alpha_A}{\alpha_B} = \frac{P_A^o / P_T}{P_B^o / P_T} > 1$$

For separation to be achieved, α_{AB} must not equal 1 and, considering the more volatile component, as α_{AB} increases above unity, y increases and the separation becomes much easier.

$$\alpha_{AB} = \frac{\alpha_A}{\alpha_B} = \frac{P_A^o}{P_B^o} = \frac{P_A / x_A}{P_B / x_B} = \frac{P_A / P_T\, x_A}{P_B / P_T\, x_B} = \frac{y_A / x_A}{y_B / x_B} = \frac{y_A / x_A}{(1 - y_A)/(1 - x_A)}$$

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1)\, x_A} \quad \ldots\ldots\ldots\ldots\ldots (*)$$

Equilibrium relation in distillation

$$x_A = \frac{y_A}{\alpha_{AB} - (\alpha_{AB} - 1) y_A}$$

Distillation

To plot x_A against y_A, we use Eq.(*) for given value of x_A between (0 – 1.0) which are arbitrary:

x_A	y_A
0	-
0.1	-
0.2	-
0.3	-
0.4	-
0.5	-
0.6	-
0.7	-
0.8	-
0.9	-
1.0	=

Distillation

3. Henry's law for non-ideal systems:

For low values of x_A, a linear relation between P_A and x_A again exists, although the proportionality factor is Henry's constant H, and not the vapour pressure P_A^o of the pure material. For a liquid solute A in a solvent liquid B, Henry's law takes the form:

$$y_A = H\, x_A$$

Where: H = is Henry's constant

or we use the equilibrium constant or we call the "distribution coefficient, **k**"

$$y_A = k_A\, x_A$$

Where: k_A = is the distribution coefficient or equilibrium constant

$k = f\,(T)$

Distillation

For non-ideal binary mixture the partial pressure may be expressed in the form:

$$P_A = \frac{\gamma_A \, P_A^o \, x_A}{P_T} \qquad \text{and} \qquad P_B = \frac{\gamma_B \, P_B^o \, x_B}{P_T}$$

Where: γ_A = is the activity coefficient for component A.

γ_B = is the activity coefficient for component B.

$$\alpha_{AB} = \frac{k_A}{k_B} = \frac{y_A/x_A}{y_B/x_B} = \frac{\gamma_A \, P_A^o}{\gamma_B \, P_B^o}$$

Distillation

Example: The following vapour pressure were obtained for phenol and ortho-cresol:

Temp. (K)	Vapour pressure of ortho-cresol (kN/m^2)	Vapour pressure of phenol (kN/m^2)
387	7.7	10
387.9	7.94	10.4
388.7	8.21	10.8
389.6	8.5	11.2
390	8.76	11.6
391.1	9.06	12.0
391	9.4	12.4
392.7	9.73	12.9
393.3	10.0	13.3

Assuming Raoult's and Dalton's laws apply. Find the following data for a total pressure of 10.0 kN/m^2.

a. A vapour-liquid equilibrium data.

b. Relative volatility against mole fraction of phenol in liquid.

Distillation

Solution:

P_A^o	P_B^o	$\alpha_{AB} = \frac{P_A^o}{P_B^o}$	x_A	$y_A = \frac{\alpha_{AB\,(avg.)} * x_A}{1 + (\alpha_{AB\,(avg.)} - 1)\, x_A}$
10	7.7	1.3	0	
10.4	7.94	1.31	0.1	
10.8	8.21	1.315	0.2	
11.2	8.5	1.318	0.3	
11.6	8.76	1.324	0.4	
12.0	9.06	1.325	0.5	
12.4	9.4	1.319	0.6	
12.9	9.73	1.326	0.7	
13.3	10.0	1.330	0.8	
			0.9	
			1.0	
		α_{AB}(average) = **1.318**		

$$\alpha_{AB\,(avg.)} = \sqrt[9]{(1.3)(1.31)(1.315)(1.318)(1.324)(1.325)(1.319)(1.326)(1.330)} = \mathbf{1.318}$$

Distillation

Methods of Distillation -Two Component Mixtures

For a binary mixture with a normal $x_A - y_A$ curve, the vapour is always richer in the more volatile component than the liquid from which it is formed. There are two main methods used in distillation practice which all rely on this basic fact. These are:

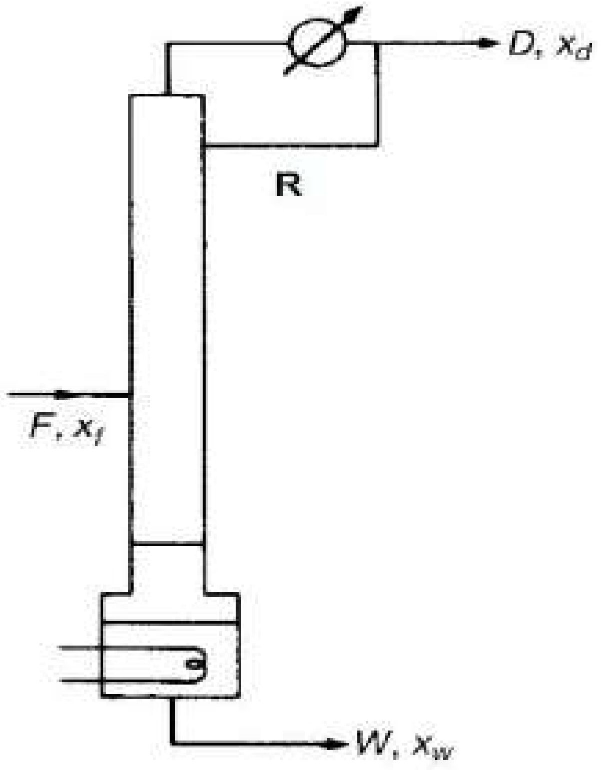
1. Continuous Distillation.

a. Rectifying (fractionation) distillation.
b. Flash (equilibrium) distillation.

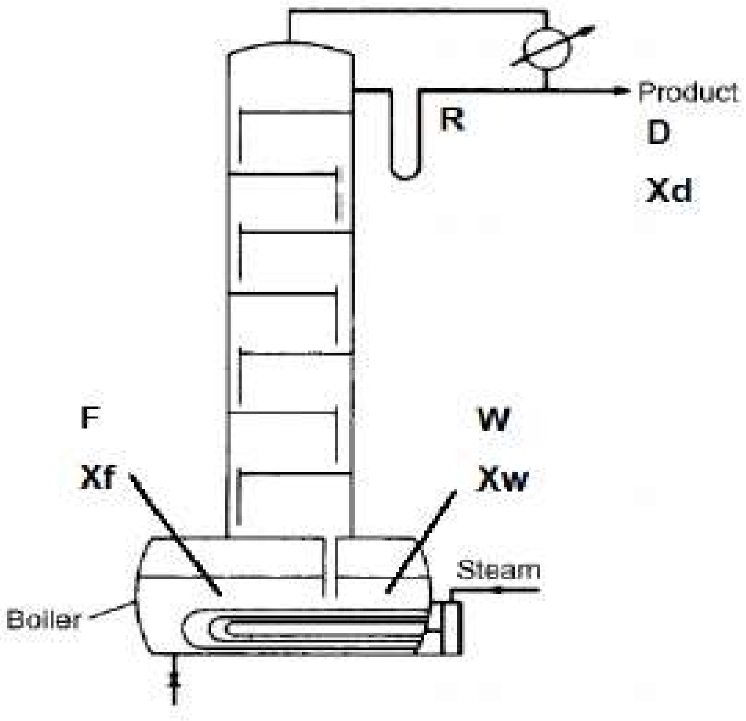
2. Non- Continuous Distillation.

a. Differential distillation.
b. Batch distillation.
   i. Operation at constant reflux ratio.
   ii. Operation at constant product composition (variable reflux)

Distillation



Rectifying distillation



Batch Distillation

Distillation

1. Differential distillation:

The simplest example of batch distillation is a single stage (differential distillation) starting with a still pot, initially full, heated at a constant rate. In this process:

1. The vapour formed on boiling the liquid is removed at once from the system.
2. Vapour is richer in the more volatile component than the liquid, and the liquid remaining becomes steadily weaker in this component, so this result that the composition of the product progressively alters.
3. The vapour formed over a short period is in equilibrium with the liquid.
4. The total vapour formed is not in equilibrium with the residual liquid. At the end of the process the liquid which has not been vaporized is removed as the bottom product

Distillation

If S_o = Numbers of moles of feed in the still initially.

S = Numbers of moles of liquid mixture in the still after concentrated.

D = Numbers of moles of product (distillate).

x_o = Mole fraction of A (more volatile component) in the feed.

x = Mole fraction of A in the waste (residue).

x_d = Mole fraction of A in the distillate (product)

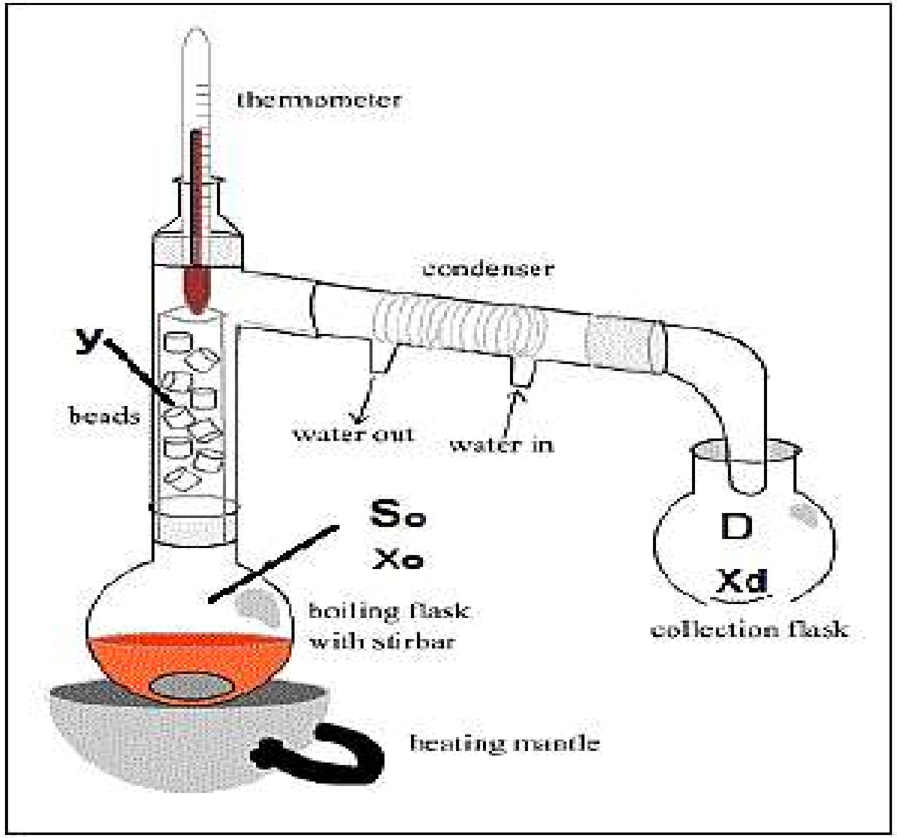
Overall material balance gives:

$S_o = D + S$

Material balance on more volatile component gives:

$(S_o)\,(x_o) = (D)\,(x_d) + (S)\,(x)$

$$x_d = \frac{(S_o)(x_o) - (S)(x)}{D}$$



Distillation

The boundary conditions of the differential distillation:

At time = o, ⟹ $S = S_o$ and $x = x_o$ (liquid condition)

At time = t, ⟹ $S = S$ and $x = x$ (variable)

In time (dt):

ds: is the amount of liquid vaporized from the still.

dx: is the concentration difference in the still.

So we take a material balance :

$$y\,dS = d\,(S\,x) = S\,dx + x\,dS$$

$$(y - x)\,dS = S\,dx$$

Distillation

$$\int_{S}^{S_0} \frac{ds}{s} = \int_{x}^{x_0} \frac{dx}{y - x}$$

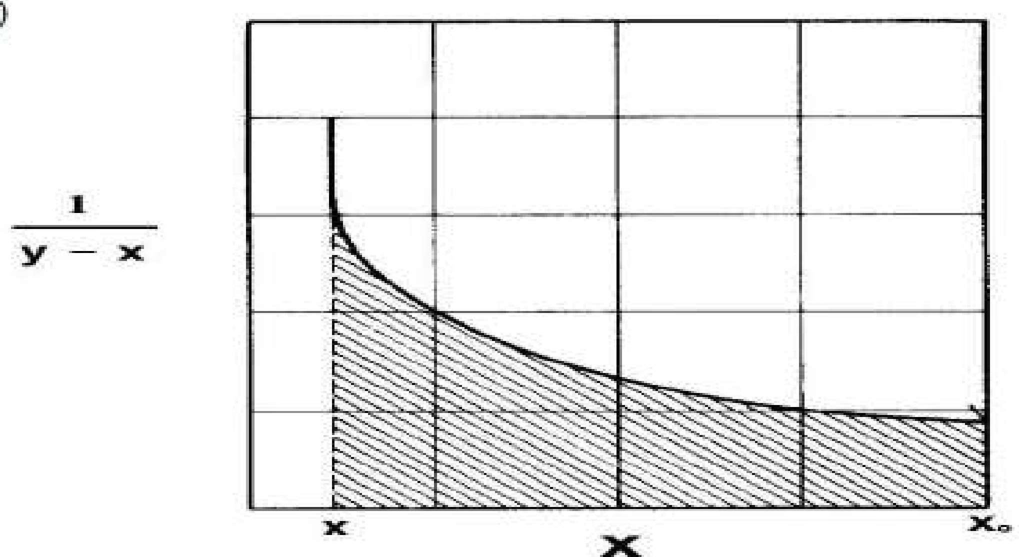
$$\ln \frac{S_0}{S} = \int_{x}^{x_0} \frac{dx}{y - x} = Area\ under\ the\ curve\ \ldots\ldots\ldots\ldots..(*)$$

The integral on the right-hand side of this equation may be solved by three ways:

1. **Graphical solution:**

Taking values of x and y from the equilibrium relationship.
(If the equilibrium data given as points)

x	y	$\frac{1}{y-x}$
-	-	-
-	-	-
-	-	-



Distillation

$$\ln\frac{S_0}{S} = \int_{x}^{x_0} \frac{dx}{y - x} = \textit{Area under the curve} \quad \ldots\ldots\ldots\ldots(*)$$

2. If the equilibrium data is a straight line of the form $\mathbf{y = m\,x + c}$

$$\ln\frac{S}{S_0} = \left(\frac{1}{m-1}\right)\ln\left[\frac{(m-1)x + c}{(m-1)x_0 + c}\right]$$

Distillation

3. If the equilibrium data is given by **volatility (α)**:

$$y = \frac{\alpha * x}{1 + (\alpha - 1)x}$$

$$\ln\frac{S}{S_0} = \int_{x_0}^{x} \frac{dx}{y - x} = \int_{x_0}^{x} \frac{dx}{\left[\frac{\alpha * x}{1 + (\alpha - 1)x}\right] - x}$$

$$\ln\frac{S}{S_0} = \left(\frac{1}{\alpha - 1}\right)\ln\left[\frac{x(1 - x_0)}{x_0(1 - x)}\right] + \ln\left[\frac{1 - x_0}{1 - x}\right]$$

Distillation

Example: 100 kmol of a mixture (A and B) is fed to a simple still. The feed contains 50 mol% of A and a remain in the still is 5 mol% of A. Calculate the quantity and the average composition of the product obtained? The equilibrium data are:

x	1.0	0.9	0.6	0.5	0.4	0.3	0.2	0.1	0.05
y	1.0	0.932	0.745	0.67	0.57	0.46	0.34	0.2	0.1

Solution:

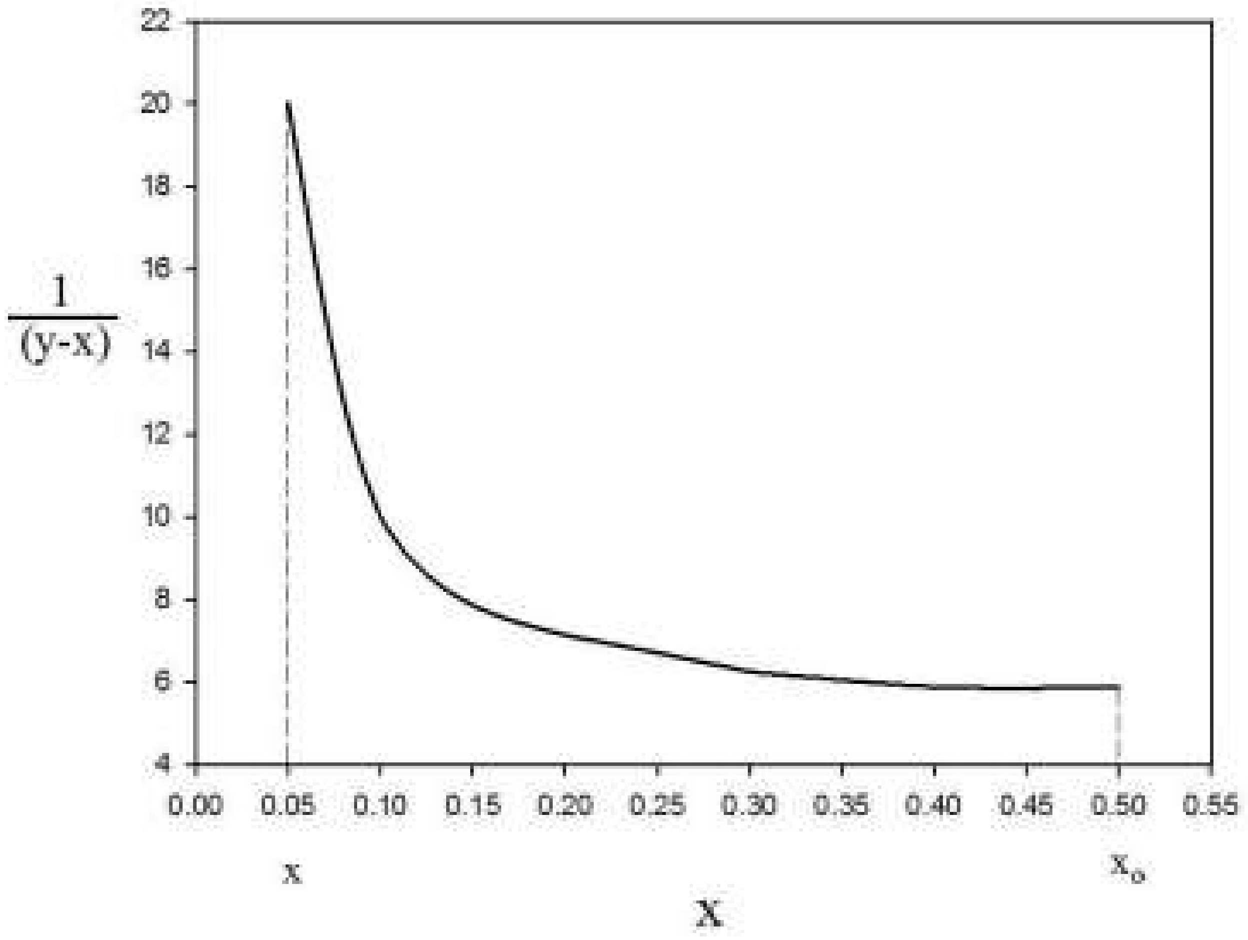
$S_o = 100$ kmol, $x_o = 0.5$ and $x = 0.05$

Since the type is differential distillation, then:

Distillation

$$\ln\frac{S_0}{S} = \int_x^{x_0} \frac{dx}{y-x} = Area\ under\ the\ curve$$

	x	y	$\frac{1}{y-x}$
$x_0 \rightarrow$	0.5	0.67	5.88
	0.4	0.57	5.88
	0.3	0.46	6.25
	0.2	0.34	7.14
	0.1	0.2	10.0
$X \rightarrow$	0.05	0.1	20.0



The No. of squares = 16

The area of one square = (Δx) (Δy) = (0.1) (2) = 0.2

The area under the curve = (No. of squares) (area of one square) = (16) (0.2) = 3.22

Distillation

From plot:

The area under the curve = 3.2 $\Longrightarrow$ $\ln \frac{S}{S_0} = 3.2$

Then, $S = 4.076$ kmol

Overall material balance:

$D = S_o - S = 100 - 4.076 = 95.92$ kmol

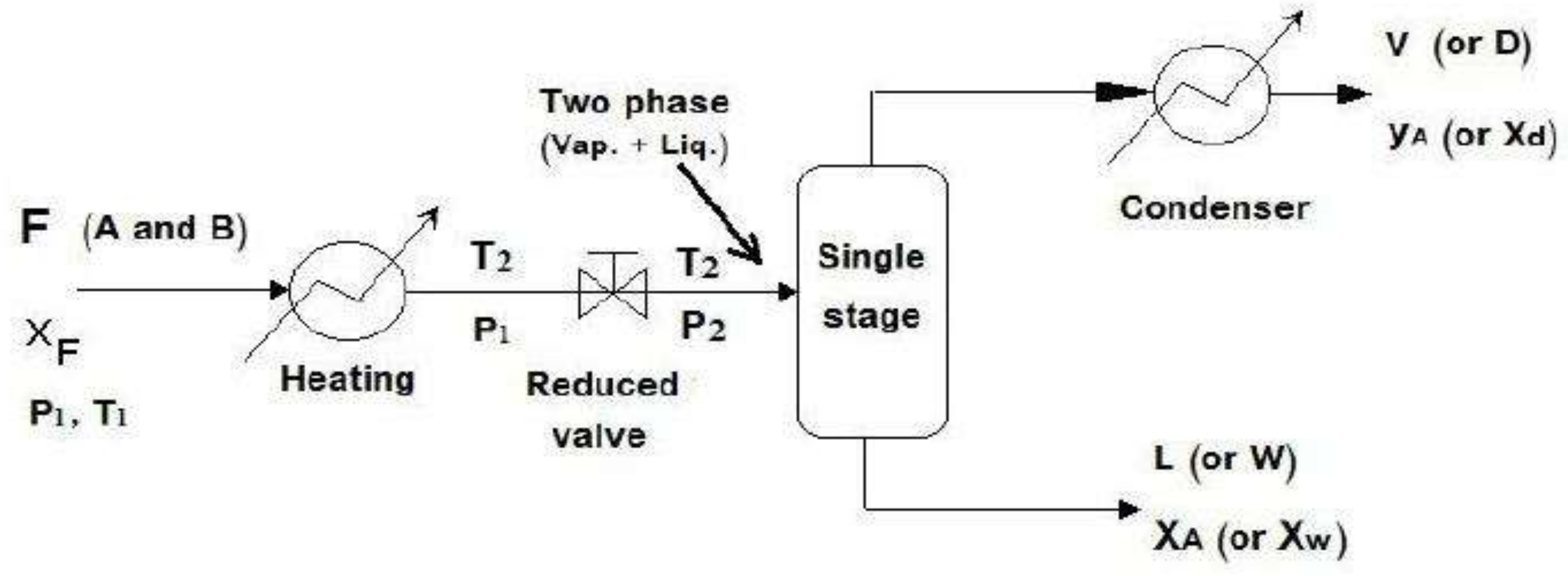
Material balance on more volatile component gives:

$(S_o)(x_o) = (D)(x_d) + (S)(x)$

$$x_d = \frac{(S_0)(x_o) - (S)(x)}{D} = \frac{(100)(0.5) - (4.076)(0.05)}{95.92} = 0.519$$

2. Flash or equilibrium distillation:

Flash or equilibrium distillation, frequently carried out as a continuous process, consists of vaporizing a definite fraction of the liquid feed in such a way that the vapour evolved is in equilibrium with the residual liquid. The feed is usually pumped through a fired heater and enters the still through a valve where the pressure is reduced. The still is essentially a separator in which the liquid and vapour produced by the reduction in pressure have sufficient time to reach equilibrium. The vapour is removed from the top of the separator and is then usually condensed, while the liquid leaves from the bottom.



Distillation

1. **Overall mass balance gives:**

$$F = V + L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

2. **Material balance on more volatile component gives:**

$$(F)(x_f) = (V)(y_A) + (L)(x_A) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The values of $\mathbf{x_A}$ and $\mathbf{y_A}$ required must satisfy, not only the equation, but also the appropriate equilibrium data. Thus these values may be determined depends on the equilibrium relationship:

a. **Graphically using an x – y diagram.**

x	-	-	-	-	-
y	-	-	-	-	-

First, plot the equilibrium data (x, y), then assume the value of $\mathbf{x_A}$ and find the value of $\mathbf{y_A}$ from the equilibrium plot. Substitute the assumed value of $\mathbf{x_A}$ and the calculated value of $\mathbf{y_A}$ in Eq.(2). If the right side of Eq.(2) equal to the left side then the assumed $\mathbf{x_A}$ and the calculated $\mathbf{y_A}$ represents the mole fraction of more volatile component in liquid and vapour phase, respectively. If not, repeat the assumption.

Distillation

b. Analytically if the equilibrium relationship is linear:

$$y_A = m\, x_A \qquad \text{.................................(3)}$$

Substitute Eq.(3) into Eq.(2) to find x_A then the calculated value of x_A substitute into the equilibrium relation Eq.(3) to find y_A.

c. Analytically if the equilibrium relationship is given by the relative volatility (α_{AB}):

$$y_A = \frac{\alpha_{AB} * x_A}{1+(\alpha_{AB} - 1)\, x_A} \qquad \text{.................................(4)}$$

Substitute Eq.(4) into Eq.(2) to find x_A then the calculated value of x_A substitute into the equilibrium relation Eq.(4) to find y_A.

Distillation

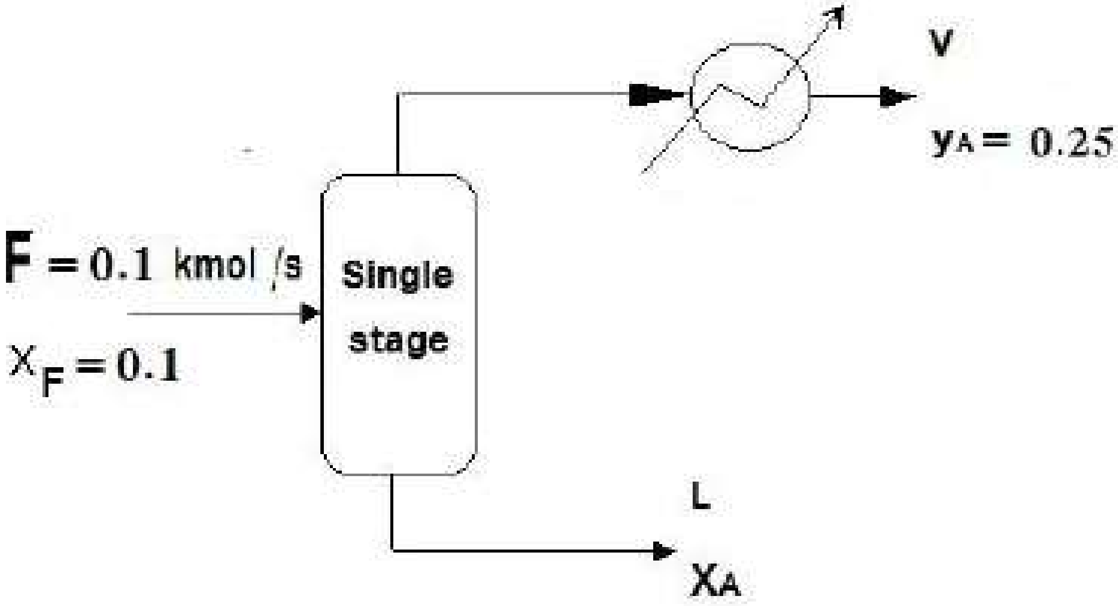
Example (1): An aqueous solution at its boiling point containing 10 mol% of ammonia is fed to the flash distillation to produce a distillate containing 25 mol% of ammonia. At equilibrium, the mole fraction of ammonia in the vapour phase is 6.3 times that in the liquid phase and the feed flow rate is 0.1 kmol/s. Calculate the number of moles distillate obtainable from the flash distillation.

Solution

Overall material balance:

$F = V + L = 0.1$

$V = 0.1 - L$ ....(1)



Distillation

Material balance on more volatile component:

$(F)\ (x_f) = (V)\ (y_A) + (L)\ (x_A)$

$(0.1)\ (0.1) = (0.1 - L)\ (y_A) + (L)\ (x_A)$ ....(2)

Equilibrium relationship:

$y_A = 6.3\ x_A$ ....(3)

at $y_A = 0.25$ $\Longrightarrow$ $x_A = \dfrac{0.25}{6.3} = 0.0346$

Substitute x_A into Eq.(2) to gain L:

$(0.1)\quad (0.1) = (0.1 - L)\ (0.25) + (L)\ (0.0346)$

$L = 0.0712$ kmol/s

$V = 0.1 - 0.0721 = 0.0287$ kmol /s

Distillation

Example (2): A liquid mixture containing 40 mol% of n-heptane and 60 mol% of n-octane is to be continuously flash vaporized at 1 atm. The product vapour is 70% of the feed. What will be the composition of the vapour and liquid. Given α_{AB}=2.16.

Solution

$F = 100$ kmol/s and $V = 70$ kmol/s

Overall material balance:

$$F = V + L$$

$100 = 70 + L \Longrightarrow L = 30$ kmol/s

Material balance on more volatile component:

$$(F)\,(x_f) = (V)\,(y_A) + (L)\,(x_A)$$

$$(100)\,(0.4) = (70)\,(y_A) + (30)\,(x_A) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Distillation

Equilibrium relationship:

$$y_A = \frac{\alpha_{AB} * x_A}{1+(\alpha_{AB}-1)\, x_A} = \frac{2.16\ x_A}{1+(2.16-1)\, x_A}$$

$$\boxed{y_A = \frac{2.16\ x_A}{1+1.16\, x_A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substitute Eq.(2) into Eq.(1) to get:

$$34.8\, x_A^2 + 134.8\, x_A - 40 = 0$$

$$x_A = \frac{-134.8 \mp \sqrt{(1384.)^2 - 4(134.8)(-40)}}{2\,(34.8)}$$

$$x_A = 0.276$$

Substitute x_A value into Eq.(2) to get y_A:

$$y_A = 0.452$$

Distillation

3. Continuous (Rectification) distillation:

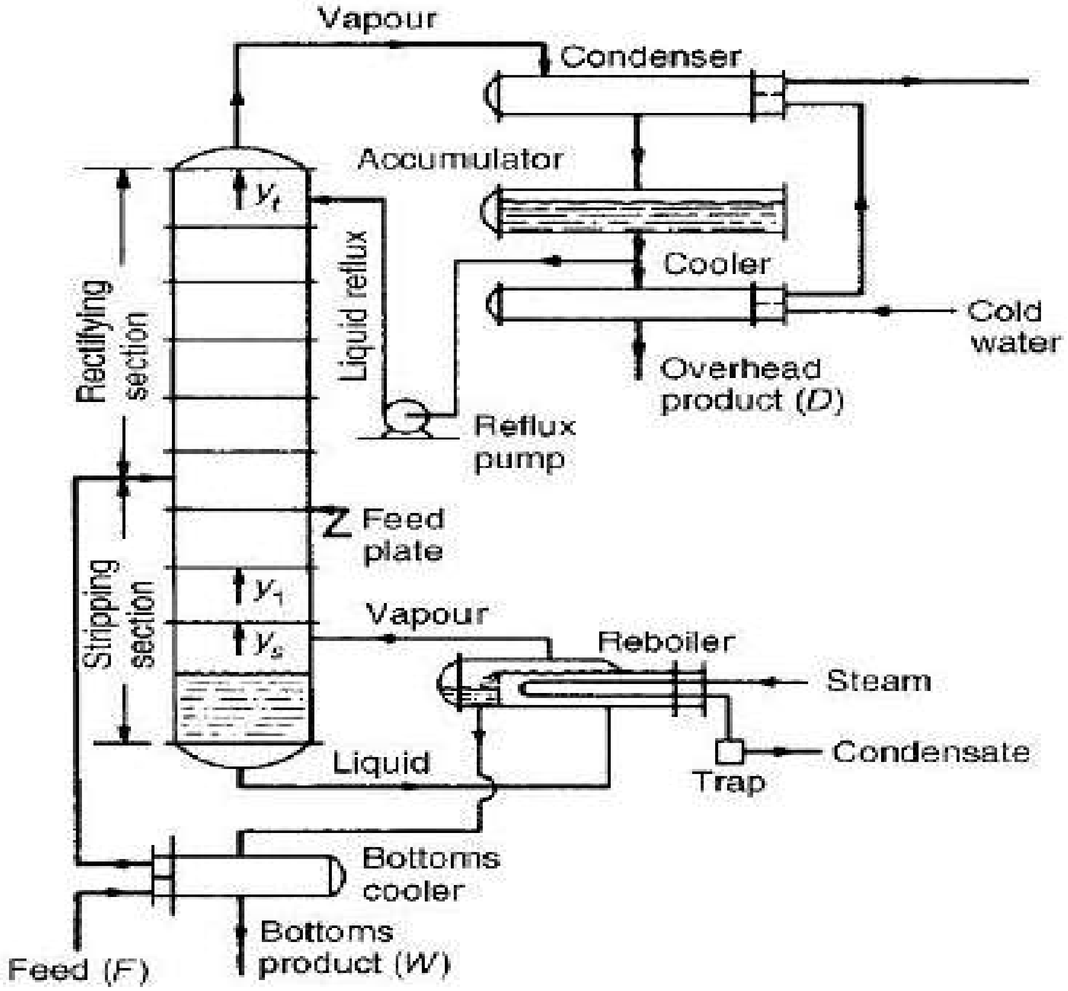


Fig: Continuous fractionating column with rectifying and stripping sections

Distillation

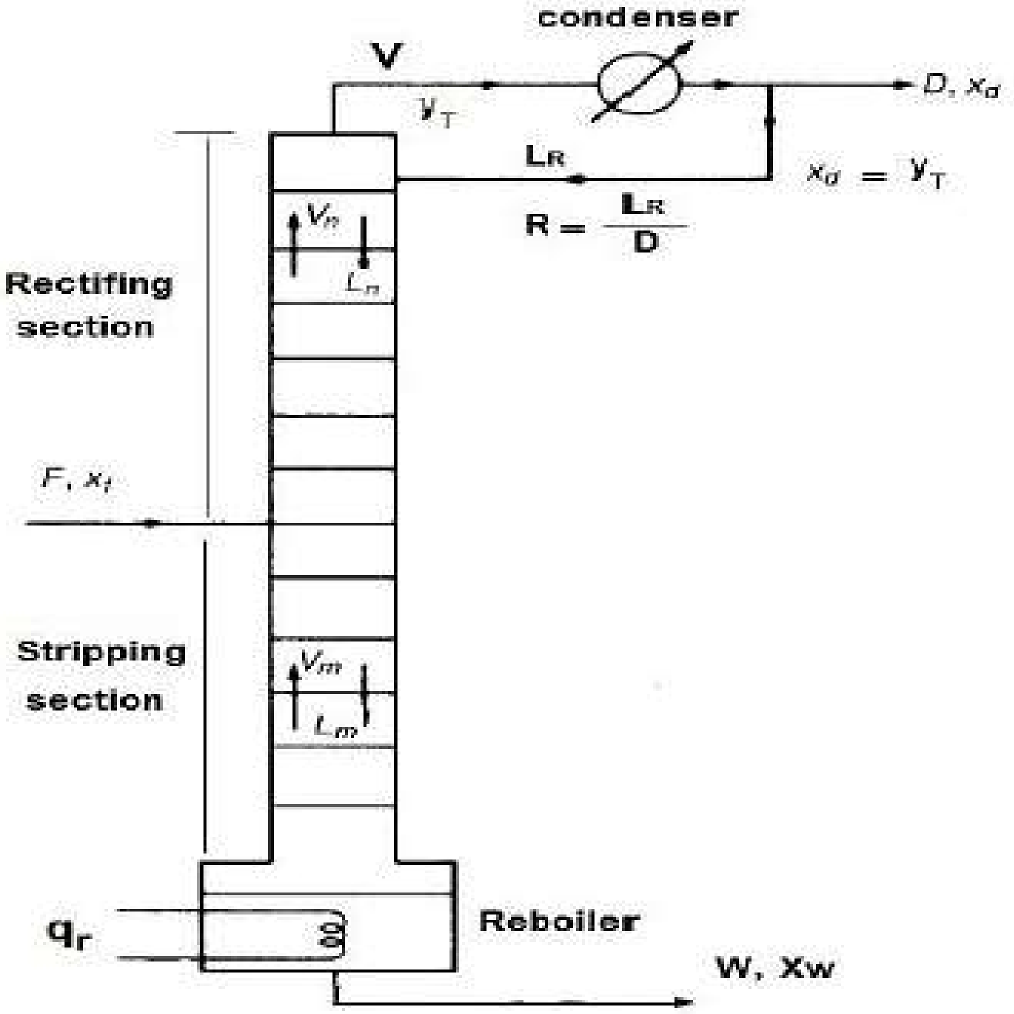


Fig: Continuous fractionating column with rectifying and stripping sections

Distillation

We must know some main points in this tower (fractionating tower):

1. The temperature various along the tower ($T_w > T_f > T_t$).

2. The feed differs from each process were it could be:

a. Cold liquid (subcooled).

b. Liquid at boiling point (saturated liquid).

c. Vapour at boiling point (saturated vapour).

d. Partially vaporized.

e. Supper heated vapour.

3. Whenever we increase L_R the tower height will decrease.

4. We have a reflux ratio of: $R = \frac{L_R}{D}$

5. Whenever we find a process with a reboiler means that the tower used is a distillation tower.

6. The feed is pumped from anywhere:

a. Form up or middle or bottom of the tower.

b. From reboiler.

Distillation

If the xf is small, the feed is pumped from the bottom tower.

If the xf is large, the feed is pumped from the top tower.

 When the feed pumped from the reboiler, all the tower is rectifying tower.

 When the feed pumped from the top, all the tower is stripping tower.

7. Reboiler is a single mass transfer stages with 100% efficiency.

No. of stages = No. of plates + 1

Continuous distillation can be divided depends on the number of components in the feed stream into:

1. Binary mixture.
2. Multi-component mixtures.

Distillation

The most common things needs to be calculated in the distillation column tower are:

1. Actual and minimum number of plates.

2. Reflux ratio and minimum reflux ratio.

3. The heat added in the boiler (Qr).

4. The heat removed in the condenser (Qc).

Distillation

The McCabe-Thiele Method

The simplifying assumption for the McCabe-Thiele method is that :

latent heat of vaporization of component A ≠ latent heat of vaporization of component B

$$\lambda_A \neq \lambda_B$$

i. Rectifying section operating line equation:

Overall material balance between plate (n)

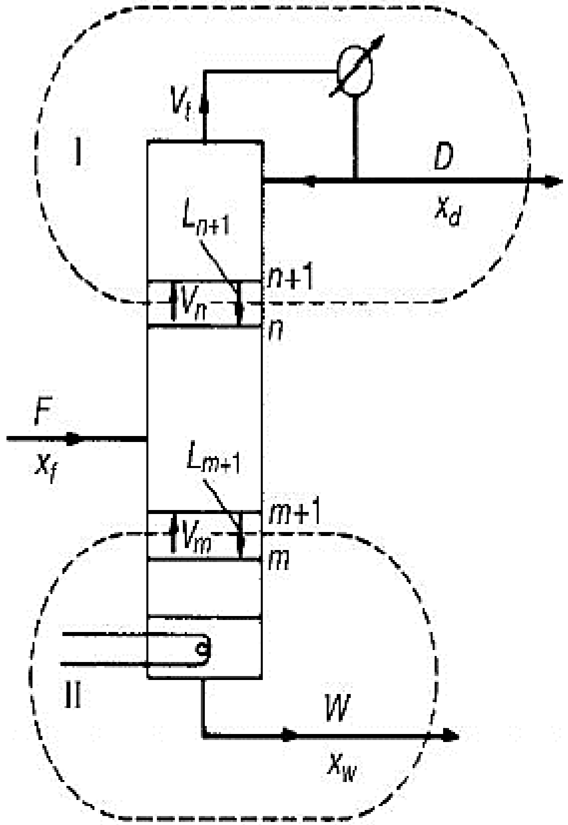
and the top product indicated by the loop I:

$$V_n = L_{n+1} + D$$

Since the molar liquid and vapour overflow is constant:

$$L_n = L_{n+1} = L_{n-1} = L_R$$

$$V_n = V_{n+1} = V_{n-1} = V$$



Distillation

Then the overall material balance equation becomes:

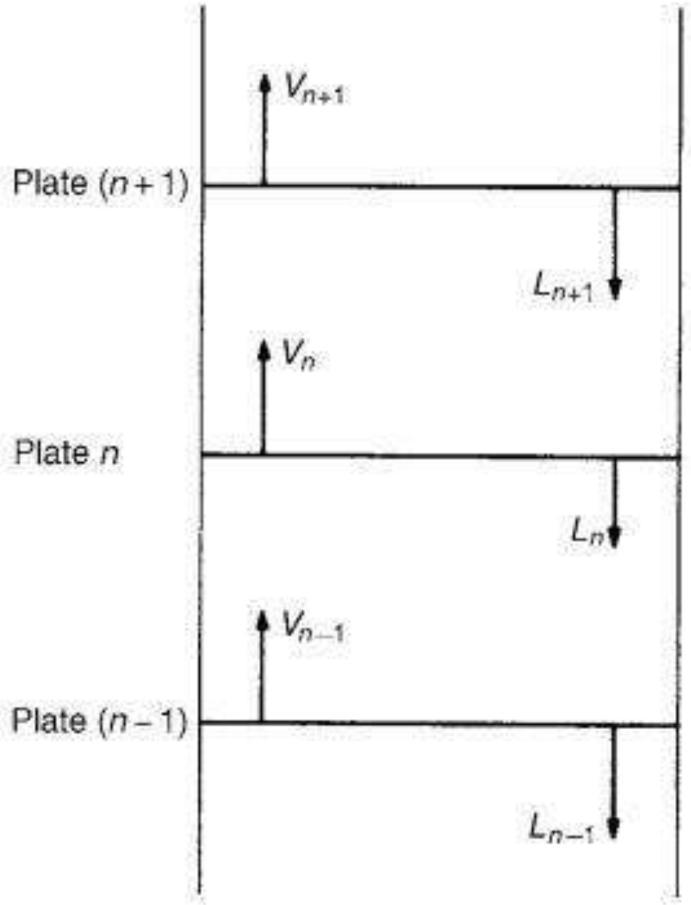
$$V_n = L_n + D$$

Material balance on (M.V.C) between plate (n) and the top product indicated by the loop I:

$$V_n\, y_n = L_{n+1}\, x_{n+1} + D\, x_d$$

$$y_n = \frac{L_{n+1}}{V_n}\, x_{n+1} + \frac{D\, x_d}{V_n}$$

$$\boxed{y_n = \frac{L_n}{V_n}\, x_{n+1} + \frac{D\, x_d}{V_n}} \quad \ldots\ldots\ldots(1)$$



Distillation

We can write Eq.(1) in the form of reflux ratio as follows:

$$\mathbf{R = \frac{L_n}{D}}$$

$$y_n = \frac{RD}{RD + D} x_{n+1} + \frac{D\ x_d}{RD + D}$$

$$\boldsymbol{y_n = \frac{R}{R+1} x_{n+1} + \frac{x_d}{R+1}} \quad \ldots\ldots\ldots\ldots(2)$$

We can plot the **rectifying (top) operating line** from the slope and intercept in the Eq.(1) and Eq.(2):

Distillation

- **slope** $= \frac{L_n}{V_n} = \frac{R}{R+1}$
- **intercept** $= (0\ ,\ \frac{D\ x_d}{V_n})$ and $(0\ ,\ \frac{x_d}{R+1})$

Or we can plot the top operating line from two points:

- $(0\ ,\ \frac{D\ x_d}{V_n})$ and $(\ x_d\ ,\ \ x_d\ )$
- $(0\ ,\ \frac{x_d}{R+1})$ and $(\ x_d\ ,\ \ x_d\ )$

Distillation

ii. Stripping section operating line equation:

Overall material balance between plate (m) and the bottom product indicated by the loop II:

$$L_{m+1} = V_m + W$$

Since the molar liquid and vapour is constant:

$$L_m = L_{m+1} = L_{m-1}$$

$$V_m = V_{m+1} = V_{m-1}$$

Then the overall material balance equation becomes:

Distillation

$$L_m = V_m + W$$

Material balance on (M.V.C) between plate (m) and the bottom product indicated by the loop II:

$$V_m\, y_m = L_{m+1}\, x_{m+1} - W\, x_w$$

$$\boxed{y_m = \frac{L_m}{V_m}\, x_{m+1} - \frac{W\, x_w}{V_m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Distillation

We can plot the **stripping (bottom) operating line** from the slope and intercept in the Eq.(1):

- **slope** $= \frac{L_m}{V_m}$
- **intercept** $= (0\,,\ \frac{-W\ x_w}{V_m})$

iii. <u>The q- line equation:</u>

If the two operating lines intersect at a point with coordinates $(\mathbf{x_q}\,,\mathbf{y_q})$, then from equations (top and bottom operating lines):

$$V_n\, y_q = L_n\ x_q + D\ x_d \quad \text{.....................................(1)}$$

$$V_m\, y_q = L_m\, x_q - W\ x_w \quad \text{.....................................(2)}$$

Distillation

Subtraction Eq.(2) from Eq.(1):

$$y_q(V_m - V_n) = (L_m - L_n)\,x_q - (D\,x_d + W\,x_w) \quad \text{...}(*)$$

A material balance over the feed plate gives:

$$F + L_n + V_m = L_m + V_n$$

$$V_m - V_n = L_m - L_n - F \quad \text{...}(**)$$

To obtain a relation between L_n and L_m, it is necessary to make an enthalpy balance over the feed plate, and to consider what happens when the feed enters the column. If the feed is all in the form of liquid at its boiling point, then:

$$L_m = L_n + F$$

Distillation

If the feed is a liquid at a temperature T_f ,that is less than the boiling point, ***so*** ***rising from the plate below will condense to provide sufficient heat to bring the f*** ***to the boiling point.***

If: H_f = is the enthalpy per mole of feed.

H_{fs} is the enthalpy of one mole of feed at its boiling point.

The heat to be supplied to bring feed to the boiling point = $F(H_{fs} - H_f)$.

The number of moles of vapour to be condensed to provide this heat $= \dfrac{F\,(H_{fs} - H_f)}{\lambda}$

Where: λ is the molar latent heat of the vapour.

The reflux liquor is then:

$$L_m = L_n + F + \frac{F\,(H_{fs} - H_f)}{\lambda}$$

$$L_m = L_n + F\left(\frac{\lambda + H_{fs} - H_f}{\lambda}\right)$$

$$L_m = L_n + qF$$

Distillation

Where:

$$q = \frac{\text{heat to vaporaize 1 mole of feed}}{\text{molar latent heat of feed}}$$

Thus, from Eq.(**)

$$V_m - V_n = q\,F - F$$

A material balance of the more volatile component over the whole column

gives: $F\,x_f = D\,x_d + W\,x_w$

Thus, from Eq.(*):

$$F(q-1)\,y_q = q\,F\,x_q - F\,x_f$$

$$y_q = \left(\frac{q}{q-1}\right)x_q - \left(\frac{x_f}{q-1}\right) \quad \text{................................(q-line equation)}$$

Distillation

Thus, the point of intersection of the two operating lines lies on the straight line of slope $\left(\frac{q}{q-1}\right)$ passing through the point (x_f , x_f).

*** We can plot the q-line from the point (x_f , x_f) with slope of $\left(\frac{q}{q-1}\right)$.**

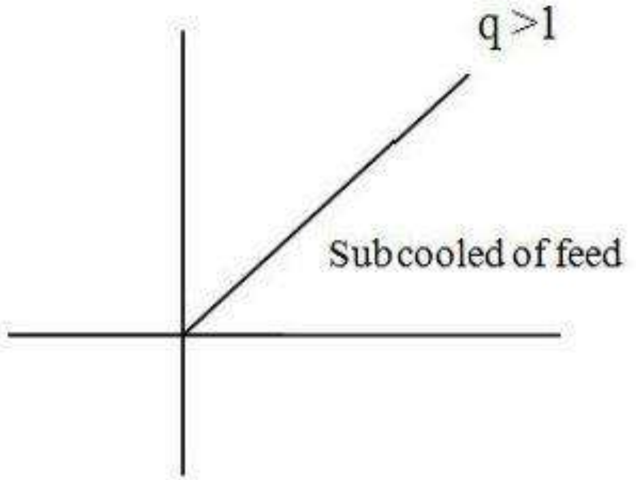
From the definition of ***q****, it follows that the slope of the* ***q-line*** *is governed by the nature of the feed as follows*:

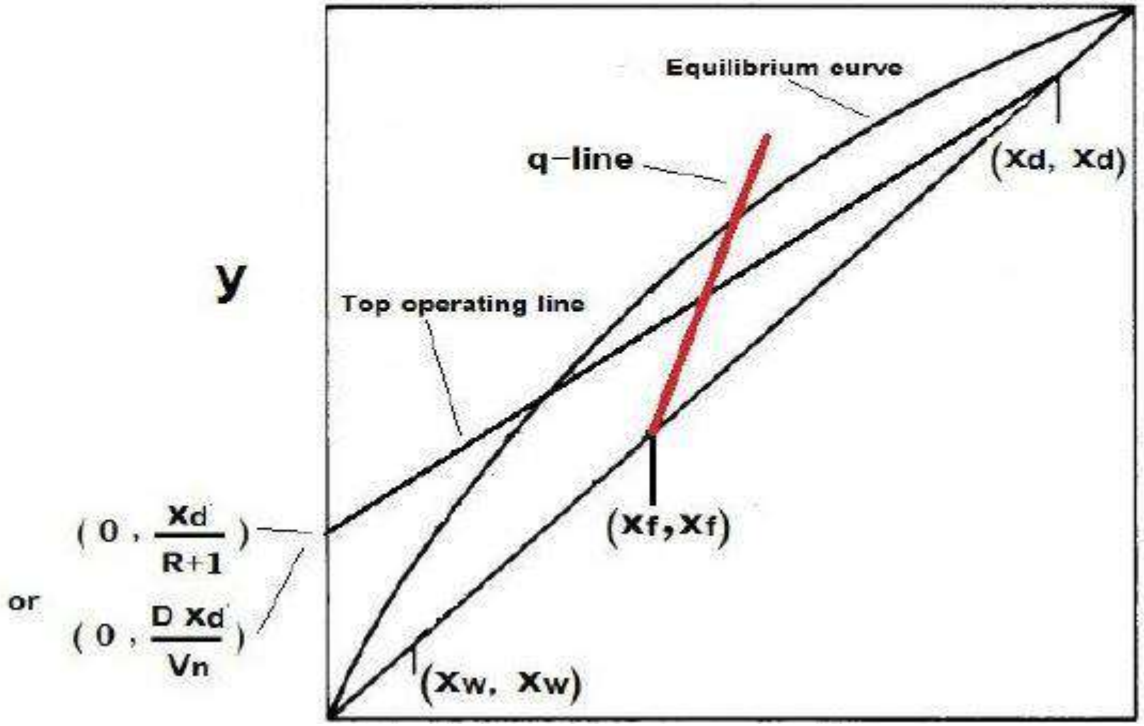
Distillation

1. Cold feed as liquor (the feed is subcooled) $T_f < T_{BP}$

$$q = \frac{C_P\,(T_{BP} - T_f) + \lambda}{\lambda} > 1$$

$$\text{Slope} = \frac{q}{q-1} = \frac{+\text{ve}}{+\text{ve}}$$



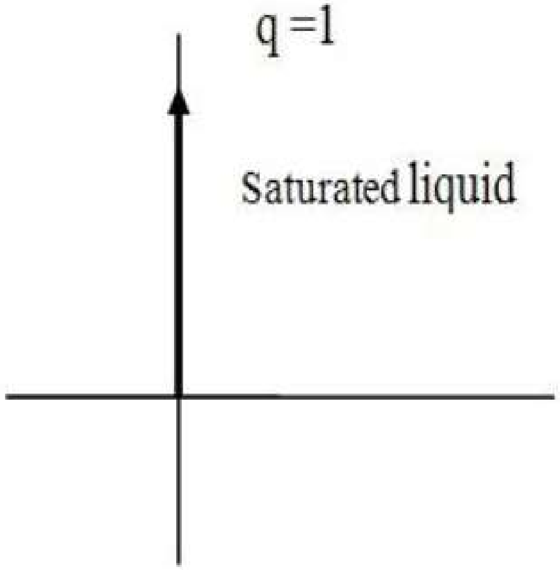


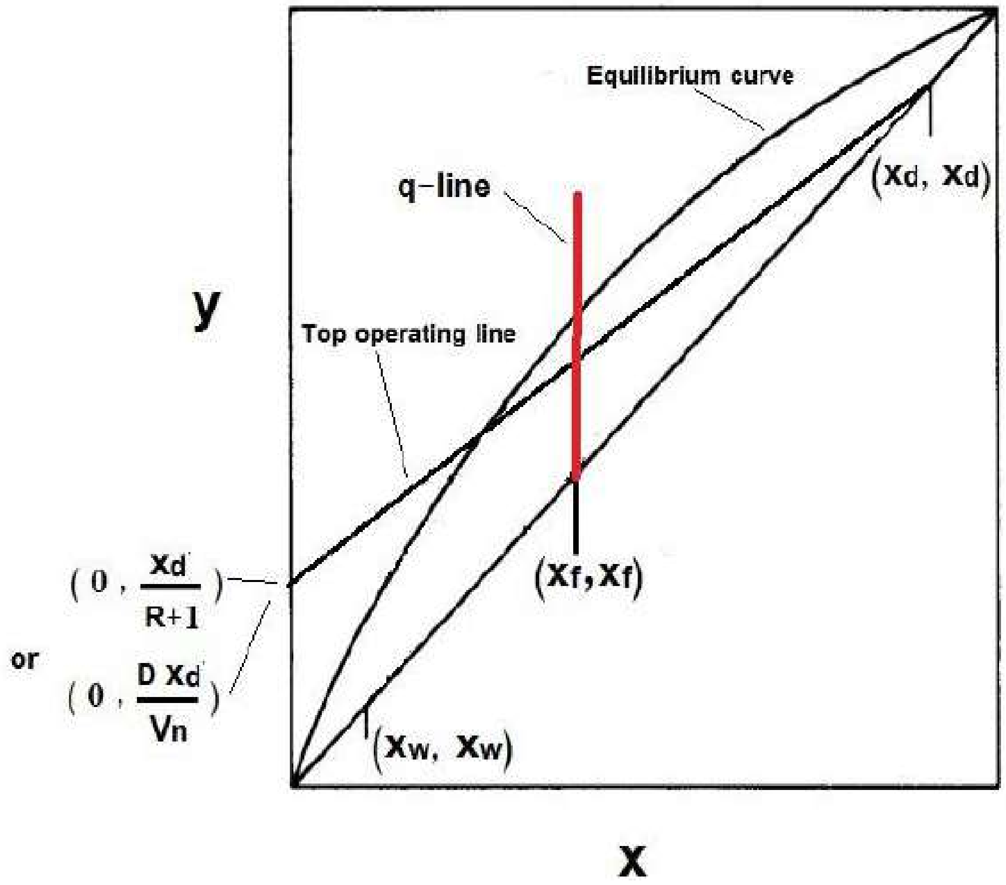
Distillation

2. Feed at boiling point (saturated liquid) $T_f = T_{BP}$

$$q = \frac{0 + \lambda}{\lambda} = 1$$

$$\text{Slope} = \frac{q}{q-1} = \frac{1}{0} = \infty$$





Distillation

3. Feed partly vapour (partially vaporized feed) $T_f = T_{BP}$

Two phase vapour and liquid feed quality = 20% vapour, this means that:

saturated vapour = 20%

saturated liquid = 80%

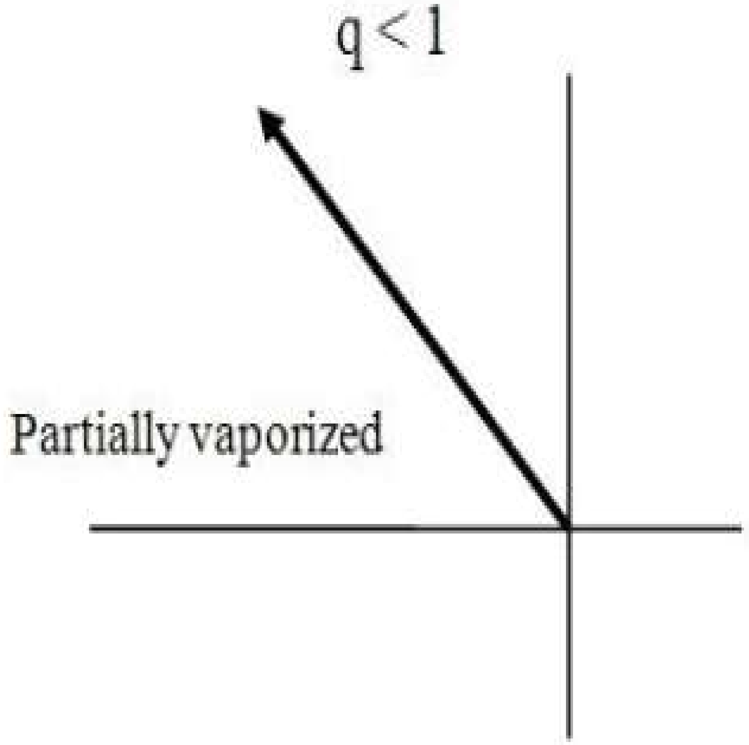
$$q = \frac{(\text{Fraction of liquid})\ \lambda}{\lambda} = 0.8 < 1$$

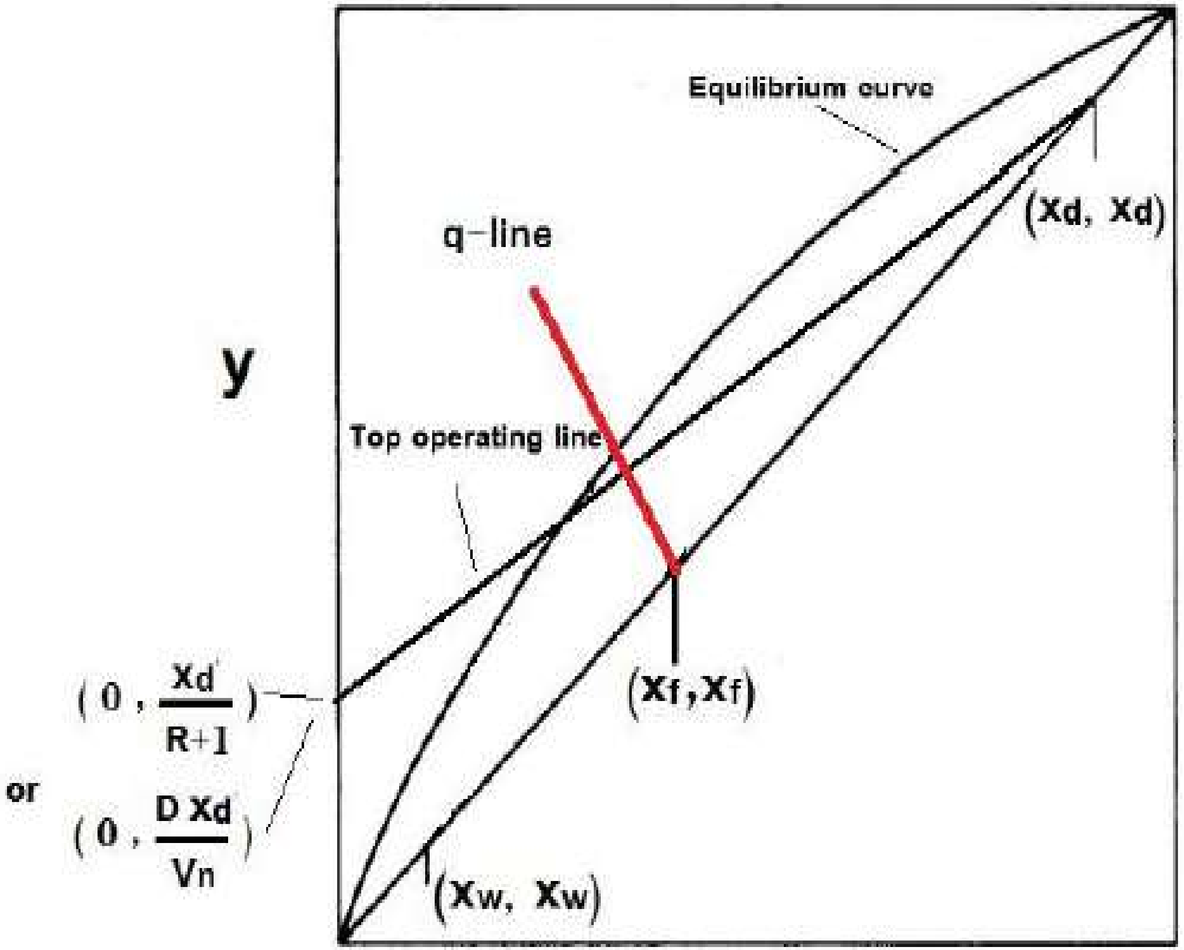
$$\boxed{q = \text{fraction of liquid}}$$

$$\text{Slope} = \frac{q}{q-1} = \frac{0.8}{0.8-1} = \frac{0.8}{-0.2} = -4$$

$$\boxed{\text{Slope} = \frac{\text{fraction of liquid}}{-\ \text{fraction of vapour}}}$$

Distillation



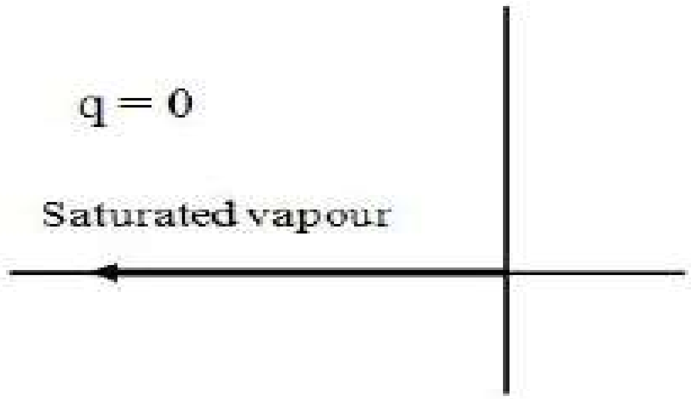


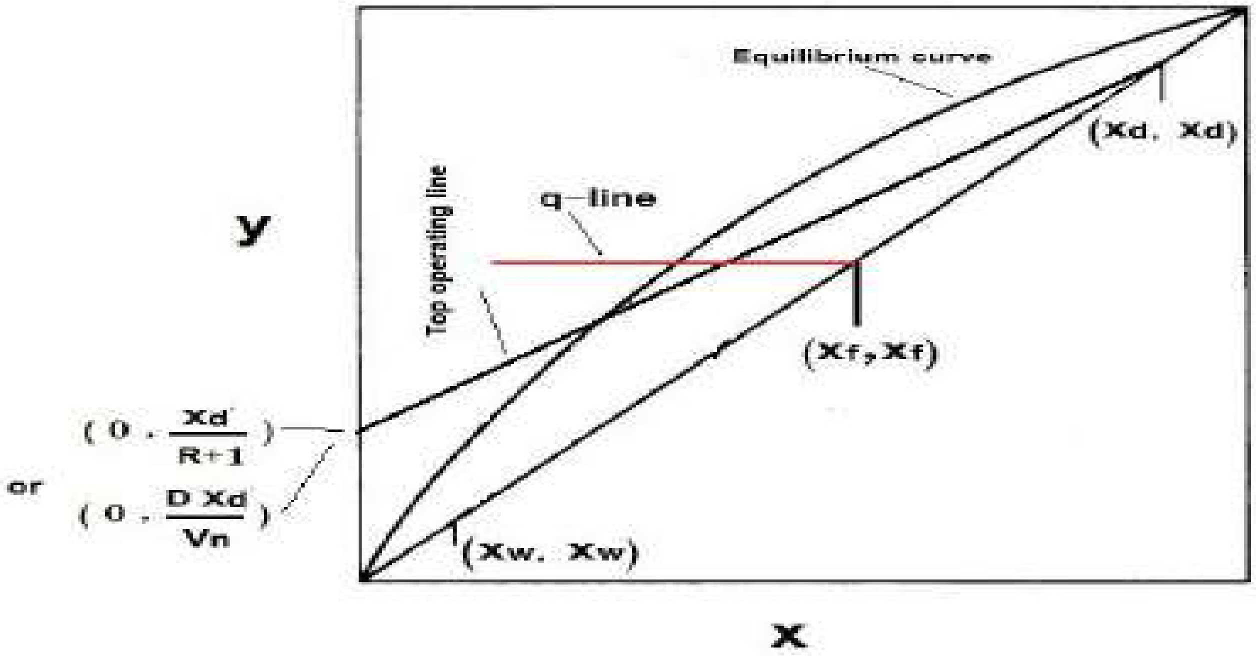
Distillation

4. Feed saturated vapour (single vapour phase): $T_f = T_{BP}$

$$q = \frac{0}{\lambda} = 0$$

$$\text{Slope} = \frac{q}{q-1} = \frac{0}{-1} = 0$$



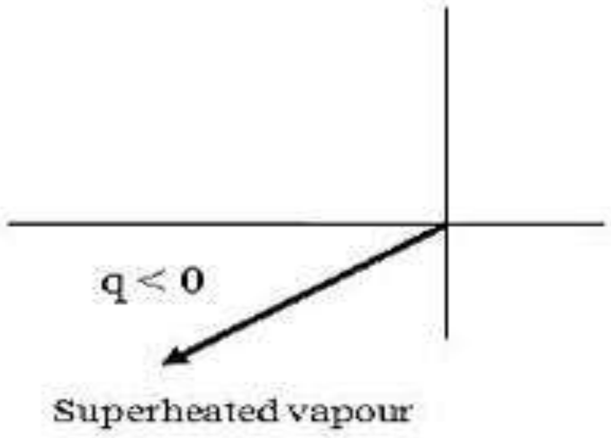


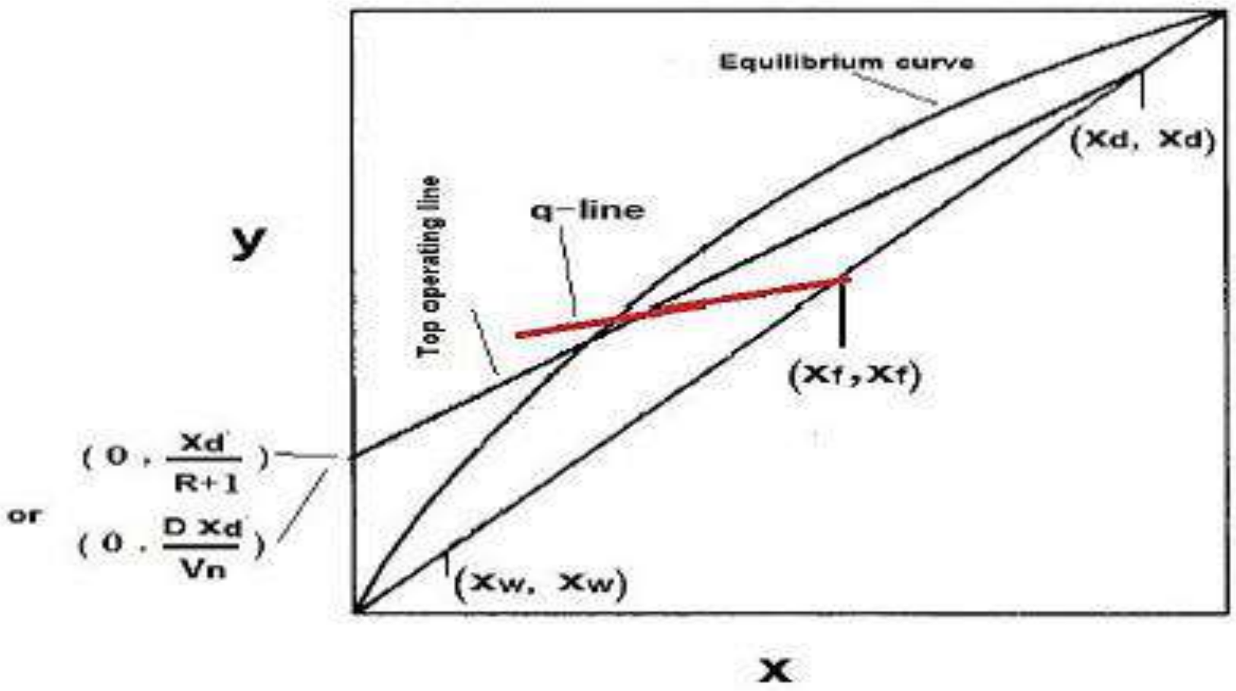
Distillation

5. Feed superheated vapour: $T_f > T_{BP}$

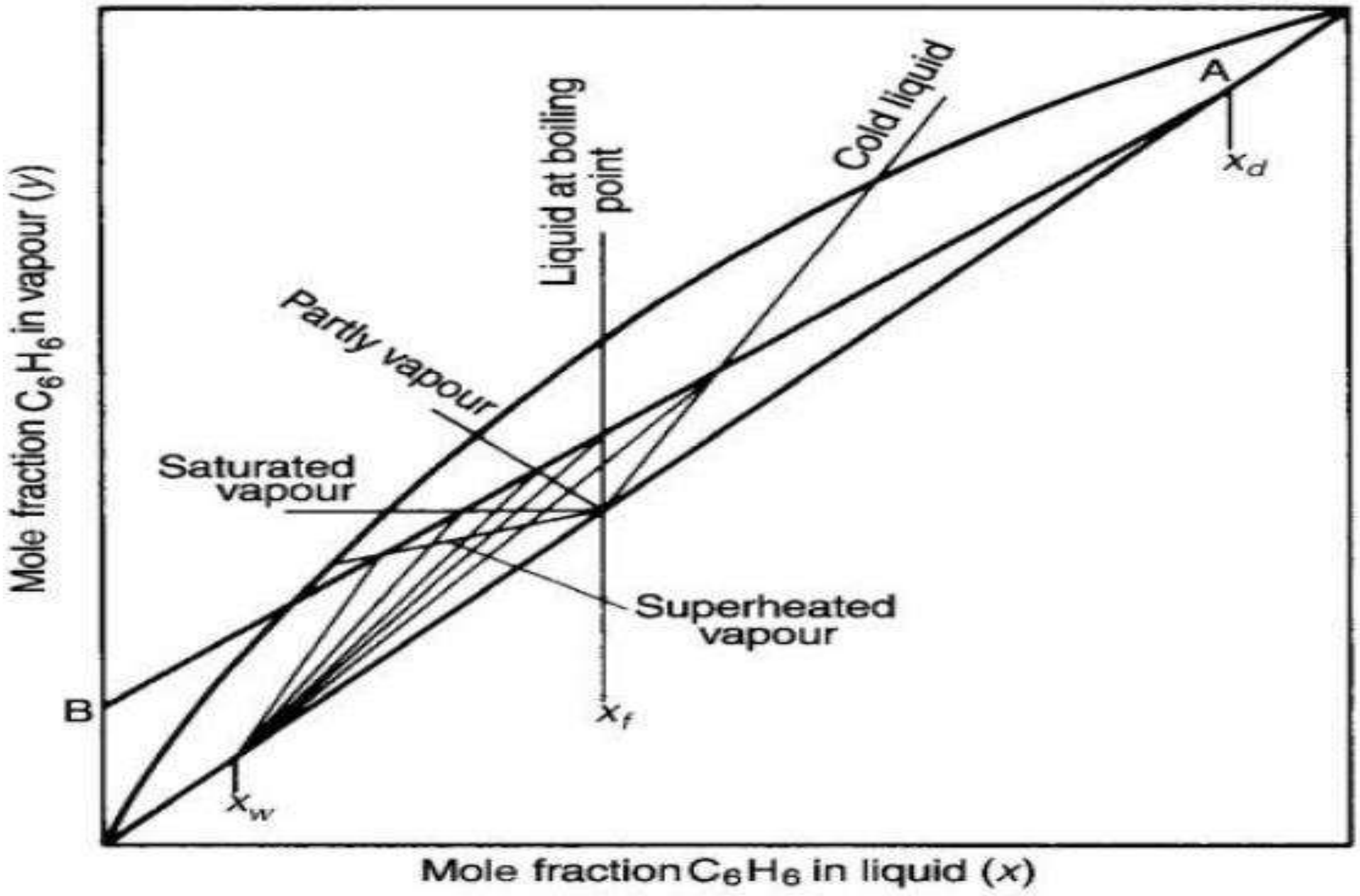
$$q = \frac{[C_P\,(T_{BP} - T_f) - \lambda] + \lambda}{\lambda} < 0$$

$$\text{Slope} = \frac{q}{q-1} = \frac{-ve}{-ve}$$





Distillation



Effect of the condition of the feed on the intersection of the operating lines for a fixed reflux ratio.

Distillation

Calculation of the theoretical number of plates of the continuous distillation column by McCabe-Thiele Method (graphically):

1. We draw the top operating line from two points:

$$\left[(x_d \ , \ x_d) \, and \, \left(0 \, , \frac{D \ x_d}{V_n}\right)\right] \qquad \text{or} \qquad \left[(x_d \ , \ x_d) \, and \, \left(0 \, , \frac{x_d}{R+1}\right)\right]$$

$$x_d \ , \ x_d) \text{ and slope} = \frac{L_n}{V_n} = \frac{R}{R+1}$$

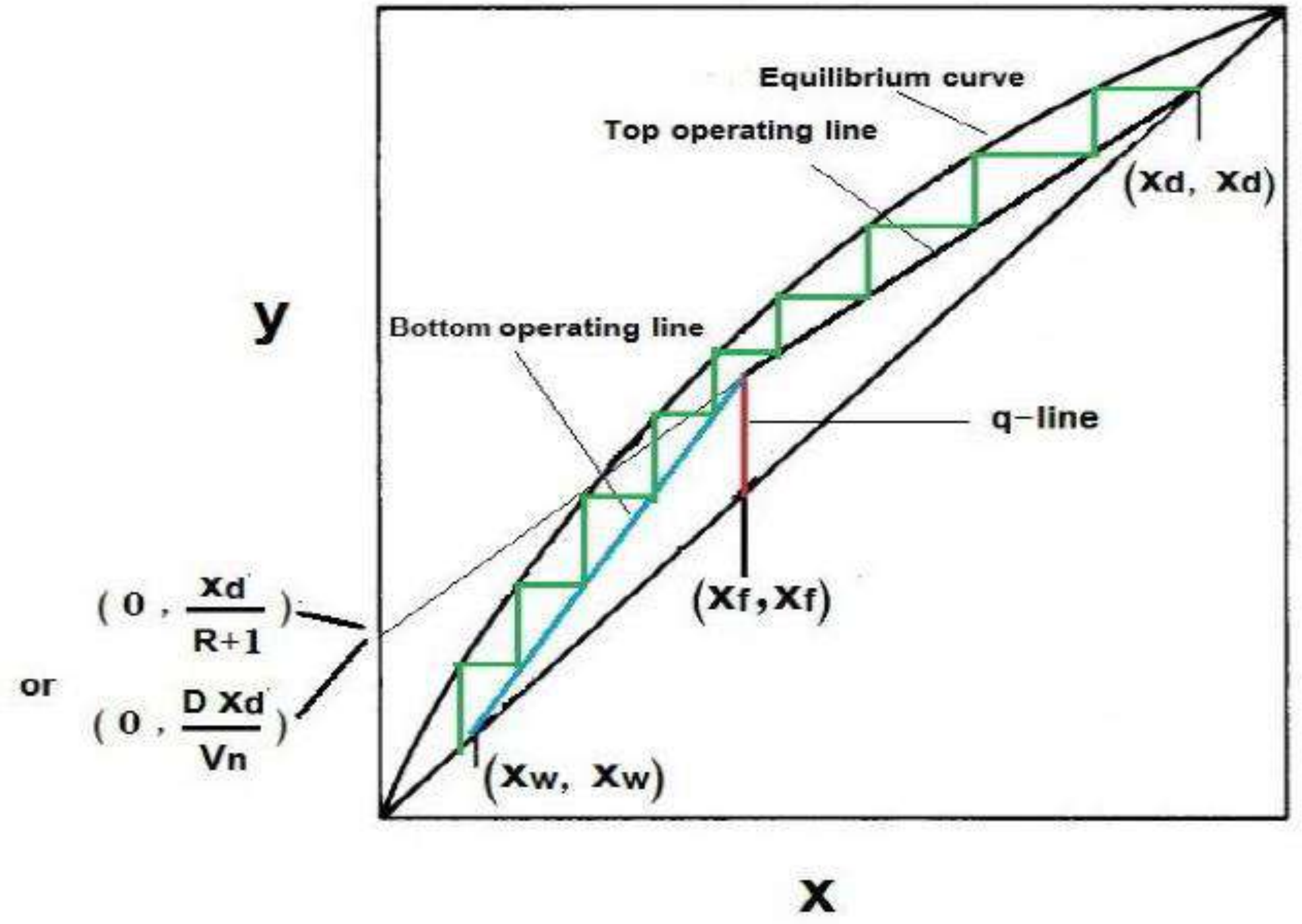
2. We draw the q-line from $(x_f \ , \ x_f)$ until its intersect with the top operating at $(x_q \ , \ x_q)$.

3. We draw the bottom operating line by joining point $(x_q \ , \ x_q)$ to point $(x_w \ , \ x_w)$.

4. We draw the vertical and horizontal lines from $(x_d \ , \ x_d)$ to $(x_w \ , \ x_w)$ which represents **the number of stages.**

No. of plates = No. of stages − 1

Distillation



Distillation

Example: A continuous rectifying column handles a mixture consisting of 40 per cent of benzene by mass and 60 per cent of toluene at the rate of 4 kg/s, and separates it into a product containing 97 per cent of benzene and a liquid containing 98 per cent toluene. The feed is liquid at its boiling-point.

(a) Calculate the mass flows of distillate and waste liquor.
(b) If a reflux ratio of 3.5 is employed, how many plates are required in the rectifying part of the column?

Mole fraction of benzene in liquid	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mole fraction of benzene in vapour	0.22	0.38	0.51	0.63	0.7	0.78	0.85	0.91	0.96

Distillation

Solution:

Mole fraction of benzene in feed, $x_f = \dfrac{(40/78)}{(40/78) + (60/92)} = 0.440$

Similarly: $x_d = 0.974$ and $x_w = 0.024$

As the feed is a liquid at its boiling-point, the q-line is vertical and may be drawn at $x_f = 0.44$.

(a) A mass balance over the column and on the more volatile component in terms of the mass flow rates gives:

$$4.0 = W' + D'$$

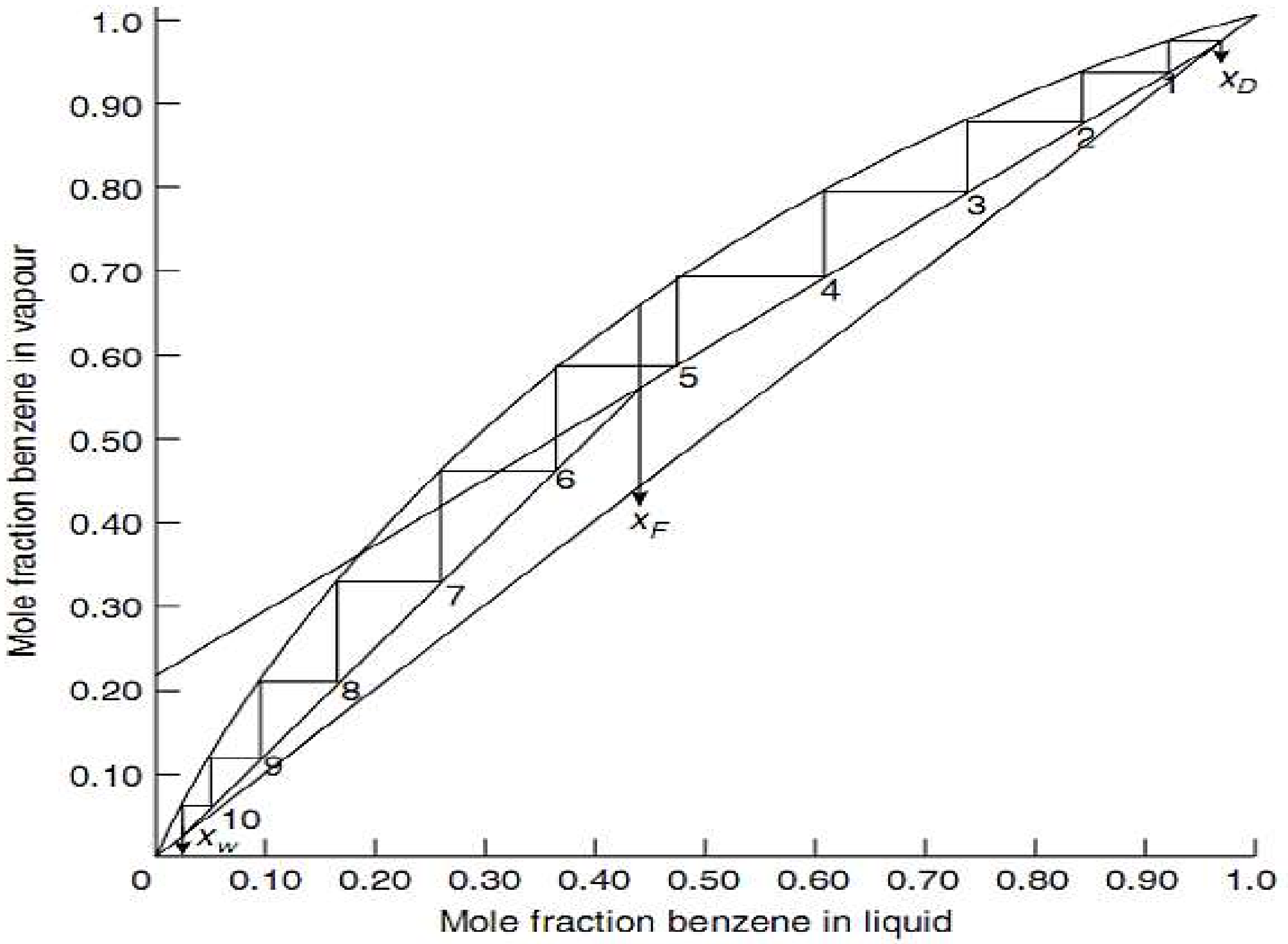
$$(4 \times 0.4) = 0.02W' + 0.97D'$$

from which: bottoms flowrate, $W' = \underline{\underline{2.4 \text{ kg/s}}}$

and: top product rate, $D' = \underline{\underline{1.6 \text{ kg/s}}}$

(b) If $R = 3.5$, the intercept of the top operating line on the y-axis is given by $x_d/(R+1) = (0.974/4.5) = 0.216$, and thus the operating lines may be drawn as shown in Figure 11h. The plates are stepped off as shown and 10 theoretical plates are required.

Distillation

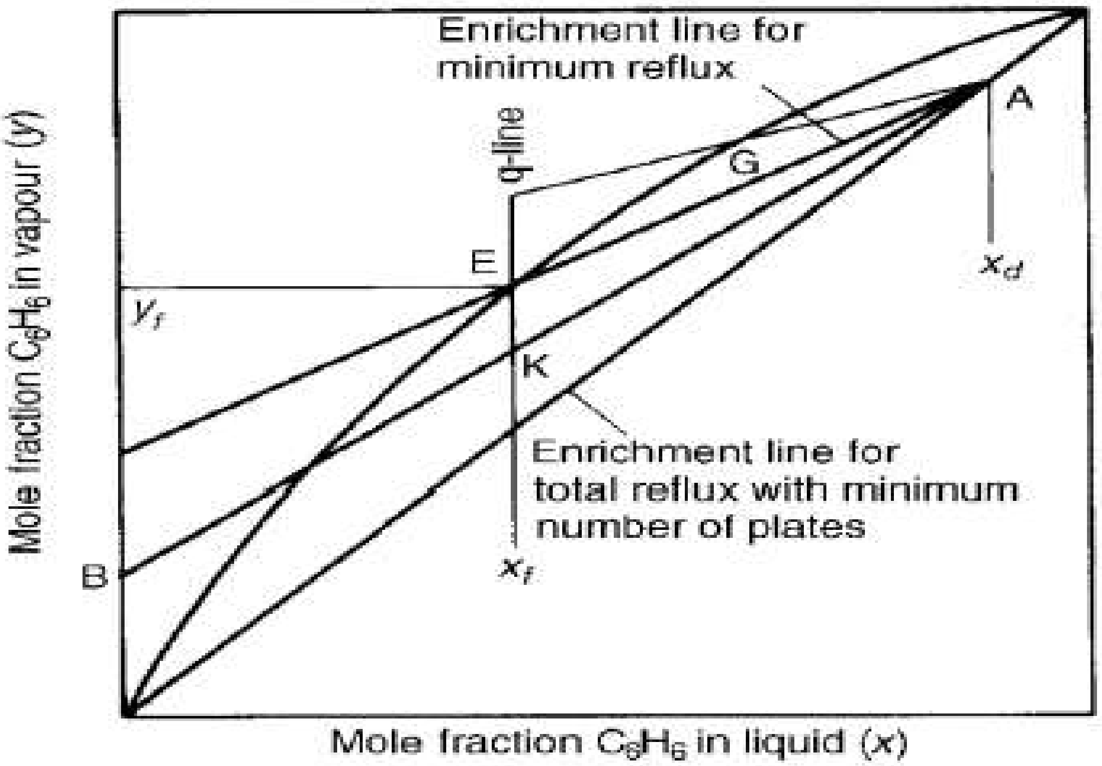


Distillation

Minimum Reflux ratio (R_{min})

If the reflux ratio is reduced, the slope of the operating line is reduced and more stages are required to pass from x_f to x_d, as shown by the line AK in Figure 11.17. Further reduction in R will eventually bring the operating line to AE, where an infinite number of stages is needed to pass from xd to x_f . This arises from the fact that under these conditions the steps become very close together at liquid compositions near to x_f, and no enrichment occurs from the feed plate to the plate above. These conditions are known as minimum reflux (R_{min}).

Distillation



Influence of reflux ratio on the number of plates required for a given separation

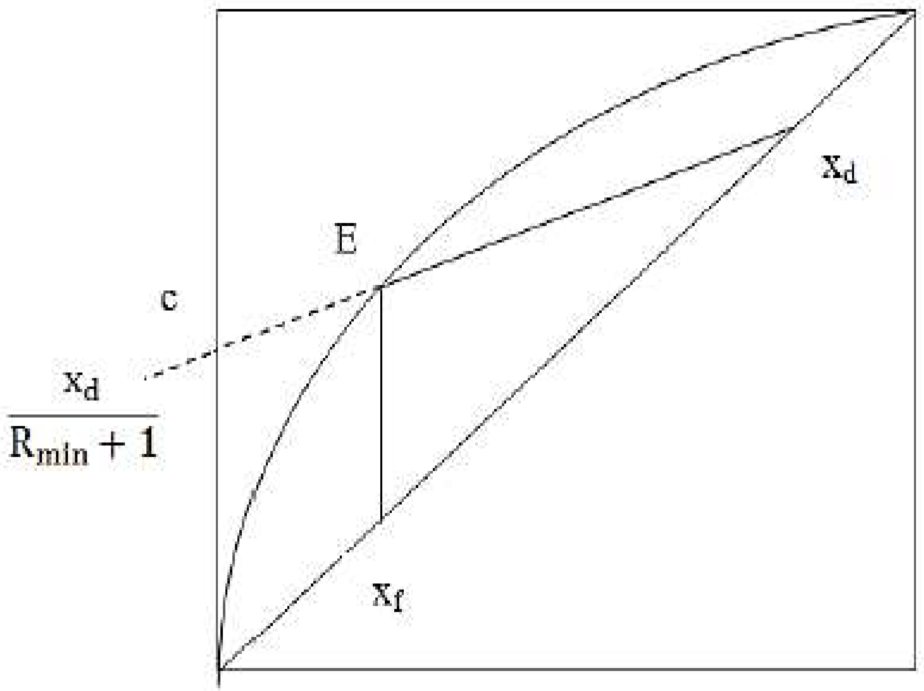
Here: The number of plates (from this figure) will equal ($N = \infty$) because the triangles will reach point E, and will not come out of that point.

Distillation

There are two methods to estimate (R_{min}):

1. Calculation of (R_{min}) by graphical method.

(R_{min}) is obtained when the top operating line (Rectifying line) intersects the equilibrium curve at the feed point.



Distillation

We read (R_{min}) from the y-axis, let's say (C) so:

$$C = \frac{x_d}{R_{min} + 1} \Longrightarrow R_{min} = \text{any number}$$

2. Calculation of (R_{min}) by equation from volatility:

$$\mathbf{R_{min} = \frac{1}{(\alpha - 1)}\left[\frac{x_d}{x_f} - \frac{\alpha\,(1 - x_d)}{(1 - x_f)}\right]}$$

$$R_{act.} = (1.1 - 1.5)\, R_{min}$$

* If we have different α_{AB} in the feed, waste and distillate product, so we take the average α_{AB}.

$$\alpha_{AB} = [(\alpha_{AB})_w \cdot (\alpha_{AB})_f \cdot (\alpha_{AB})_d]^{1/3}$$

Distillation

Minimum number of stages (Nmin)

1. Calculation of (Nmin) by graphical method.

If no product is withdrawn from the still, that is **D = 0**, then the column is said to **operate under conditions of total reflux** and the top operating line has its maximum slope of unity, and coincides with the line x = y. If the reflux ratio is reduced, the slope of the operating line is reduced and more stages are required to pass.

Distillation

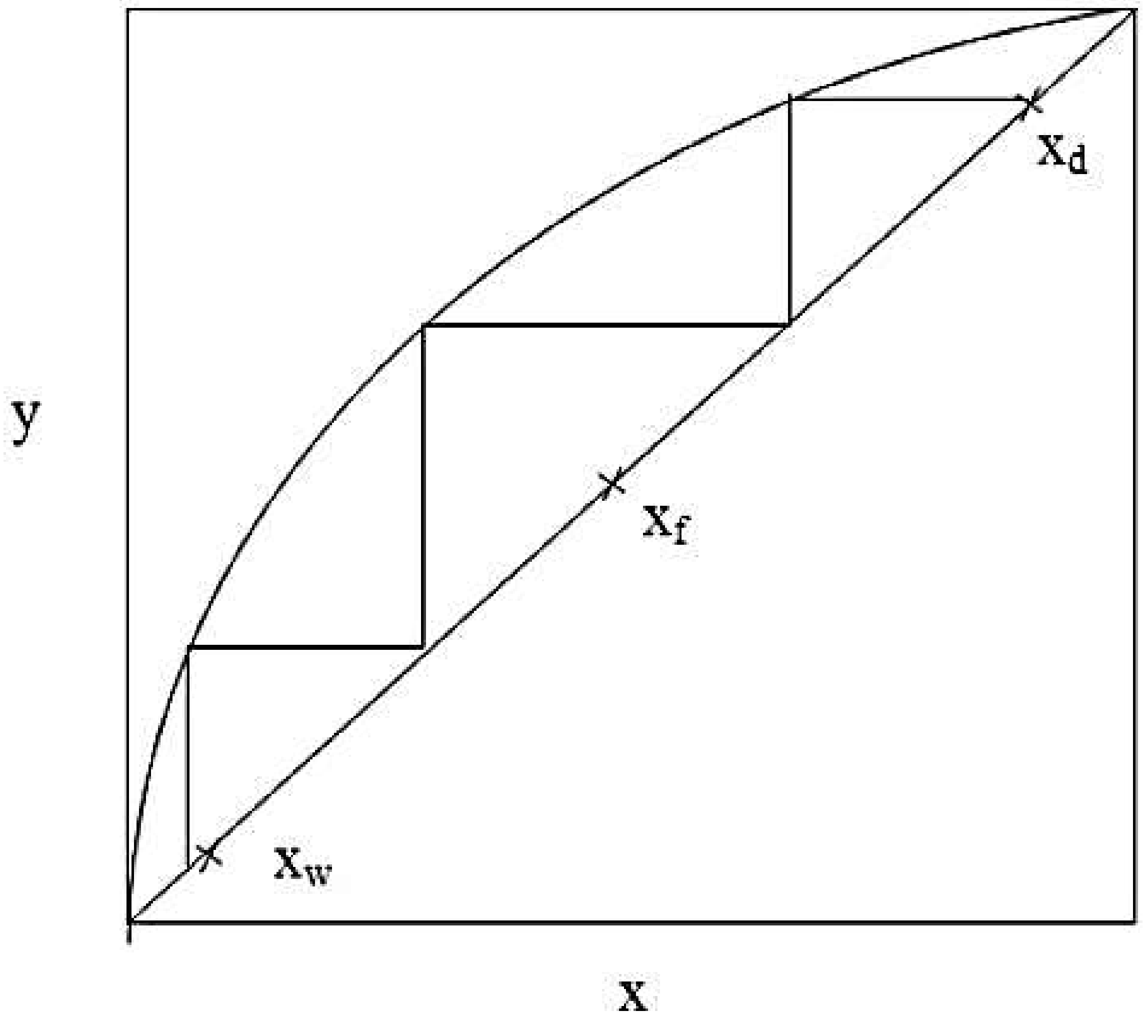
When $D = 0 \Longrightarrow R_{max} = \frac{L_R}{D} = \frac{L_R}{0} = \infty$ (at total reflux)

$$\text{Slope} = \frac{R}{R+1} = \frac{\infty}{\infty + 1} = 1.0$$

$$\text{Interccept} = \frac{x_d}{R+1} = \frac{x_d}{\infty + 1} = 0$$

$$\mathbf{N_{min}} = 3\ stages$$

$$= 2\ plates$$



2. **Calculation of (N_{min}) by Fenske equation.**

$$N_{min} + 1 = \frac{\log\left[\left(\frac{x_d}{1 - x_d}\right) \times \left(\frac{1 - x_w}{x_w}\right)\right]}{\log(\alpha_{av.})}$$

Efficiency of Column

1. **Overall efficiency (η_c):**

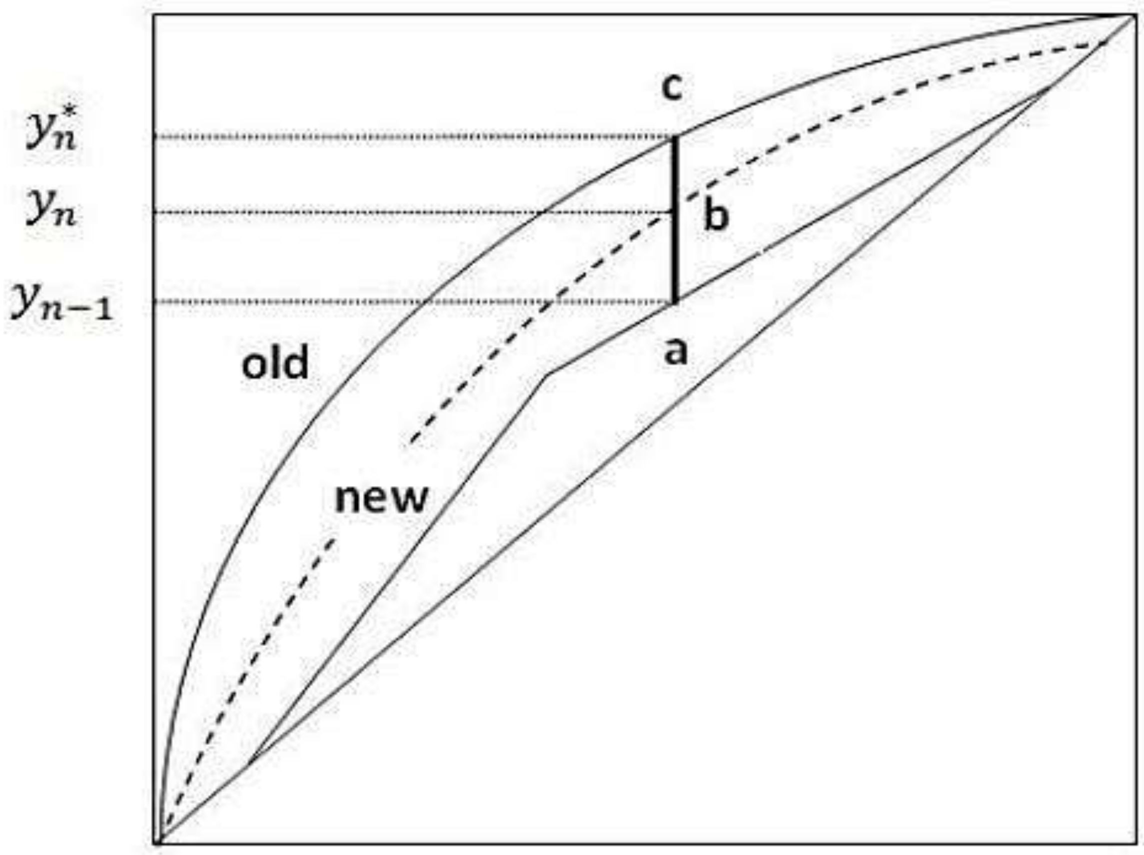
$$\eta_c = \frac{Theoritial\ No.}{Actual\ No.}$$

Distillation

2. Plate efficiency:

i. Efficiency based on vapour phase (Emv).

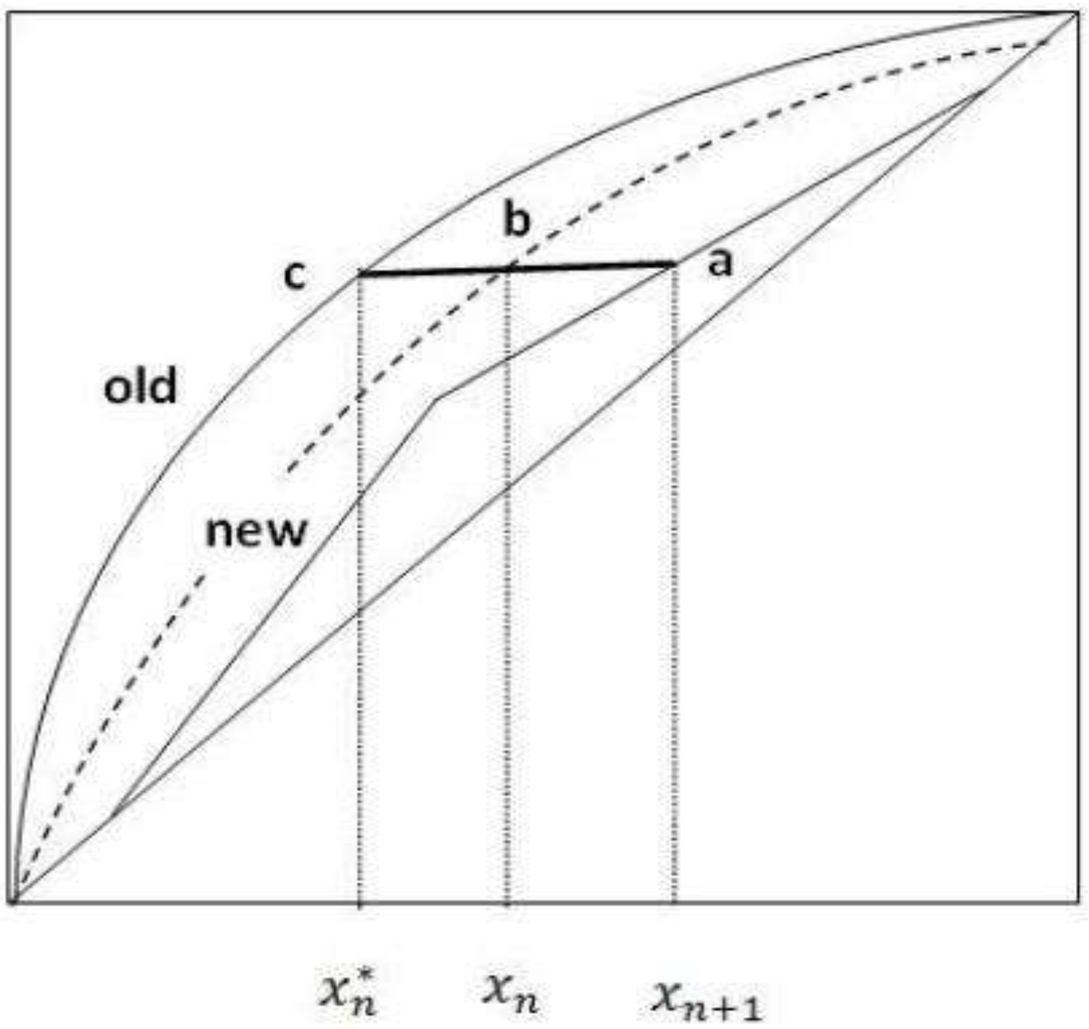
$$\text{Emv} = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}} = \frac{\text{ab}}{\text{ac}}$$



Distillation

ii. Efficiency based on liquid phase (Eml).

$$\text{Eml} = \frac{x_{n+1} - x_n}{x_{n+1} - x_n^*} = \frac{\text{ab}}{\text{ac}}$$



Distillation

The Lewis–Sorel method

This method is used to calculate the mole fraction of components on the plates and the number of plates by stage to stage calculations for binary mixture with one feed only. There are two types of calculations:

1. Calculations from top to bottom section:

a. $x_d = y_t$

b. x_t is to be found from the equilibrium data (Graphically or by equation):

$$x_t = \frac{y_t}{\alpha_{AB} - (\alpha_{AB} - 1)y_t}$$

Distillation

c. y_{t-1} is to be calculated from rectifying operating line equation:

$$y_{t-1} = \frac{L_n}{V_n} x_t + \frac{D\ x_d}{V_n} \qquad \text{or} \qquad y_{t-1} = \frac{R}{R+1} x_t + \frac{x_d}{R+1}$$

d. x_{t-1} is to be found from the equilibrium data (Graphically or by equation):

$$x_{t-1} = \frac{y_{t-1}}{\alpha_{AB} - (\alpha_{AB} - 1) y_{t-1}}$$

e. y_{t-2} is to be calculated from rectifying operating line equation:

$$y_{t-2} = \frac{L_n}{V_n} x_{t-1} + \frac{D\ x_d}{V_n} \qquad \text{or} \qquad y_{t-2} = \frac{R}{R+1} x_{t-1} + \frac{x_d}{R+1}$$

a. Continue until reach x_f, then use stripping operating line equation to calculate the mole fraction in the vapour (y) and stop the calculation when reach x_w.

Distillation

2. Calculations from bottom to top section:

b. $x_w = x_1$

c. y_1 is to be found from the equilibrium data (Graphically or by equation):

$$y_1 = \frac{\alpha_{AB} * x_1}{1 + (\alpha_{AB} - 1) x_1}$$

d. x_2 is to be calculated from stripping operating line equation:

$$y_1 = \frac{L_m}{V_m} x_2 - \frac{W x_w}{V_m}$$

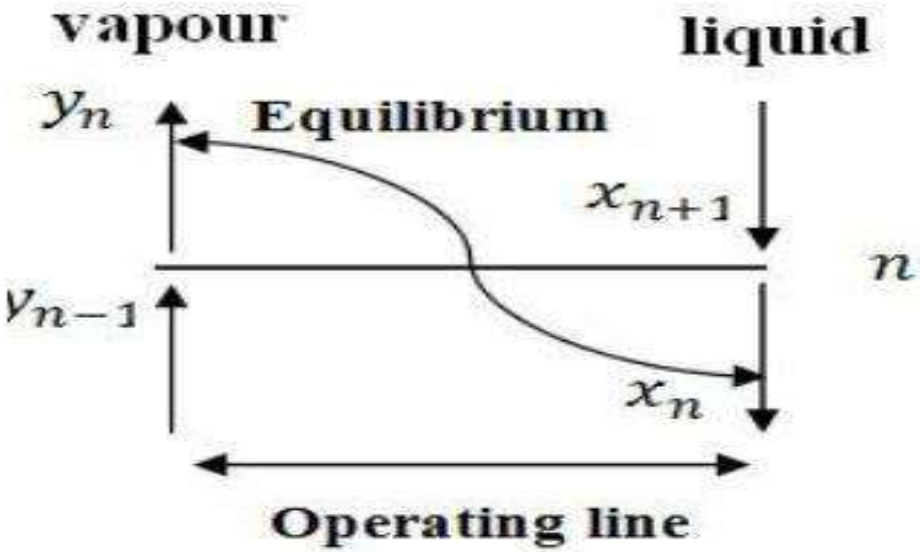
Distillation

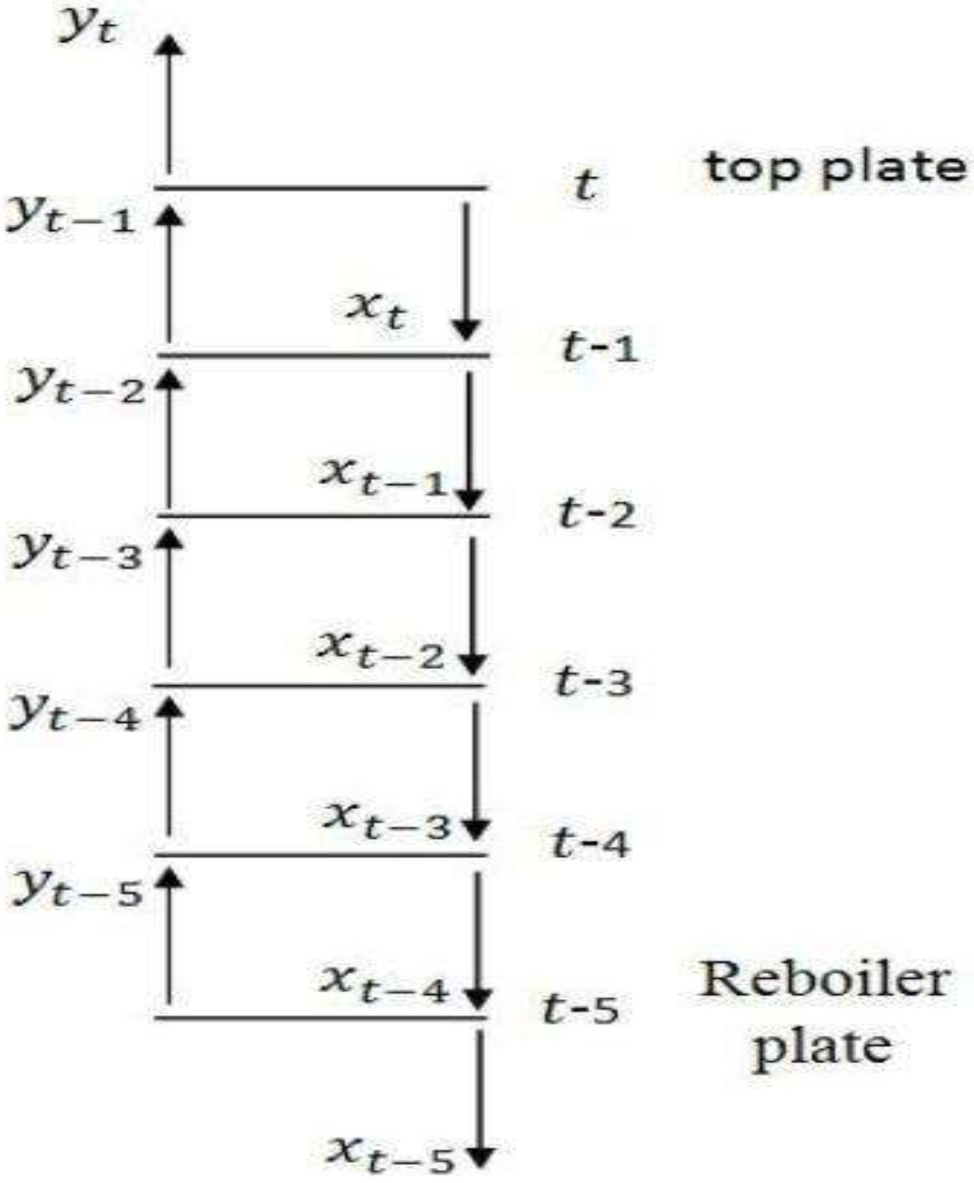
e. y_2 is to be found from the equilibrium data (Graphically or by equation):

$$y_2 = \frac{\alpha_{AB} * x_2}{1 + (\alpha_{AB} - 1)\, x_2}$$

f. Continue until reach x_f, then use rectifying operating line equation to calculate the mole fraction in the liquid (x) and stop the calculation when reach x_d.

Distillation





Distillation

Example: A mixture of benzene and toluene containing 50 mole per cent benzene is to be separated to give a product containing 90 mole per cent benzene at the top, and a bottom product containing not more than 10 mole per cent benzene. The feed enters the column at its boiling point. It is proposed to operate the unit with an (L_n/D) ratio of 3.5:1 kmol/kmol product. It is required to find the composition of the liquid on the third theoretical plate from top and on the third theoretical plate from bottom. Take the relative volatility as 2.16.

Solution:

Basis: 100 kmol/hr of feed

$F = D + W$

$100 = D + W$ ……………………………………….(1)

$F\,(x_f) = D\,(x_d) + W\,(x_w)$

$100\,(0.5) = D\,(0.9) + W\,(0.1)$ …………………….(2)

From Eq.(1) & Eq.(2) :

$D = 50$ kmol/hr, $W = 50$ kmol/hr

Distillation

$L_n = R\,D = 3.5\,(50) = 175$ kmol/hr

$V_n = (R+1)\,D = (3.5+1)\,(50) = 225$ kmol/hr

Top operating line equation:

$$y_n = \frac{L_n}{V_n}\, x_{n+1} + \frac{D\ x_d}{V_n}$$

$$y_{t-1} = 0.778\ x_t + 0.2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equilibrium relation:

$$x_A = \frac{y_A}{\alpha_{AB} - (\alpha_{AB} - 1) y_A} = \frac{y_A}{2.16 - 1.16\, y_A}$$

Distillation

Calculations from top to bottom section:

$$y_t = x_d = 0.9$$

From equilibrium relation:

$$x_t = \frac{y_t}{2.16 - 1.16\, y_t} = \frac{0.9}{2.16 - 1.16\,(0.9)} = 0.806$$

Distillation

From top operating line equation:

$$y_{t-1} = 0.778\ x_t + 0.2 = 0.778\,(0.806) + 0.2 = 0.827$$

From equilibrium relation:

$$x_{t-1} = \frac{y_{t-1}}{2.16 - 1.16\,y_{t-1}} = \frac{0.827}{2.16 - 1.16\,(0.827)} = 0.688$$

From top operating line equation:

$$y_{t-2} = 0.778\ x_{t-1} + 0.2 = 0.778\,(0.688) + 0.2 = 0.735$$

From equilibrium relation:

$$x_{t-2} = \frac{y_{t-2}}{2.16 - 1.16\,y_{t-2}} = \frac{0.735}{2.16 - 1.16\,(0.735)} = 0.562$$

Distillation

Calculations from bottom to top section:

$L_m = L_n + q\,F$ (for saturated liquid $q = 1$)

$L_m = 175 + (1)\,100 = 275$ kmol/hr

$V_m = V_n + (q-1)\,F$

$V_m = V_n = 225$ kmol/hr

Bottom operating line equation:

$$y_m = \frac{L_m}{V_m}\,x_{m+1} - \frac{W\;x_w}{V_m}$$

$$y_1 = 1.22\;x_2 - 0.022$$

E ium relation:

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1)\,x_A}$$

Distillation

$$y_1 = \frac{2.16\ x_1}{1 + 1.16\ x_1}$$

$$x_1 = x_w = 0.1$$

From equilibrium relation:

$$y_1 = \frac{2.16\ x_1}{1 + 1.16\ x_1} = \frac{2.16\ (0.1)}{1 + 1.16\ (0.1)} = 0.193$$

From bottom operating line equation:

$$y_1 = 1.22\ x_2 - 0.022$$

$$0.193 = 1.22\ x_2 - 0.022$$

$$x_2 = 0.173$$

Distillation

From equilibrium relation:

$$y_2 = \frac{2.16\ x_2}{1 + 1.16\ x_2} = \frac{2.16\ (0.173)}{1 + 1.16\ (0.173)} = 0.315$$

From bottom operating line equation:

$$y_2 = 1.22\ x_3 - 0.022$$
$$0.315 = 1.22\ x_3 - 0.022$$
$$x_3 = 0.276$$

From equilibrium re

$$y_3 = \frac{2.16\ x_3}{1 + 1.16\ x_3} = \frac{2.16\ (0.276)}{1 + 1.16\ (0.276)} = 0.451$$

From bottom operating line equation:

$$y_3 = 1.22\ x_4 - 0.022$$
$$0.451 = 1.22\ x_4 - 0.022$$
$$x_4 = 0.55$$

Distillation

Batch Distillation

Similar to differential distillation with rectifying section only and reflux ratio. In batch distillation the whole of a batch is run into the boiler of the still and, on heating, the vapour is passed into a fractionation column, as shown in Figure 11.33. As with continuous distillation, the composition of the top product depends on the still composition, the number of plates in the column and on the reflux ratio used. There are two possible modes of operation in batch distillation column:

1. Operation at constant product composition (x_d), the reflux ratio maybe increased continuously [mean, constant (x_d) and variable (R)].

2. Operation at constant reflux ratio (R), the composition of the top product will decrease with time [mean, constant (R) and variable (x_d)].

Distillation

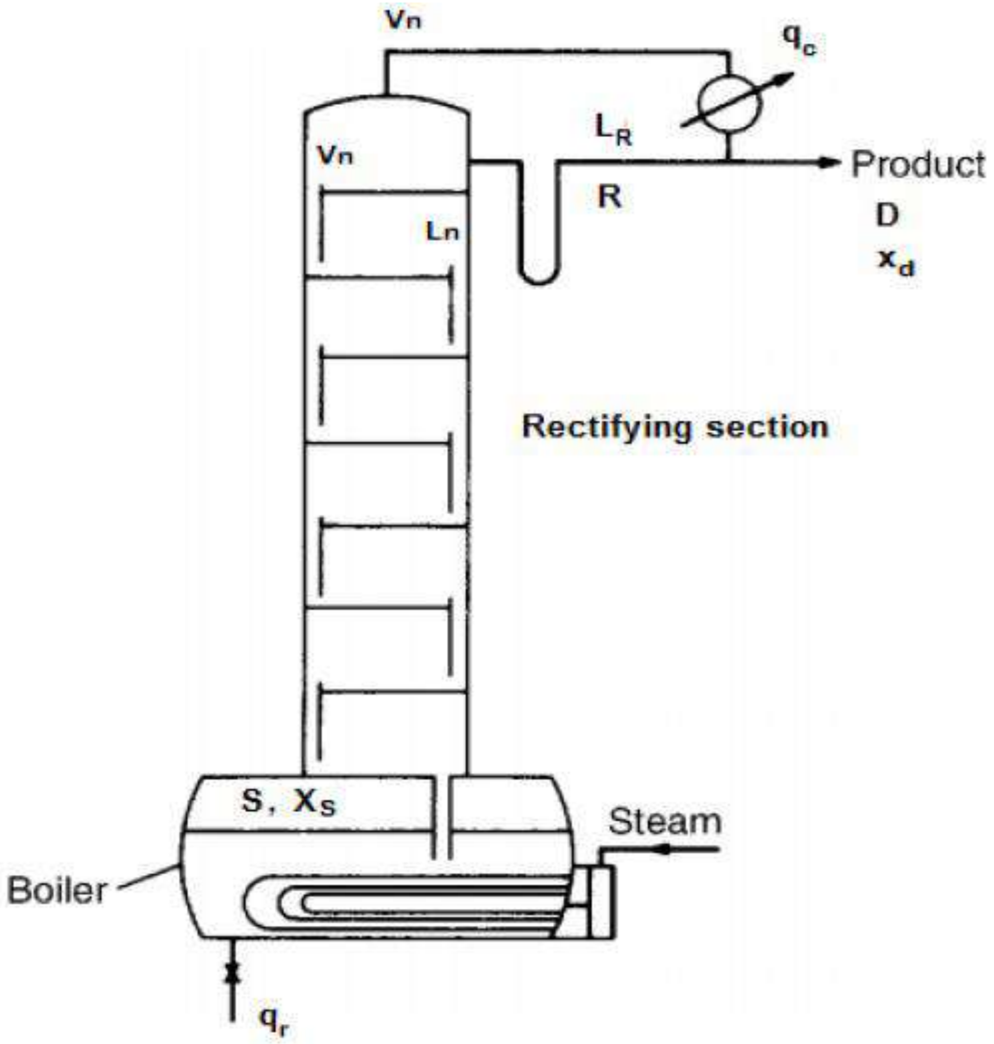


Figure 11.33. Column for batch distillation

Distillation

1. Operation at constant product composition (x_d)

The batch distillation column operates with a rectifying section only. Therefore, there is one operating line equation represents this section which is the rectifying operating line equation. There are some symbols are used in this type of distillation:

S_1: is the number of moles of feed in the still initially.

X_{S1}: is the mole fraction of more volatile component in the feed

S_2: is the number of moles of liquid mixture in the still after concentrated (distillation).

X_{S2}: is the mole fraction of more volatile component in the still after concentrated (distillation).

Over all material balance

$$S_1 - S_2 = D$$

Distillation

Over all material balances on more volatile component:

$$S_1\left(x_{s_1}\right) - S_2\left(x_{s_2}\right) = D\left(x_d\right)$$

$$S_1\left(x_{s_1}\right) - (S_1 - D)\left(x_{s_2}\right) = D\left(x_d\right)$$

$$S_1\left(x_{s_1}\right) - S_1\left(x_{s_2}\right) = D\left(x_d\right) - D\left(x_{s_2}\right)$$

$$D = S_1\left[\frac{x_{s_1} - x_{s_2}}{x_d - x_{s_2}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The heat to be supplied in the boiler to provide this reflux during the total distillation Q_R is given by:

$$Q_R = \int_0^{L_n} \lambda \; dL_n = \lambda \int_{R=R_1}^{R=R_2} R\, dD = \lambda \,.\ \text{Area} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Distillation

Where: λ is the latent heat per mole.

Q_R is the amount of heat required in the reboiler to vaporize (L_n).

$$Q_T = Q_R + \lambda \quad \ldots\ldots (3)$$

Equation (2) may be integrated graphically if the relation between **R** and ***D*** is known. For any desired value of **R**, $\mathbf{x_s}$ may be obtained by drawing the operating line, and marking off the steps corresponding to the given number of stages. The amount of product ***D*** is then obtained from equation (1) and, if the corresponding values of **R** and ***D*** are plotted, graphical integration will give the value of **R d*D***.

Distillation

Example (1): A mixture of ethyl alcohol and water with 0.55 mole fraction of alcohol is distilled to give a top product of 0.75 mole fraction of alcohol. The column has four ideal plates and the distillation is stopped when the reflux ratio has to be increased beyond 4.0. What is the amount of distillate obtained, and the heat required per kmol of product?

The equilibrium data are given as:

Mole fraction of ethyl alcohol in liquid	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mole fraction of ethyl alcohol in vapour	0	0.42	0.52	0.58	0.61	0.65	0.7	0.75	0.81	0.9

Distillation

Solution:

The distillation with constant (x_d) and variable (R).

$S_1 = 100$ kmol

N = 4 plates = 5 stages

$x_{s_1} = 0.55$, $x_{s_2} = ?$, D = ? , $x_d = 0.75$

$R_2 = 4$, $R_1 = ?$

Values of x_s are found as shown in Figure 11.35 for the two values of R of 0.85 and 4. The amount of product is then found from equation (1). Thus, for R = 4:

$$D = S_1 \left[\frac{x_{s_1} - x_{s_2}}{x_d - x_{s_2}}\right] = 100 \left[\frac{0.55 - 0.05}{0.75 - 0.05}\right] = 41.4 \text{ kmol}$$

Distillation

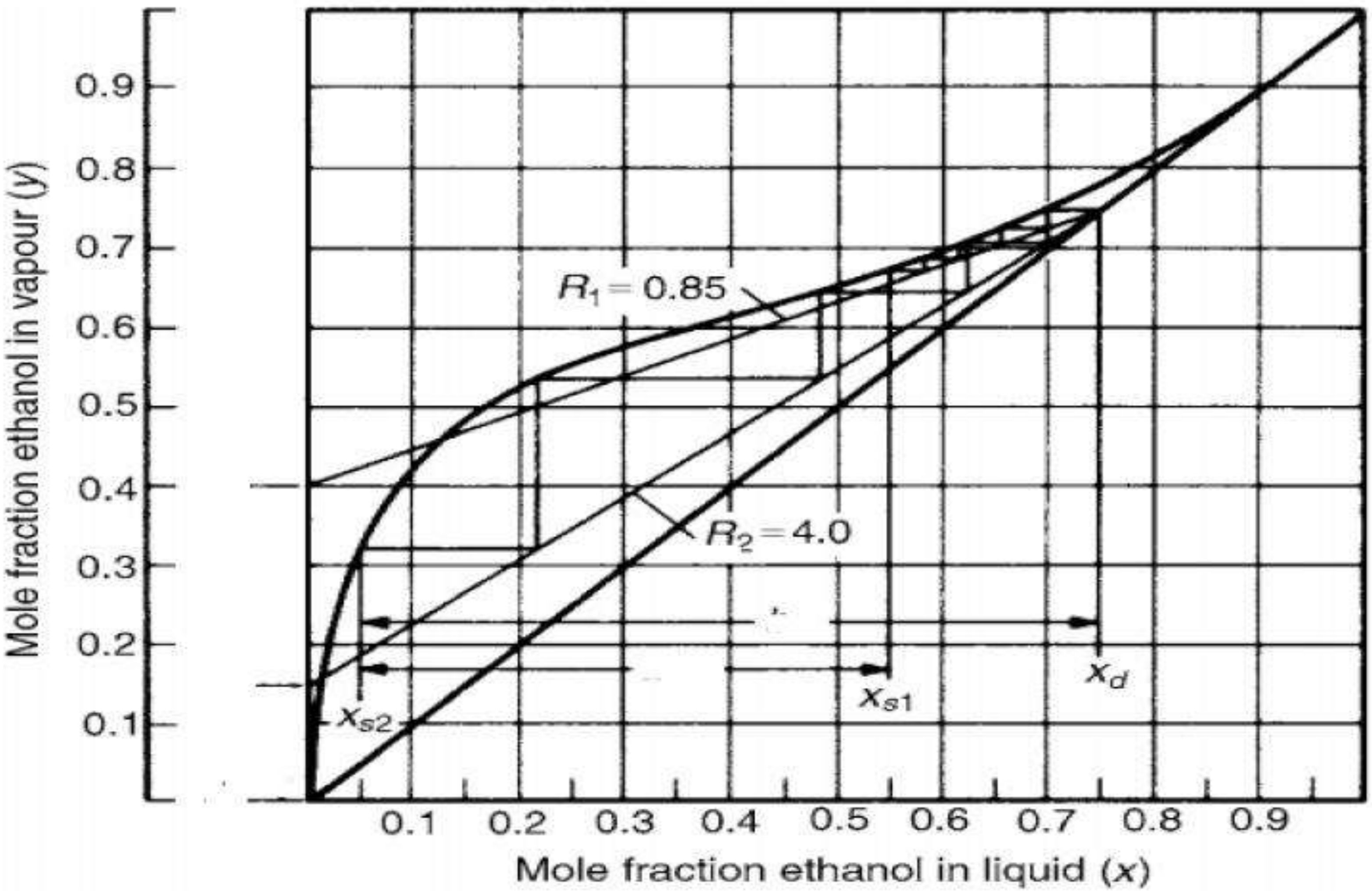
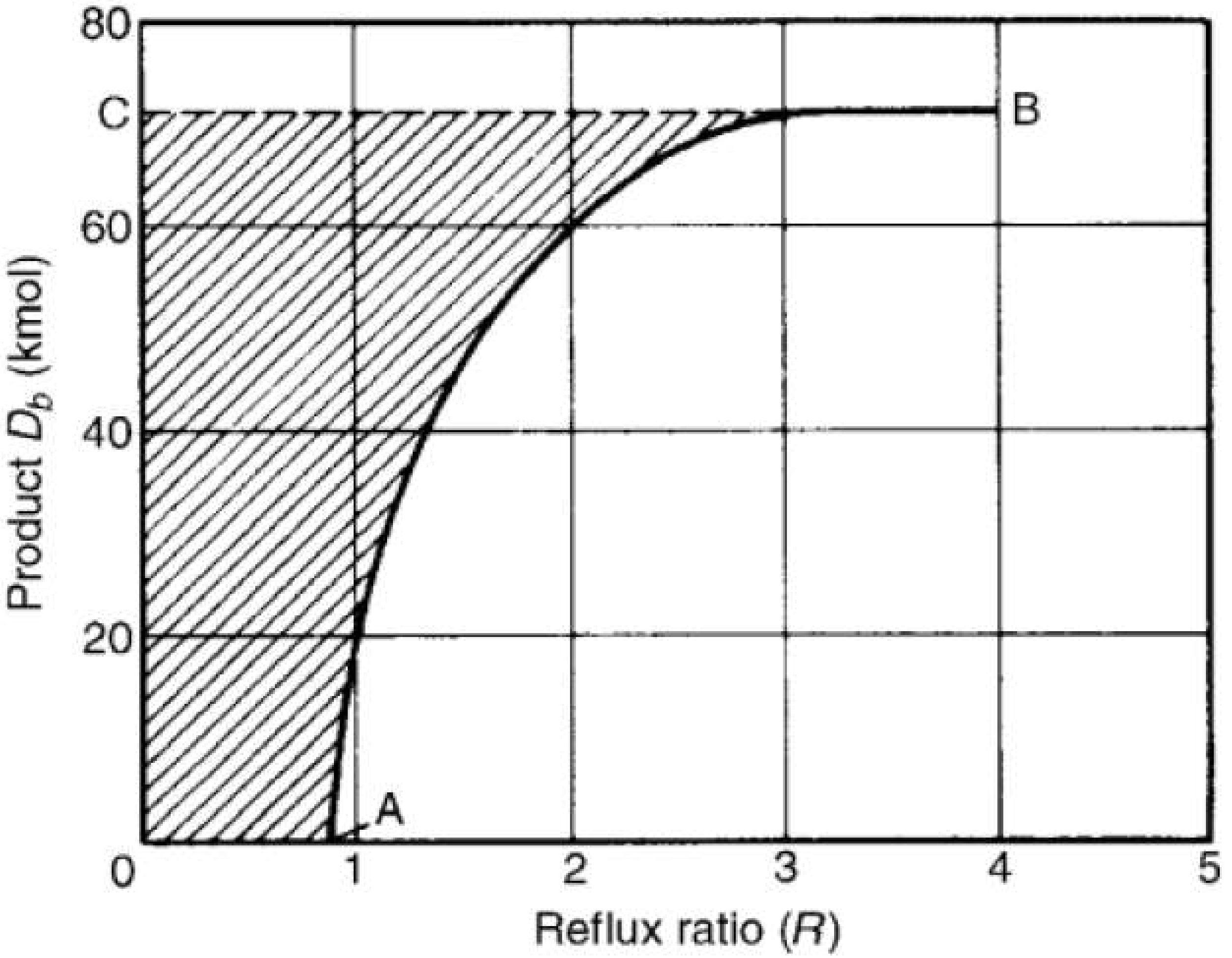


Figure 11.35. Batch distillation–constant product composition

Distillation

values of D found in this way are:

R	x_s	D
0.85	0.55	0
1.0	0.50	20.0
1.5	0.37	47.4
2.0	0.20	63.8
3.0	0.075	70.5
4.0	0.05	71.4



The area under the curve = 96 kmol

$$Q_R = \lambda \int_{R=R_1}^{R=R_2} R\, dD \;=\; \lambda .\ \text{Area} = (4000)(96) = 380000\ \text{kJ} = 380\ \text{MJ}$$

The heat to be supplied to provide the reflux per kmol of product is then $(380/71.4) = 5.32$ MJ and the total heat is $(5.32 + 4.0) = 9.32$ MJ/kmol product.

2. Operation at constant reflux ratio (R)

If the same column is operated at a constant reflux ratio **(R)**, the concentration of the more volatile component in the top product will continuously fall. Over a small interval of time $\mathbf{d}t$, the top-product composition with respect to the more volatile component will change from x_d to $x_d + \mathbf{d}x_d$, where $\mathbf{d}x_d$ is negative for the more volatile component. If in this time the amount of product obtained is $\mathbf{d}D$, then a material balance on the more volatile component gives:

Distillation

Over all material balance

$$D = S_1 - S_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Over all material balances on more volatile component:

$$S_1\,(x_{s_1}) - S_2\,(x_{s_2}) = D\,(x_d)$$

$$(x_d) = \frac{S_1\,(x_{s_1}) - S_2\,(x_{s_2})}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

More volatile component removed in product $= \mathrm{d}D\left[x_d + \frac{\mathrm{d}x_d}{2}\right]$

which, neglecting second − order terms, gives: $= x_d\,\mathrm{d}D$

and: $x_d\,\mathrm{d}D = -\,\mathrm{d}\,(Sx_s)$

but: $\mathrm{d}D = -\,\mathrm{d}\,S$

Distillation

and hence: $$-x_d\, dS = -S\, d\,x_s - x_s\, dS$$

and: $$S\, d\,x_s = dS\,(x_d - x_s)$$

Thus: $$\int_{S_1}^{S_2} \frac{d\,S}{S} = \int_{x_{s1}}^{x_{s2}} \frac{d\,x_s}{(x_d - x_s)}$$

$$\ln\frac{S_1}{S_2} = \int_{x_{s2}}^{x_{s1}} \frac{d\,x_s}{(x_d - x_s)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The heat to be supplied to provide the reflux Q_R is given by:

$$Q_R = \lambda\, R\, D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$Q_T = Q_R + \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Distillation

Example (2): If the same batch as in Example (1) is distilled with a constant reflux ratio of R = 2.1, what will be the heat required and the average composition of the distillate if the distillation is stopped when the composition in the still has fallen to 0.105 mole fraction of ethanol?

Solution:

The initial composition of the top product will be 0.78, as shown in Figure 11.37, and the final composition will be 0.74. Values of x_d, x_s, $x_d - x_s$ and of $1/(x_d - x_s)$ for various values of x_s and a constant reflux ratio are:

Distillation

x_s	x_d	$x_d - x_s$	$1/(x_d - x_s)$
0.550	0.780	0.230	4.35
0.500	0.775	0.275	3.65
0.425	0.770	0.345	2.90
0.310	0.760	0.450	2.22
0.225	0.750	0.525	1.91
0.105	0.740	0.635	1.58

Values of x_s and $1/(x_d - x_s)$ are plotted in Figure 11.38 from which $\int_{0.105}^{0.55} (\mathrm{d}x_s/(x_d - x_s)) = 1.1$.
From equation 11.103: $\ln(S_1/S_2) = 1.1$ and $(S_1/S_2) = 3.0$.
Product obtained, $D_b = S_1 - S_2 = (100 - 100/3) = 66.7$ kmol.
Amount of ethanol in product $= x_1 S_1 - x_2 S_2$

$$= (0.55 \times 100) - (0.105 \times 33.3) = 51.5 \text{ kmol}$$

Thus: average composition of product $= (51.5/66.7) =$ 0.77 mole fraction ethanol.

The heat required to provide the reflux $= (4000 \times 2.1 \times 66.7) = 560{,}380$ kJ.
Heat required to provide reflux per kmol of product $= (560{,}380/66.7) =$ 8400 kJ.

Thus in Example 2 the total heat required per kmol of product is $(5320 + 4000) = 9320$ kJ and at constant reflux ratio (Example 1) it is $(8400 + 4000) = 12{,}400$ kJ, although the average quality of product is 0.77 for the second case and only 0.75 for the first.

Distillation

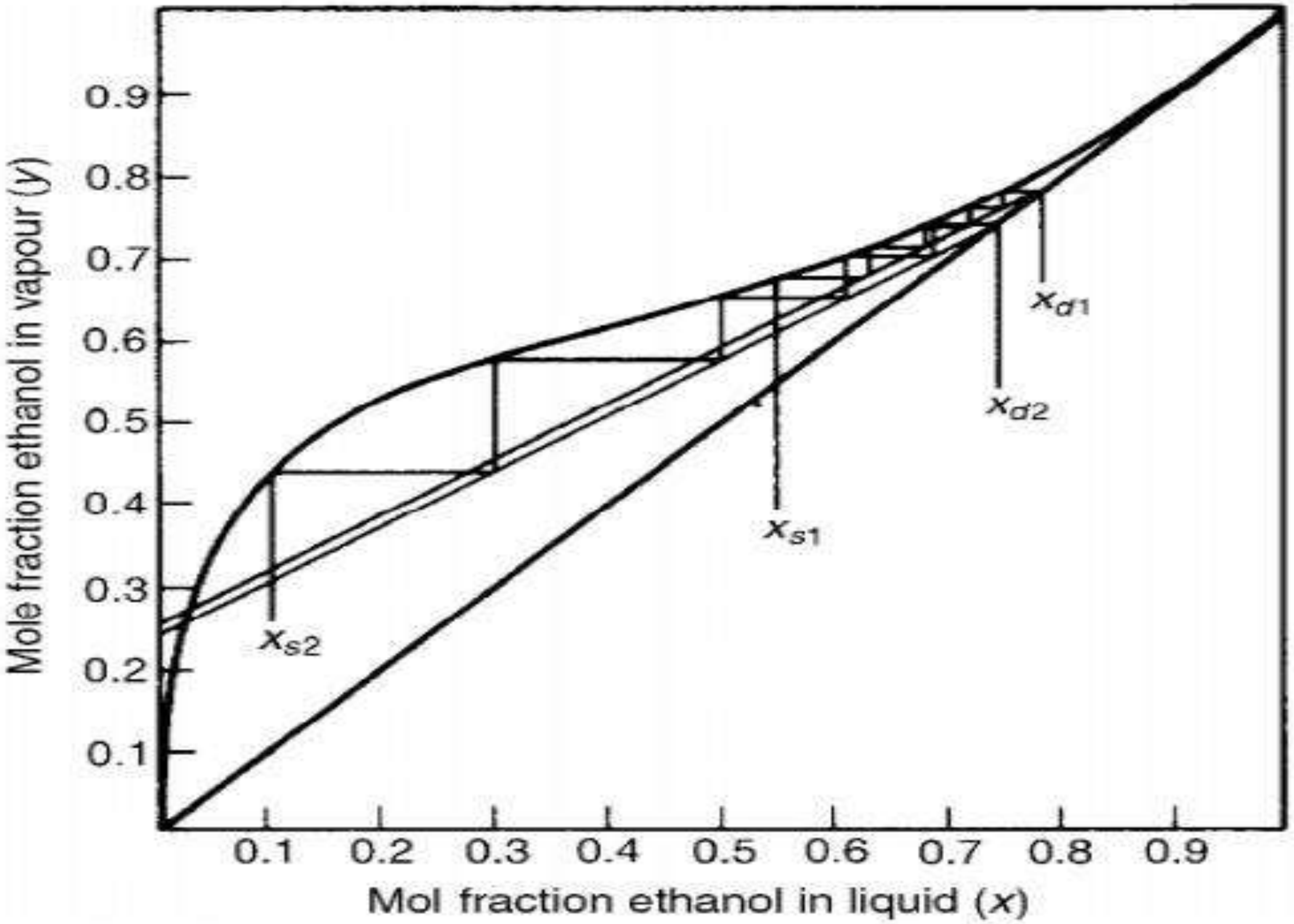


Figure 11.37. Batch distillation–constant reflux ratio (Example 11.13)

Distillation

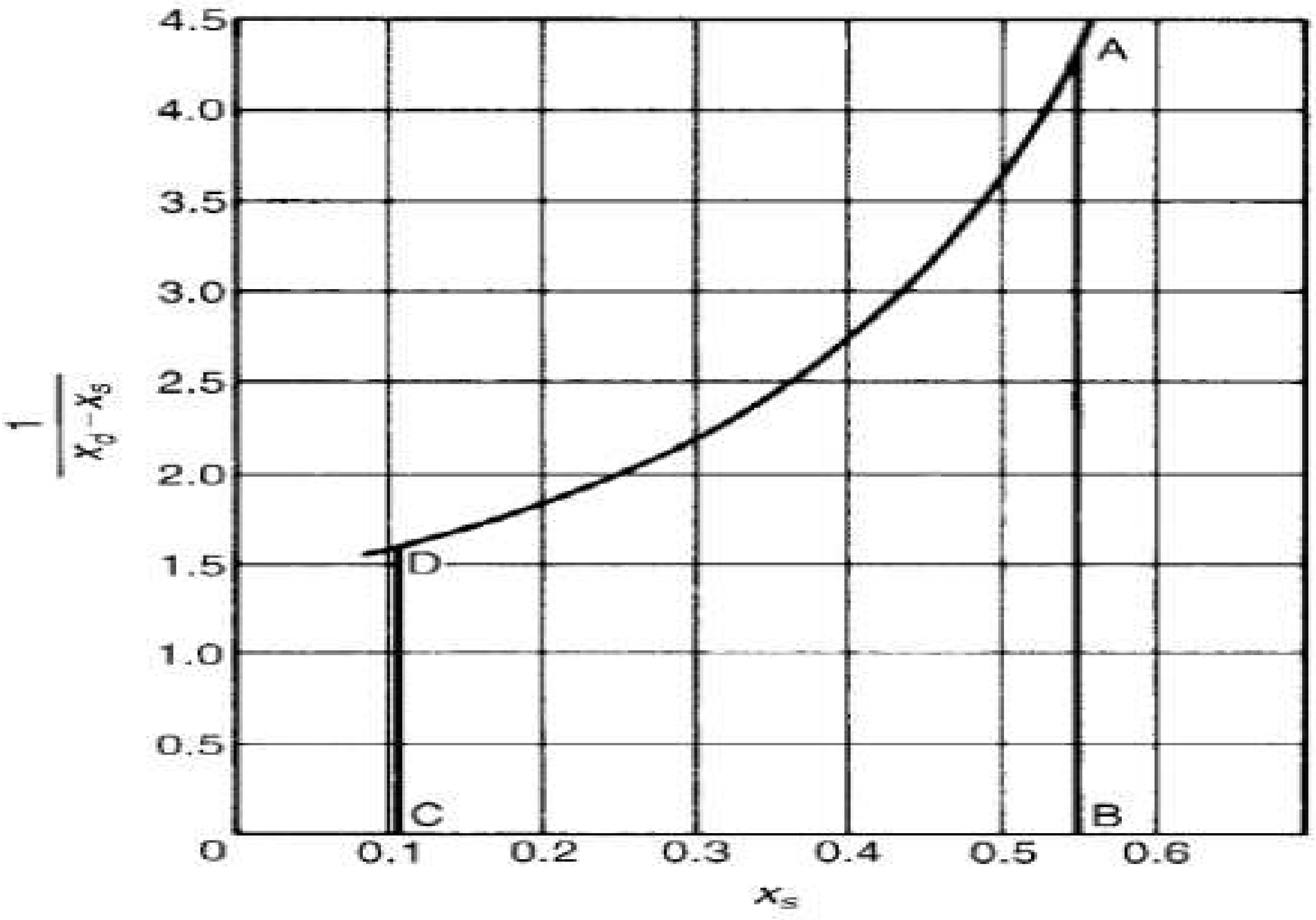


Figure 11.38. Graphical integration for Example 11.13

Absorption of Gases
((Gas – Liquid Separation))

Presenter by:

Dr. Mohammed Qader

Absorption

Absorption of Gases:- In absorption (also called gas absorption, gas scrubbing, and gas washing), a gas mixture is contacted with a liquid (the absorbent or solvent) to selectively dissolve one or more components by mass transfer from the gas to the liquid. The components transferred to the liquid are referred to as solute or absorbate.

Absorption

Absorption is used to separate gas mixture; remove impurities, contaminants, pollutants, or catalyst poisons from gas; or recovery valuable chemicals. Thus, the species of interest in the gas mixture may be all components, only the component(s) not transferred, or only the component(s) transferred. The opposite of absorption is stripping (also called desorption), wherein a liquid mixture is contacted with gas to selectively remove components by mass transfer from the liquid to the gas phase.

Absorption

There are two types of absorption processes:

1. Physical process (e.g. absorption of acetone from acetone – air mixture by water).

2. Chemical process, sometimes called chemi-sorption (e.g. absorption of nitrogen oxides by water to produce nitric acid.

Equipment:

Absorption and stripping are conducted in tray towers (plate column), packed column, spray tower, bubble column, and centrifugal contactors. The first two types of these equipment will be considered in our course for this year.

Absorption

1. Tray tower:

A tray tower is a vertical, cylindrical pressure vessel in which gas and liquid, which flow counter currently, are contacted on a series of metal trays or plates. Liquid flows across any tray over an outlet weir, and into a down comer, which takes the liquid by gravity to the tray below. The gas flows upward through opening in each tray, bubbling through the liquid on the other tray. A schematic diagram for the flow patterns inside the tray column is shown below.

Absorption

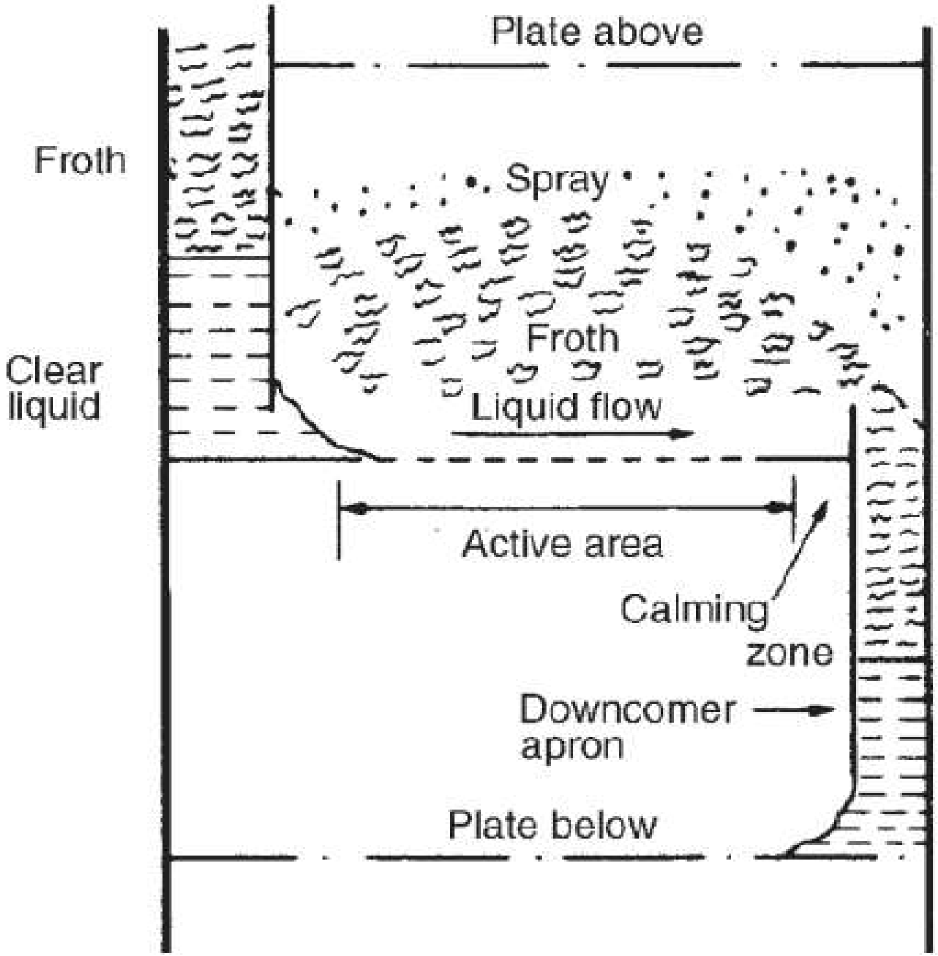


Figure : Typical cross-flow plate (sieve)

Absorption

2. Packed tower:

The packed column is a vertical, cylindrical pressure vessel containing one or more section of packing material over who's the liquid flows down wards by gravity as a film or as droplets between packing elements. Gas flows upwards through the wetted packing contacting the liquid. The sections of packing are contained between a lower gas – injection support plate, which holds the packing, and an upper grid or mish hold – down plate, which prevent packing movement. A liquid distributor, placed above the hold – down plate, ensures uniform distribution of liquid as it enters the packing section.

Absorption

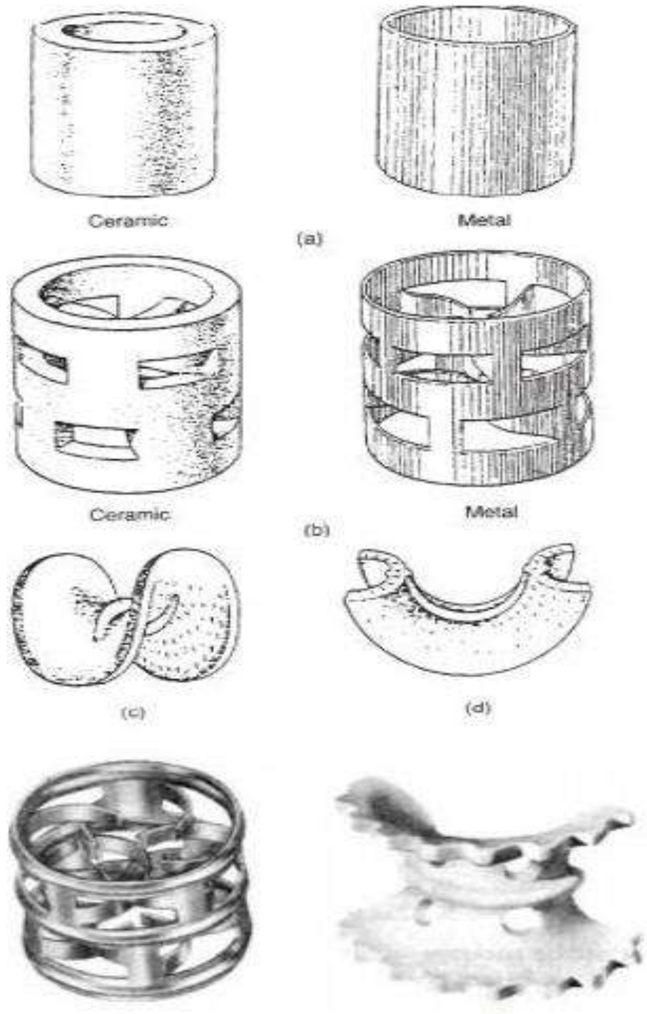


Figure: Types of packing (a) Raschig rings (b) Pall rings (c) Berl saddle ceramic (d) Intalox saddle ceramic (e) Metal Hypac (f) Ceramic, super Intalox.

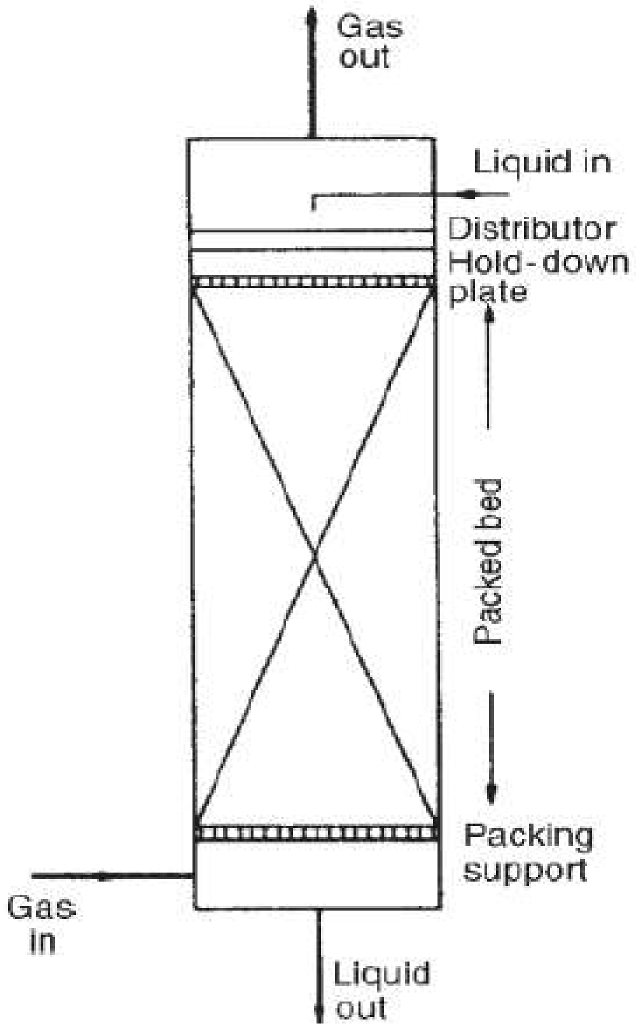


Figure: Packing absorber column.

Absorption

General Design Consideration:

Design or analysis of an absorber (or stripper) requires consideration of a number of factors, including:

1. Entering gas (liquid) flow rate, composition, temperature, and pressure.

2. Design degree of recovery (R) of one or more solutes.

3. Choice absorbent (solvent) agent.

4. Operating pressure and temperature and allowable pressure drop.

5. Minimum absorbent (solvent) agent flow rate and actual solvent flow rate as a multiple of the minimum rate needed to make the separation.

6. Number of equilibrium stages.

7. Heat effects and need for cooling (heating).

8. Type of absorber (stripper) equipment.

9. Height of absorber (stripper) column.

10. Diameter of absorber (stripper) column.

Absorption

The ideal absorbent (solvent) should have:

a. High solubility for the solute(s) to minimize the need for absorbent (solvent).

b. A low volatility to reduce the loss of absorbent (solvent) and facilitate separation of absorbent (solvent) from solute(s).

c. Be stable to maximize absorbent (solvent) life and reduce absorbent makeup requirement.

d. Be non – corrosive to permit use of common material of construction.

e. Have a low viscosity to provide low pressure drop and high mass and heat transfer rates.

f. Be non – foaming when contacted with gas so as to make it unnecessary.

g. Be non – toxic and non – flammable to facilitate its safe use.

h. Be available, if possible.

The most widely absorbent (solvent) used are water, hydrocarbon oils, and aqueous solutions of acids and bases. While the most common stripping agents used are water vapor, air, inert gases, and hydrocarbon gases.

Absorption

Equilibrium Relations Between Gas and Liquid Phases:

The equilibrium of any gas-liquid system can be expressed as:

Non-ideal system (Henry's law):

$P_A = H\, x_A$ divided by (P_T)

$$\frac{P_A}{P_T} = \frac{H}{P_T} x_A$$

$$\mathbf{y_A = m\, x_A}$$

Ideal system (Raoult's law):

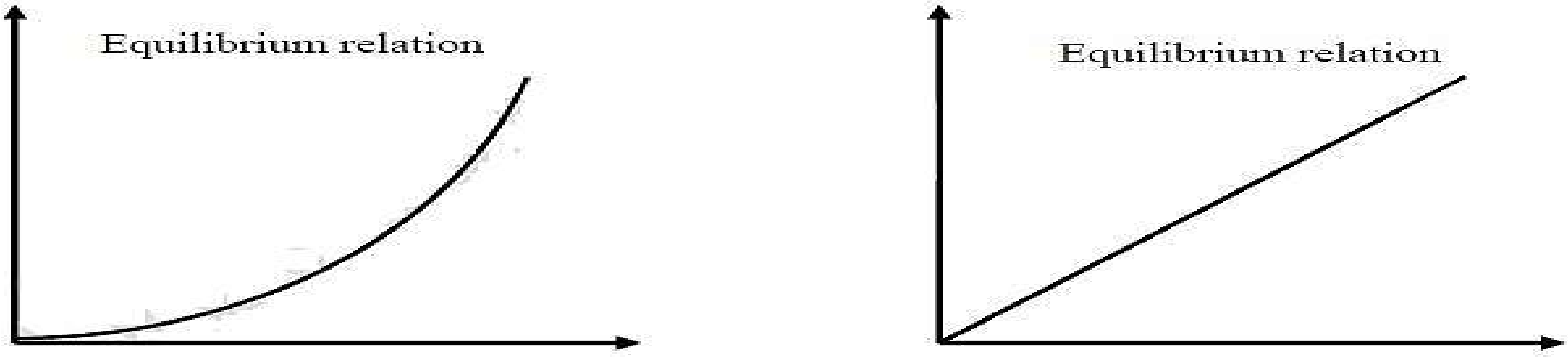
$P_A = P_A^0\, x_A$ divided by (P_T)

$$\frac{P_A}{P_T} = \frac{P_A^0}{P_T} x_A$$

$$\mathbf{y_A = m\, x_A}$$

Absorption

* في عمليات الامتصاص من المهم جدا معرفة طبيعة علاقة التعادل (وهي العلاقة بين تركيز المذاب (A) في الغاز (Y_A) مع تركيز المذاب (A) في السائل (X_A)) . فقد تكون علاقة التعادل بين (X_A) و (Y_A) علاقة خطية أو علاقة غير خطية اعتمادا على طبيعة المواد وتركيز المذاب.



Where:

X_A : is the mole ratio of solute in liquid phase (A/C).

Y_A : is the mole ratio of solute in gas phase (A/B).

Absorption

Notes:

The equilibrium relation is the ratio between the *mole ratio* of solute in gas phase (Y_A) and the *mole ratio* of solute in liquid phase (X_A). The equilibrium relation may be linear or no linear.

* إذا كانت علاقة التعادل خطية فتعطى بالشكل التالي ($Y_A = m\,X_A$).

* أما إذا أعطيت علاقة التعادل بشكل بيانات كما في أدناه:

X_A	-	-	-	-	-	-
Y_A	-	-	-	-	-	-

ففي هذه الحالة لمعرفة طبيعة علاقة التعادل فيتم رسم البيانات أولا فإذا كان الرسم بين X_A و Y_A هو خط مستقيم عند ذاك سوف يتم اخذ الميل من الرسم فقط وتكوين علاقة التعادل ($Y_A = m\,X_A$).

أما إذا كان الرسم الناتج بين X_A و Y_A بشكل منحني فعند ذاك سيكون الحل بالرسم.

* في بعض الأحيان تعطى علاقة التعادل بين الضغط الجزيئي (P_A) والنسبة المولية (x_A) كما في علاقة راؤول أو هنري ففي هذه الحالة يجب تحويلها الى علاقة بين (X_A , Y_A).

Absorption

The relation between the mole fraction and mole ratio:

$$Y_A = \frac{y_A}{1 - y_A}$$

and

$$X_A = \frac{x_A}{1 - x_A}$$

Where:

x_A and y_A : are the mole fractions of solute (A) in liquid and gas phases, respectively.

X_A and Y_A : are the mole ratio of solute (A) in liquid and gas phases, respectively.

Absorption

The relation between the mole fraction and weight fraction:

$$\mathbf{wt.\%} = \frac{(\mathbf{mol\%}) * (\mathbf{M.wt})}{\sum[(\mathbf{mol\%}) * (\mathbf{M.wt})]}$$

$$\mathbf{mol\%} = \frac{(\mathbf{wt.\%})\ /(\mathbf{M.wt})}{\sum[(\mathbf{wt.\%})/(\mathbf{M.wt})]}$$

Where:

wt. % : is the weight fraction.

mol% : is the mole fraction.

M. wt : is the molecular weight.

Absorption

Symbols used in the absorption processes:

A solute (A) in a mixture (A, B) shall be absorbed in Liquid (C), the inert gas (B) is insoluble in solvent (C). The following symbols will be used:

G : is the mole rate of the gas mixture (A + B), kmol/s.

$\mathbf{G_s}$: is the mole rate of the inert (insoluble) gas (B), kmol/s.

$\mathbf{\overline{G}}$: is the mole flux of the gas mixture (A + B), $kmol/m^2.s$.

$\mathbf{\overline{G}_s}$: is the mole flux of the inert (insoluble) gas (B), $kmol/m^2.s$.

L : is the mole rate of the liquid mixture (A + C), kmol/s.

$\mathbf{L_s}$: is the mole rate of the liquid solvent only (C), kmol/s.

$\mathbf{\overline{L}}$: is the mole flux of the liquid mixture (A + C), $kmol/m^2.s$.

$\mathbf{\overline{L}_s}$: is the mole flux of the liquid solvent only (C), $kmol/m^2.s$.

$\mathbf{x_A}$: is the mole fraction of solute (A) in liquid, (A /A+C).

$\mathbf{y_A}$: is the mole fraction of solute (A) in gas, (A /A+B).

$\mathbf{X_A}$: is the mole ratio of solute (A) in liquid, (A / C).

$\mathbf{Y_A}$: is the mole ratio of solute (A) in gas, (A / B).

Absorption

Calculation of Tower Height

The physical absorption process can be carried out in countercurrent flow process, which may be carried out in packed or tray column:

Packed Tower

$$Z = HOG * NOG$$

Where:

HOG: is the height of transfer unit (HTU) based on gas phase, and it can be calculated from the equation below:

$$HOG = \frac{\overline{G}_s}{KoG.\, a \cdot p} \quad \text{in (meter)}$$

Tray Tower

$$Z = H * N$$

Where:

H : is the distance between two trays, and it is given (0.3 - 0.7 m)

Absorption

NOG: is the number of transfer unit (NTU) based on gas phase, and it can be calculated based on equilibrium data:

- If the equilibrium data are ***linear***, then **NOG** will be calculated using a suitable equation.
- If the equilibrium data are ***non-linear***, then **NOG** will be calculated graphical method.

N : is the number of trays, and it can be calculated based on equilibrium data:

- If the equilibrium data are ***linear***, then **N** will be calculated using a suitable equation.
- If the equilibrium data are ***non-linear***, then **N** will be calculated graphical method.

Absorption

1. Packed tower:

Absorption and stripping are frequently conducted in packed columns, particularly when:

(1) the required column diameter is less than 0.6 m.

(2) the pressure drop must be low, as for a vacuum service.

(3) corrosion consideration favor the use of ceramic or polymeric material.

(4) low liquid holdup is desirable.

The gas liquid contact in a packed bed column is continuous, not stage-wise, as in a plate column. The liquid flows down the column over the packing surface and the gas or vapour, counter-currently, up the column. In some gas-absorption columns co-current flow is used. The performance of a packed column is very dependent on the maintenance of good liquid and gas distribution throughout the packed bed, and this is an important consideration in packed-column design.

Absorption

Calculations of the packing height based on gas phase:

Overall material balance on the solute (A) over an element (∂z) based on gas phase:

$$G_S\, dY = L_S\, dX = N_A \cdot A = K_oG . a(P_G - P^*) . S\, \partial z$$

$$N_A = G_s\, Y - G_s\left(Y + \frac{dY}{dZ}\partial z\right) = (KoG)(a\; S\, \partial z)(Y - Y^*) \cdot P$$

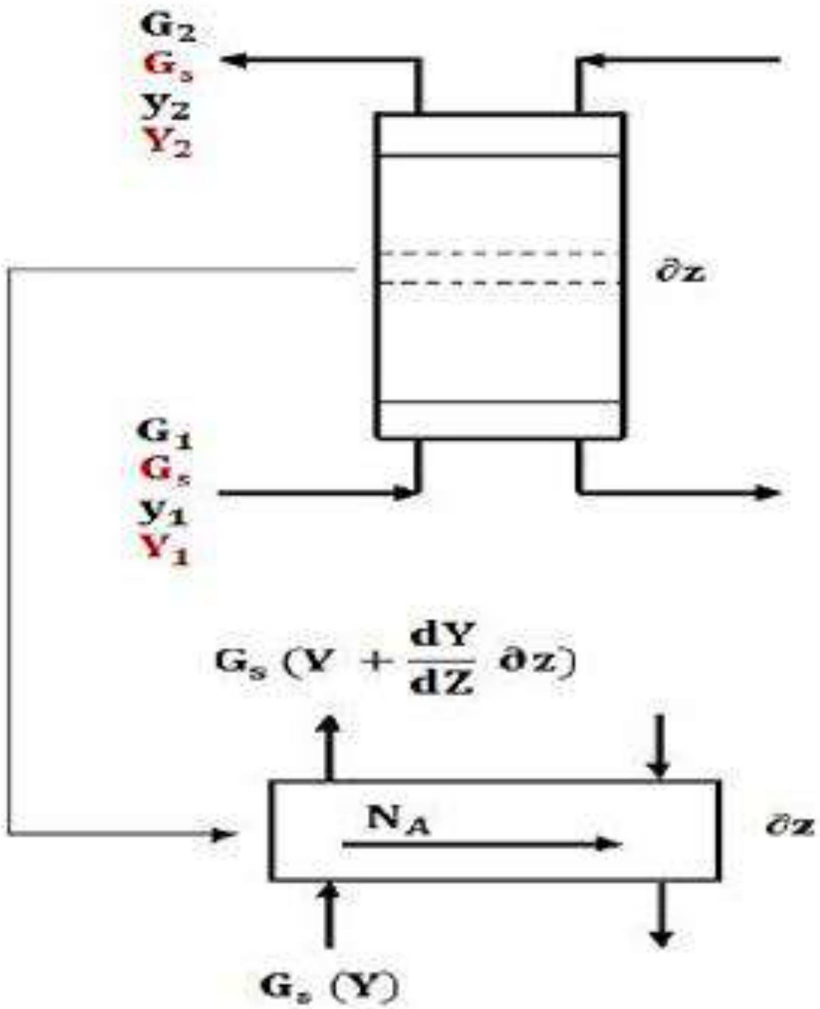
Where:

The interficial area for transfer = a dV = a S ∂z

S: is the cross-sectional area of column (m^2).

a: is the surface area of interface per unit volume of column (m^2/m^3).

$$-G_s\left(\frac{dY}{dZ}\partial z\right) = (KoG . a)(S . \partial z)(Y - Y^*) \cdot P$$



Absorption

$$G_s \frac{dY}{dZ} = -(KoG.a) \;.S\; (Y - Y^*)\,_{\cdot P}$$

$$\int_0^Z dZ = \frac{-G_s}{(KoG.a).S\,_{\cdot P}} \int_{Y_1}^{Y_2} \frac{dY}{(Y - Y^*)}$$

$$Z = \frac{(G_s/S)}{KoG.a\,_{\cdot P}} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z = \frac{\overline{G}_s}{KoG.a\,_{\cdot P}} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z = HOG * NOG = HTU * NTU$$

Where:

$HOG = \frac{\overline{G}_s}{KoG.a\,_{\cdot P}}$: heiht of transfer unit (HTU) based on gas phase, with the units of (m).

$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$: number of transfer unit (NTU) based on gas phase, without units.

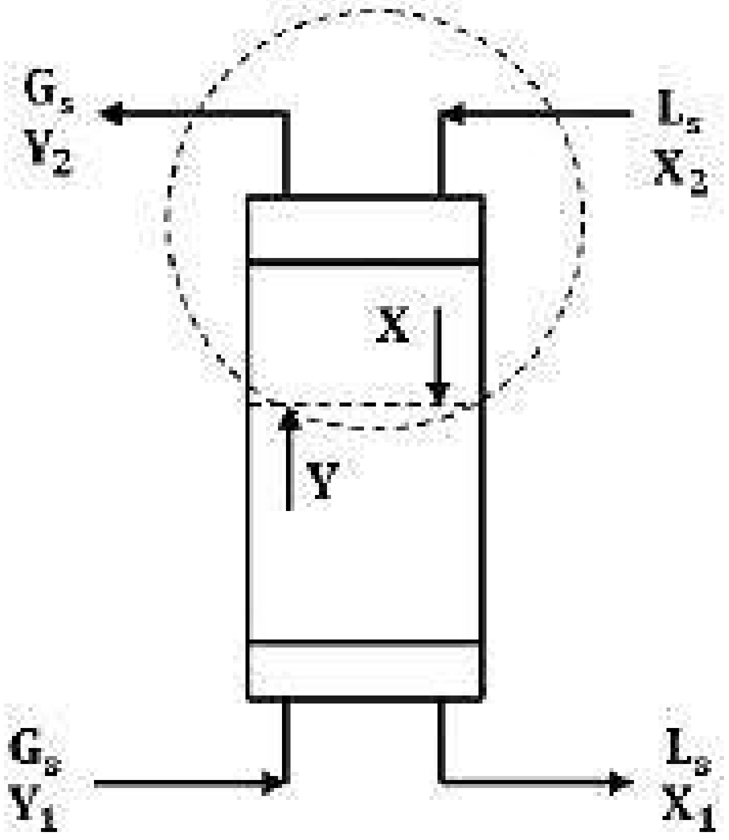
Absorption

Equation of the operating line:

Solute material balance between one end of the column and any point will give:

$$G_s\,(Y - Y_2) = L_s\,(X - X_2)$$

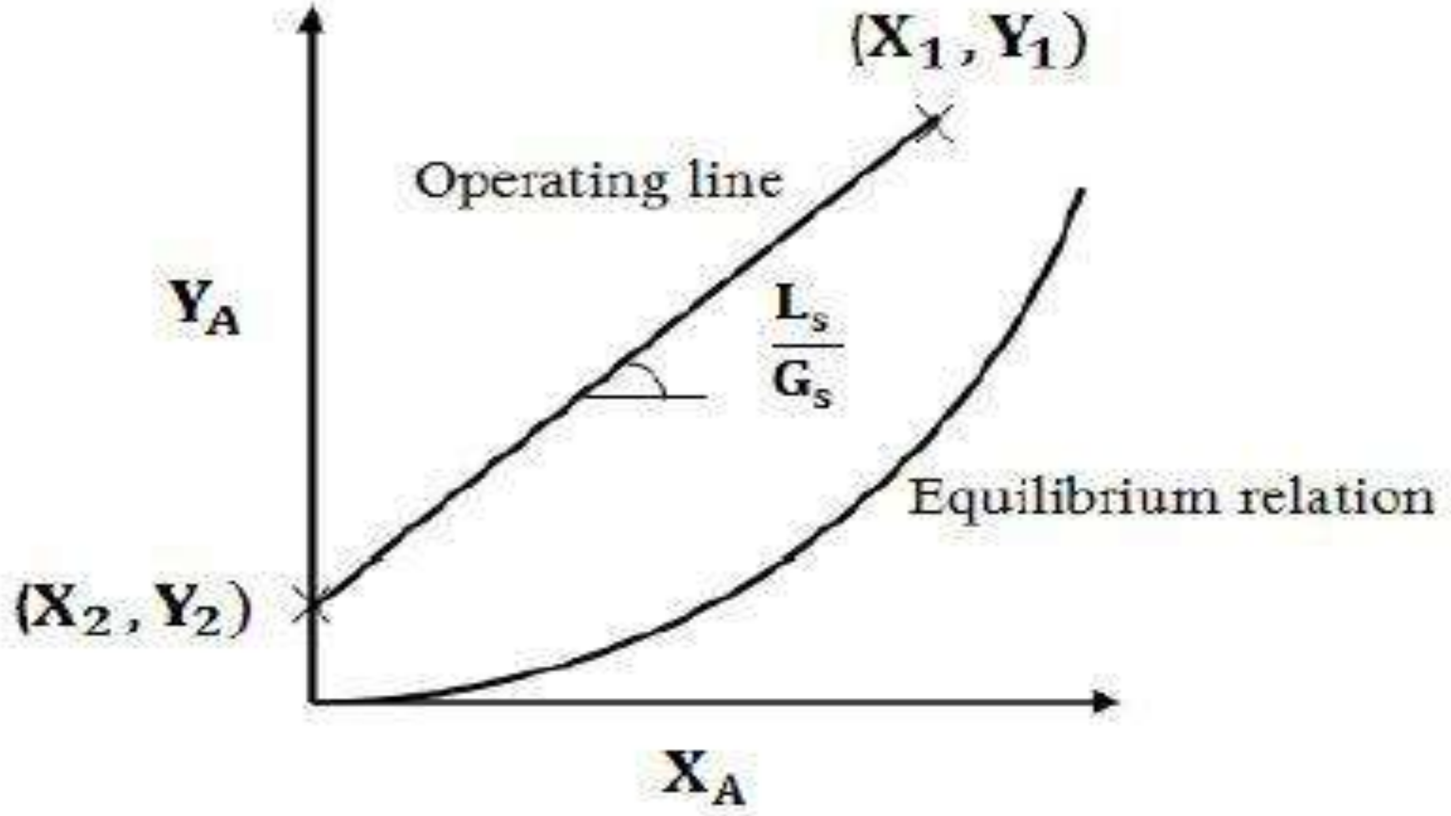
$$\boxed{Y = \frac{L_s}{G_s}(X - X_2) + Y_2}$$



* The equation of operating line is a relation between mole ratio of solute in gas phase (Y) and the mole ratio of solute in liquid phase (X).

Absorption

* The operating line can be draw from two points (X_1, Y_1) and (X_2, Y_2), or from its slope $(\frac{L_S}{G_S})$ and one of the two points.



Absorption

Calculation of Number of Transfer Unit (NOG):

A. For Linear Equilibrium Relationship ($Y^* = m X$):

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)} \qquad \text{... (1)}$$

$$Y^* = m X \qquad \text{... (2)}$$

$$G_s (Y - Y_2) = L_s (X - X_2) \qquad \text{... (3)}$$

$$\Longrightarrow \quad X = \frac{G_s}{L_s}(Y - Y_2) + X_2$$

For pure liquid solvent used then, $X_2 = 0$

$$X = \frac{G_s}{L_s}(Y - Y_2) \qquad \text{... (4)}$$

Substitution Eq.(4) into Eq.(2) to get:

$$Y^* = \frac{m\, G_s}{L_s}(Y - Y_2) \qquad \text{... (5)}$$

Absorption

Substitution Eq.(5) into Eq.(1) to get:

$$\mathbf{NOG} = \int_{Y_2}^{Y_1} \frac{dY}{\left(Y - \frac{m\,G_s}{L_s}(Y - Y_2)\right)}$$

Let: $$\frac{m\,G_s}{L_s} = \phi = \frac{\text{Slope of equilibrium line}}{\text{Slope of operating line}} = \frac{m}{L_s/G_s} < 1.0$$

$$\mathbf{NOG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - \phi Y + \phi Y_2}$$

$$\mathbf{NOG} = \int_{Y_2}^{Y_1} \frac{dY}{(1-\phi)Y + \phi Y_2}$$

$$\mathbf{NOG} = \frac{1}{(1-\phi)} \ln\left[\frac{(1-\phi)Y_1 + \phi Y_2}{(1-\phi)Y_2 + \phi Y_2}\right]$$

Absorption

B. For Non-linear Equilibrium Relationship:

In this case the integration [$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y-Y^*)}$] will be solved using graphical method or numerical method (Simpson rule) following steps below:

1. Draw the given equilibrium data.
2. Draw the operating line, from two points (X_1, Y_1) and (X_2, Y_2) or one point and slope of ($\frac{L_s}{G_s}$).
3. Create the table below by calculated (Y^*) from the plot as below:

Absorption

Y Assume points between (Y_1 - Y_2)	**Y*** Calculated from plot	$\frac{\mathbf{1}}{(\mathbf{Y - Y^*})}$
Y_1	- calculated	$\sqrt{}$ = f_0
- (assumed)	- calculated	$\sqrt{}$ = f_1
- (assumed)	- calculated	$\sqrt{}$ = f_2
- (assumed)	- calculated	$\sqrt{}$= f_3
Y_2	- calculated	$\sqrt{}$ = f_n

Absorption

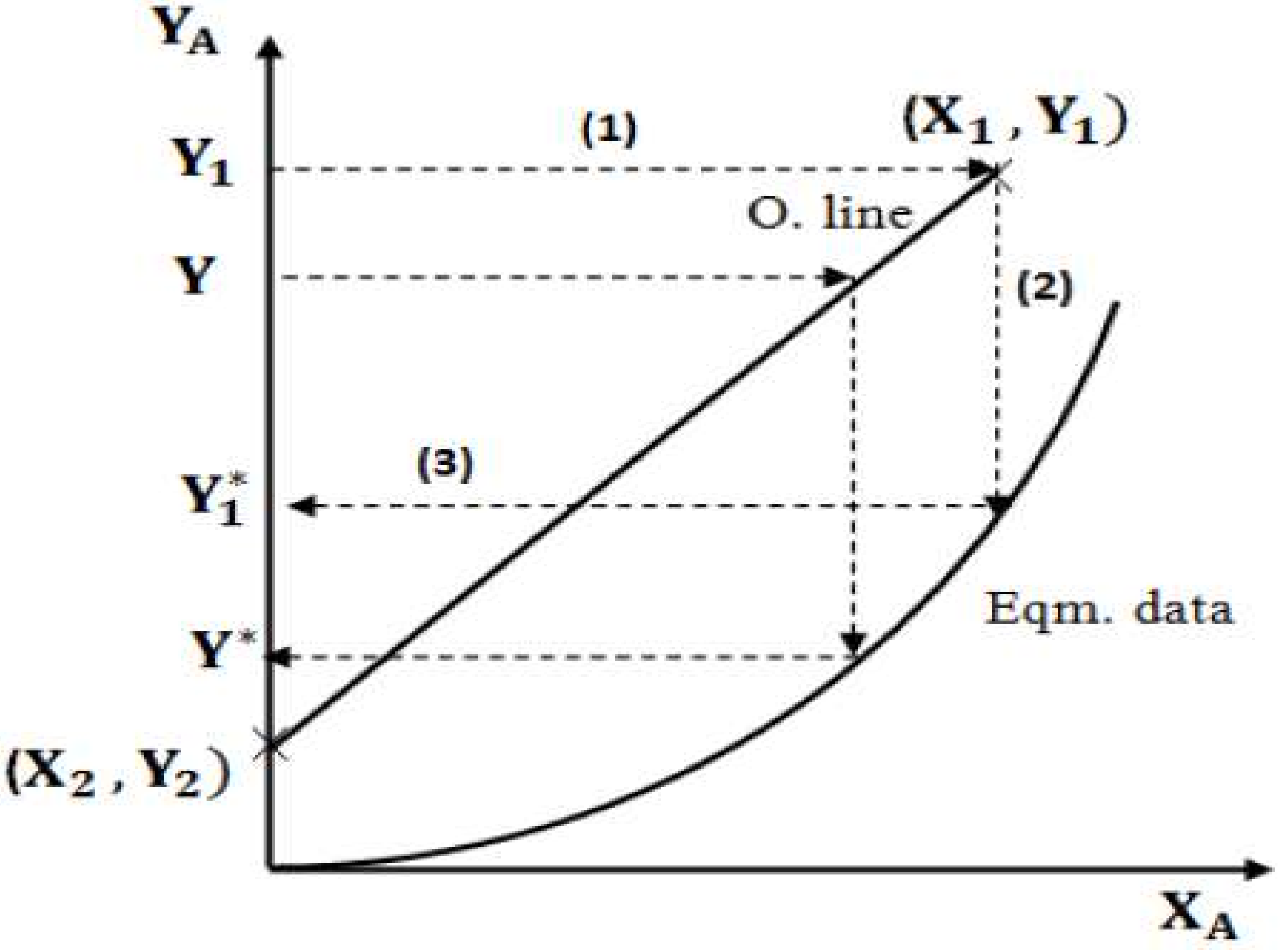
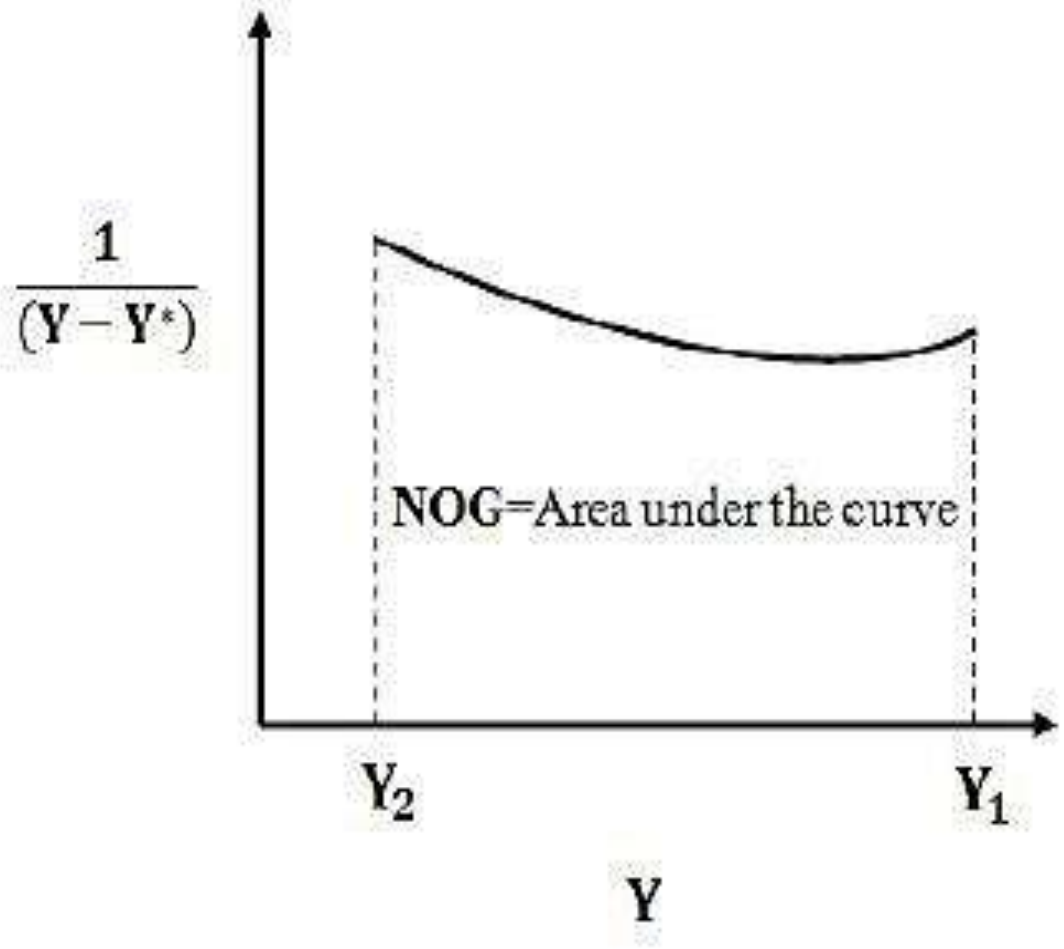


Figure: Calculation of (Y^*) for packed column.

Absorption

4. To calculate **NOG** we draw $[\frac{1}{(Y-Y^*)}]$ Vs. [Y] to find the area under the curve:



Where:

NOG = Area under the curve

Absorption

Simpson rule for calculation of NOG:

NOG = Area under the curve

$$NOG = \frac{h}{3}\left[f_0 + f_n + 2\sum f_{even} + 4\sum f_{odd}\right]$$

Where:

$$h = \frac{Y_1 - Y_2}{n} \quad , \quad n = 2, 4, 6, 8, \dots \dots . . etc.$$

Notes:

* If the entering solute concentration is dilute (Y < 5%), then:

$$Y_A = y_A \quad , \quad X_A = x_A \quad , \quad G_s = G \quad , \quad L_s = L$$

* If the tower type is not mention in the problem we can take it as a packed tower.

Absorption

Example (1):

Ammonia is to be removed from a 10 percent ammonia–air mixture by countercurrent scrubbing with water in a packed tower at 293 K so that 99 percent of the ammonia is removed when working at a total pressure of 101.3 kN/m^2. If the gas rate is 0.95 kg/m^2.s of tower cross-section and the liquid rate is 0.65 kg/m^2. s, find the necessary height of the tower if the absorption coefficient KoG.a = 0.0008 kmol/ m^3.s. kPa., The equilibrium data are: Y^*=0.8 X .

Absorption

Solution:

$$y_2 = (1 - recovery)\, y_1 = (1 - 0.99)(0.1) = 0.001$$

Convert mole fraction to mole ratio:

$$\mathbf{Y_1} = \frac{y_1}{1 - y_1} = \frac{0.1}{1 - 0.1} = 0.11$$

$$\mathbf{Y_2} = \frac{y_2}{1 - y_2} = \frac{0.001}{1 - 0.001} = 0.001$$

We can see that at low conc. (mole ratio = mole fraction):

$$\text{The gas mole flux, } \bar{\mathbf{G}} = \frac{\text{gas mass flux}}{\text{average gas molecular weight}}$$

$$= \frac{0.95}{[(0.1)(17) + (0.9)(29)]} = 0.0341\ \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

Absorption

The liquid mole flux, $\bar{\mathbf{L}} = \frac{\text{liquid mass flux}}{\text{average liquid molecular weight}}$

$$= \frac{0.65}{(18)} = 0.0361 \ \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

the mole flux of the inert gas, $\bar{\mathbf{G}}_\mathbf{s} = \bar{\mathbf{G}}(\mathbf{1} - \mathbf{y_1}) = (0.0341)(1 - 0.1) = 0.0307 \ \frac{\text{kmol}}{\text{m}^2.\text{s}}$

the mole flux of the inert liquid, $\bar{\mathbf{L}}_\mathbf{s} = \bar{\mathbf{L}}(\mathbf{1} - \mathbf{x_2}) = (0.0361)(1 - 0) = 0.0361 \ \frac{\text{kmol}}{\text{m}^2.\text{s}}$

Therefore, for pure solvent: $\bar{\mathbf{L}}_\mathbf{s} = \bar{\mathbf{L}}$

$$\text{HOG} = \frac{\bar{G}_s}{\text{KoG. a. P}_T} = \frac{0.0307}{(0.0008)(101.3)} = 0.38 \ \text{m}$$

Absorption

Since the equilibrium is linear:

$$\phi = \frac{m\,\bar{G}_s}{\bar{L}_s} = \frac{(0.8)(0.0307)}{(0.0361)} = 0.68$$

$$NOG = \frac{1}{(1-\phi)} \ln\left[\,(1-\phi)\frac{Y_1}{Y_2} + \boldsymbol{\phi}\right]$$

$$NOG = \frac{1}{(1-0.68)} \ln\left[\,(1-0.68)\frac{0.11}{0.001} + 0.68\right] = 11.19$$

Z = HOG * NOG = (0.38) (11.19) = 4.25 m

Absorption

Example (2):

Ammonia is to be removed from a 10 percent ammonia–air mixture by countercurrent absorption with water in a packed tower at 293 K. The outlet gas concentration from the top of the tower is 0.1%. The absorption tower is working at a total pressure of 101.3 kN/m^2. If the inlet gas is 0.034 kmol/m^2.s and the liquid rate is 0.036 kmol/m^2. s, find the necessary height of the tower if the absorption coefficient $\overline{KoG}$.a = 0.081 kmol/m^3.s. The equilibrium data is given by the following data:

kmol NH_3/kmol water:	0.021	0.031	0.042	0.053	0.079	0.106	0.159
Partial pressure NH_3 in gas phase (kN/m^2):	1.6	2.4	3.3	4.2	6.7	9.3	15.2

Absorption

Solution:

First of all we have to convert the equilibrium data to mole ratio:

$$mole\ fraction\ of\ NH_3\ in\ gas\ phase\ , y_{NH_3} = \frac{P_A}{P_T} = \frac{1.6}{101.3} = 0.0158$$

$$mole\ ratio\ of\ NH_3\ in\ gas\ phase\ , Y_{NH_3} = \frac{y_{NH_3}}{1 - y_{NH_3}} = \frac{0.0158}{1 - 0.0158} = 0.0160$$

The equilibrium data becomes:

X_{NH_3}	0.021	0.031	0.042	0.053	0.079	0.106	0.159
Y_{NH_3}	0.0160	0.0243	0.0337	0.0433	0.0708	0.1011	0.1765

$$HOG = \frac{\overline{G}_s}{\overline{KoG}.\,a} = \frac{0.034}{0.081} = 0.419\ m$$

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

Absorption

The equilibrium data may be not linear relation, so that the integration should be solved by plotting or by Simpson's rule as follows:

1. Draw the equilibrium data:
2. Draw the operating line from two points:
   $(\mathbf{X_1}, \mathbf{Y_1})$ and $(\mathbf{X_2}, \mathbf{Y_2})$

$$\mathbf{Y_1} = \frac{y_1}{1 - y_1} = \frac{0.1}{1 - 0.1} = 0.11$$

$$\mathbf{Y_2} = \frac{y_2}{1 - y_2} = \frac{0.001}{1 - 0.001} = 0.001$$

Overall ammonia material balance:

$$\overline{\mathbf{G}}_s\,(\mathbf{Y_1} - \mathbf{Y_2}) = \overline{\mathbf{L}}_s\,(\mathbf{X_1} - \mathbf{X_2})$$

$$\mathbf{X_1} = \frac{\overline{\mathbf{G}}_s}{\overline{\mathbf{L}}_s}(\mathbf{Y_1} - \mathbf{Y_2}) + \mathbf{X_2} = \frac{\mathbf{0.034}}{\mathbf{0.036}}(\mathbf{0.11} - \mathbf{0.001}) + \mathbf{0}$$

$$\mathbf{X_1} = 0.0935$$

Absorption

Operating line:

$(X_1, Y_1) = (0.0935, 0.11) = (9.35*10^{-2},\ 10*10^{-2})$

$(X_2, Y_2) = (0, 0.001) = (0,\ 0.1*10^{-2})$

We will solve the integration by Simpson's rule:

$$\mathbf{h} = \frac{\mathbf{Y_1 - Y_2}}{\mathbf{n}} \quad , \quad We\ choose \quad n = 4$$

$$\mathbf{h} = \frac{\mathbf{0.11 - 0.001}}{\mathbf{4}} = 0.02725$$

Calculate $\mathbf{Y^*}$ from the plot as follows:

Absorption

Y Assume points between (Y_1 - Y_2)	Y^* Calculated from plot	$\frac{1}{(Y - Y^*)}$
0.11	0.088	$45.45 = f_0$
0.08275	0.061	$45.98 = f_1$
0.05550	0.0375	$55.56 = f_2$
0.02825	0.0175	$93.02 = f_3$
0.001	0.00	$1000 = f_n$

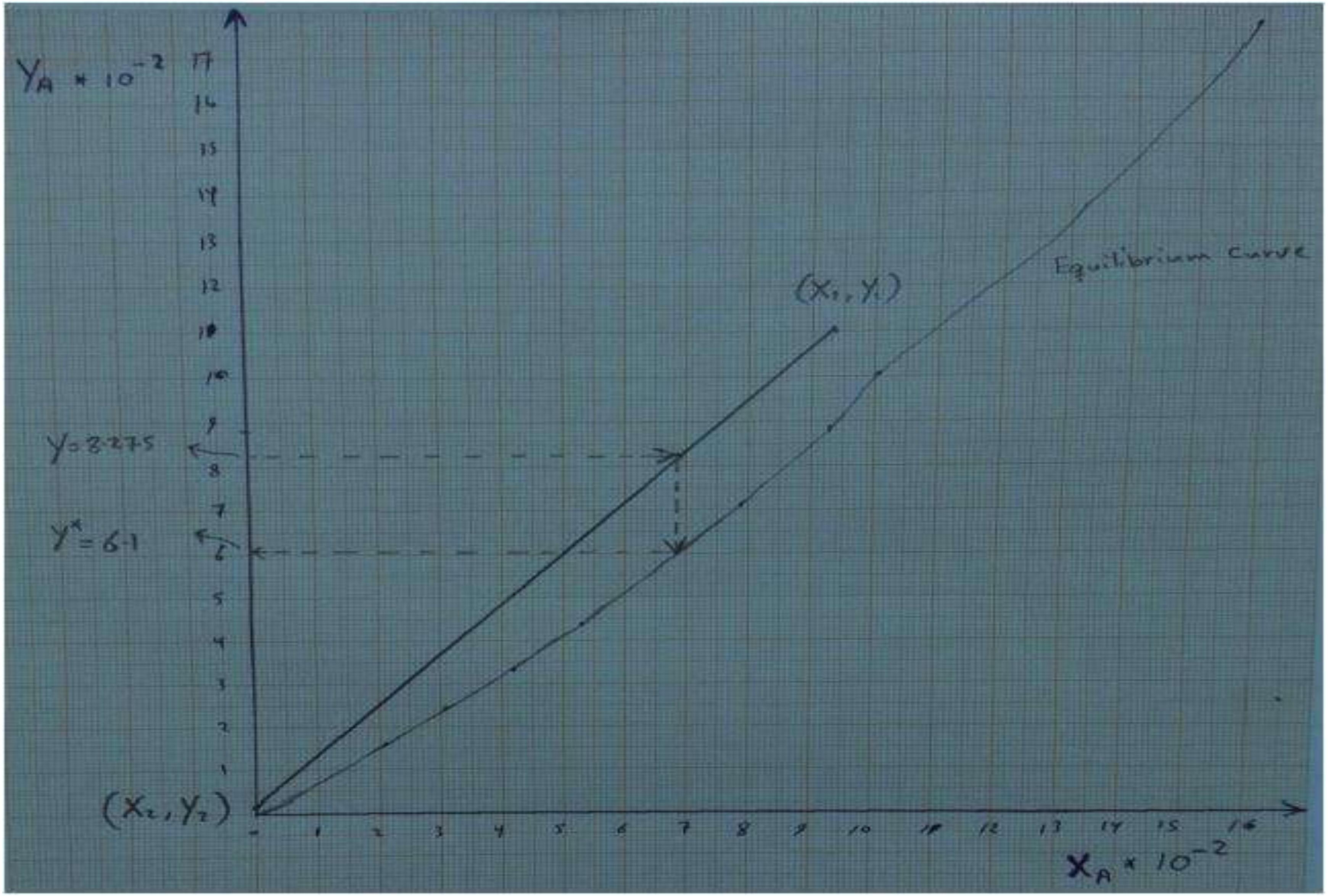
$$\mathbf{NOG} = \frac{\mathbf{h}}{\mathbf{3}} \left[\mathbf{f_0} + \mathbf{f_n} + \mathbf{2}\sum \mathbf{f_{even}} + \mathbf{4}\sum \mathbf{f_{odd}}\right]$$

$$\mathbf{NOG} = \frac{0.02725}{\mathbf{3}} \; [\mathbf{45.45} + \mathbf{1000} + \mathbf{2}(\mathbf{55.56}) + \mathbf{4}[(\mathbf{45.98}) + (\mathbf{93.02})]]$$

$$\mathbf{NOG} = 15.56$$

$$\mathbf{Z = HOG * NOG} = (0.419)\,(15.56) = 6.52 \text{ m}$$

Absorption



Calculation of Minimum Liquid Flow Rate:

The minimum liquid (solvent) flow rate is calculated when the exit solvent concentration from the absorber (X_1) is ***in equilibrium*** with the entering gas concentration to the absorber (Y_1). However, this calculations based on the equilibrium relationship natural:

A. If the equilibrium relationship is linear ($Y^* = m\,X$):

The exit solvent concentration from the absorber (X_1) is calculated from the equilibrium relationship as below:

$$\mathbf{Y_1 = m\,X_1}$$

$$\rightarrow \boxed{\mathbf{X_1 = \frac{Y_1}{m}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (1)$$

Absorption

Overall solute material balance on the absorber column:

$$G_s\,(Y_1 - Y_2) = L_s\,(X_1 - X_2)$$

$$\frac{L_s}{G_s} = \frac{Y_1 - Y_2}{X_1 - X_2}$$

For pure solvent ($X_2 = 0$):

$$\boxed{\frac{L_s}{G_s} = \frac{Y_1 - Y_2}{X_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

To calculate minimum liquid flow rate $\left[\left(\frac{L_s}{G_s}\right)_{min}\right]$ we subsititute Eq. (1) into Eq. (2):

Absorption

$$\left(\frac{L_s}{G_s}\right)_{min} = \frac{Y_1 - Y_2}{\frac{Y_1}{m}} = m\,\frac{Y_1 - Y_2}{Y_1} = m\left(1 - \frac{Y_2}{Y_1}\right)$$

$$\boxed{\left(\frac{L_s}{G_s}\right)_{min} = m\left(1 - \frac{Y_2}{Y_1}\right)}$$

Where:

$$\boxed{\left(\frac{L_s}{G_s}\right)_{actual} = (1.1 - 1.5)\left(\frac{L_s}{G_s}\right)_{min}}$$

Absorption

B. If the equilibrium relationship is non-linear:

The exit solvent concentration from the absorber (X_1) is calculated from the equilibrium relationship as below:

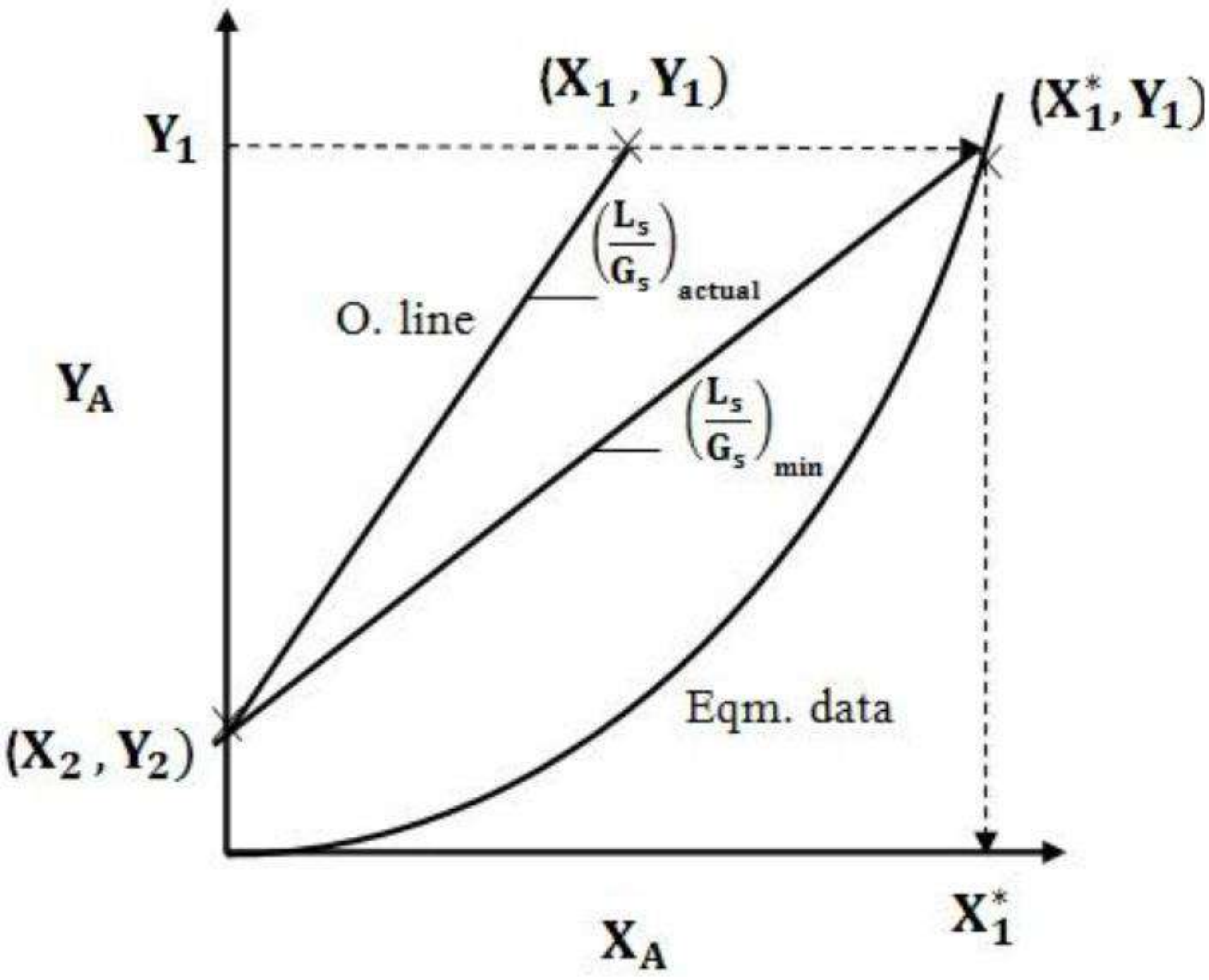
$$\left(\frac{L_s}{G_s}\right)_{min} = \frac{Y_1 - Y_2}{X_1^* - X_2}$$

For pure solvent ($X_2 = 0$):

$$\left(\frac{L_s}{G_s}\right)_{min} = \frac{Y_1 - Y_2}{X_1^*}$$

Where: X_1^* the exit liquid concentration which is in equilibrium with (Y_1) is calculated from the plot as show bellow:

Absorption



(X_1, Y_1)
(X_1^*, Y_1)
Y_1
$\left(\frac{L_s}{G_s}\right)_{actual}$
O. line
Y_A
$\left(\frac{L_s}{G_s}\right)_{min}$
Eqm. data
(X_2, Y_2)
X_A
X_1^*

Example (3):

A solute gas is absorbed from a dilute gas-air mixture by counter current scrubbing with a solvent in a packed tower. The equilibrium relation is **Y = m X**. Show that the number of transfer units (**NOG**) required is given by the following equation:

$$\mathrm{NOG} = \frac{1}{(1-\phi)} \ln\left[\frac{(1-\phi)Y_1 + \phi Y_2}{(1-\phi)Y_2 + \phi Y_2}\right]$$

If (99%) of the solute is to be recovered using a liquid rate of 1.75 times the minimum and the height of transfer unit is (1 m). What the height of packing will be required.

Absorption

Solution:

Z = HOG * NOG

For linear equilibrium relationship:

$$\left(\frac{L_s}{G_s}\right)_{min} = m\left(1 - \frac{Y_2}{Y_1}\right)$$

$$\mathbf{Y_2 = (1 - Recovery)\, Y_1} = (1 - 0.99)\, Y_1 = 0.01\, Y_1$$

$$Y_2 = 0.01\, Y_1$$

$$\left(\frac{L_s}{G_s}\right)_{min} = m\left(1 - \frac{0.01\, Y_1}{Y_1}\right) = 0.99\ m$$

$$\left(\frac{L_s}{G_s}\right)_{actual} = 1.75\ \left(\frac{L_s}{G_s}\right)_{min} = (1.75)\,(0.99\ m) = 1.7325\ m$$

$$\phi = \frac{m\, G_s}{L_s} = \frac{m}{1.7325\ \ m} = 0.577$$

Absorption

$$NOG = \frac{1}{(1-\phi)} \ln[\,(1-\phi)\frac{Y_1}{Y_2} + \boldsymbol{\phi}]$$

$$NOG = \frac{1}{(1-0.577)} \ln[\,(1-0.577)\frac{Y_1}{0.01\,Y_1} + 0.577] = 8.8$$

$$NOG = 8.88$$

$$Z = HOG * NOG = (1)\,(8.8) = 8.8 \text{ m}$$

Absorption

E-operator

يستفاد من هذا الموضوع الرياضي لاشتقاق العلاقات الرياضية التي من خلالها يتم حساب عدد المراحل (الصواني) في الأبراج ذات الصواني وكما يلي:

$$Y_{n+1} = E\ Y_n$$

$$Y_{n+2} = E^2\ Y_n$$

$$Y_{n-1} = E^{-1}\ Y_n$$

$$Y_{n-2} = E^{-2}\ Y_n$$

عند عمل موازنة مادة على البرج ذو الصواني سوف يظهر تركيز المذاب على الصواني بالشكل التالي:

$[Y_n , Y_{n-1} , X_{n+1} , X_n]$ ولكي نوحد هذه المتغيرات على مرحلة واحدة (n) فسوف نستخدم المعامل (E-operator) والذي بدوره سوف يساعدنا على تحويل المعادلة بدلالة متغير واحد (Y_n). ثم لإيجاد الحل لتلك المعادلة (أي إيجاد جذور المعادلة) يتم استبدال (E) بمتغير جديد هو (ρ) ومن ثم نقوم بالتحليل لإيجاد جذور المعادلة فإذا كانت الجذور مختلفة فسيكون الحل كالتالي:

Absorption

$$Y_n = c_1 \rho_1^n + c_2 \rho_2^n$$

Where:

ρ_1, ρ_2 : are roots of equation.

Y_n : is the concentration (mole ratio) of solute on tray (n).

n : is the number of trays.

c_1, c_2 : are equation constants.

To find the equation constants we will use the boundary conditions at:

$n = 0$

$n = 1$

Then we will have two equations, we can solve them simultaneously to find c_1 and c_2 .

Absorption

2. Tray or plate tower:

The plate column is a common type of absorption equipment for large installations. Bubble-cap columns or sieve trays are sometimes used for gas absorption, particularly when the load is more than can be handled in a packed tower of about 1 m diameter and when there is any probability of deposition of solids which would quickly choke a packing. Plate towers are particularly useful when the liquid rate is sufficient to flood a packed tower. Phase equilibrium is assumed to be achieved at each tray between the vapor and liquid streams leaving the tray. That is, each tray is treated as equilibrium stage. Assume that the only component transferred from one phase to the other is solute A.

Absorption

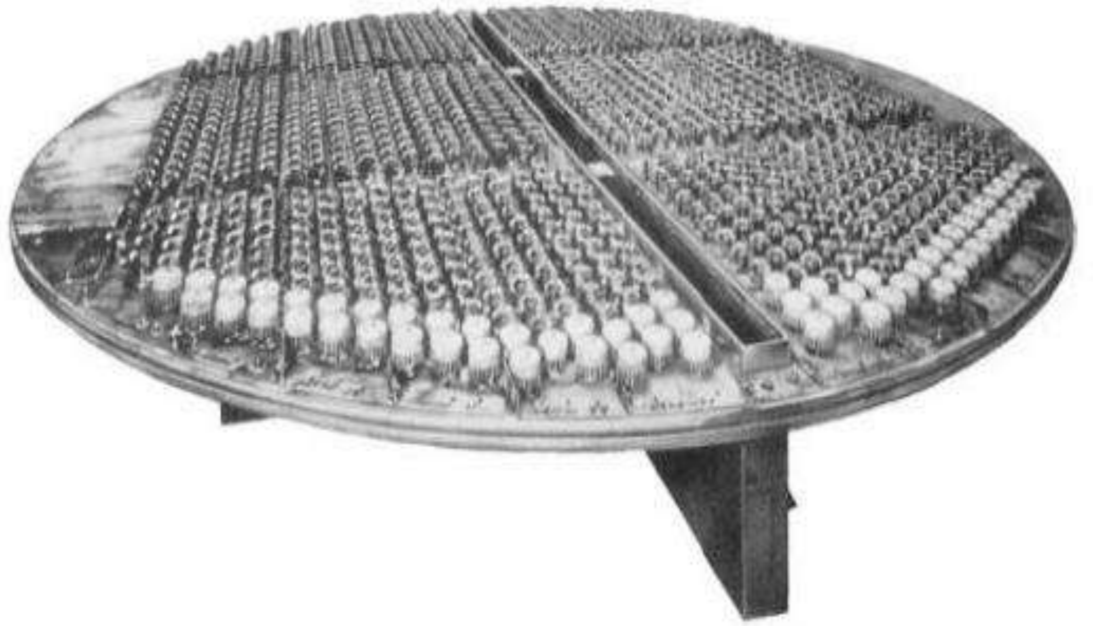


Figure: A perforated or sieve tray.

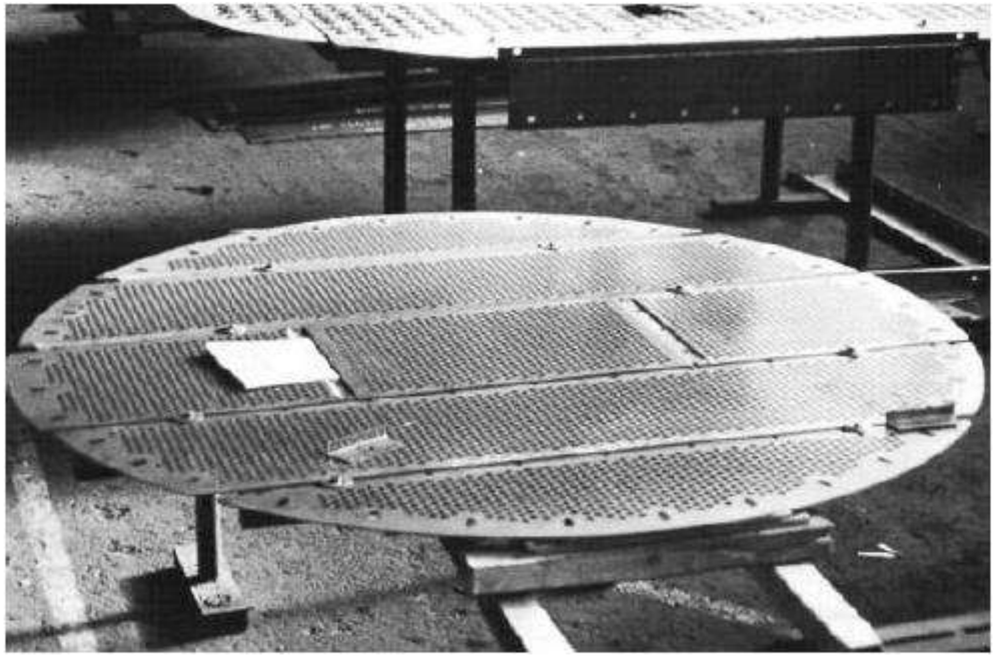


Figure: A bubble tray.

Absorption

The height of tray tower can be obtained by using the following equation:

Z = H * N

Where:

H : is the distance between two trays, and it is given (0.3 - 0.7 m) and usually used (0.5 m).

N : is the number of trays, and it can be calculated based on equilibrium data.

Absorption

Calculation of Number of theoretical Trays (N):

A. For Linear Equilibrium Relationship (Y = m X):

Solute material balance over tray (n):

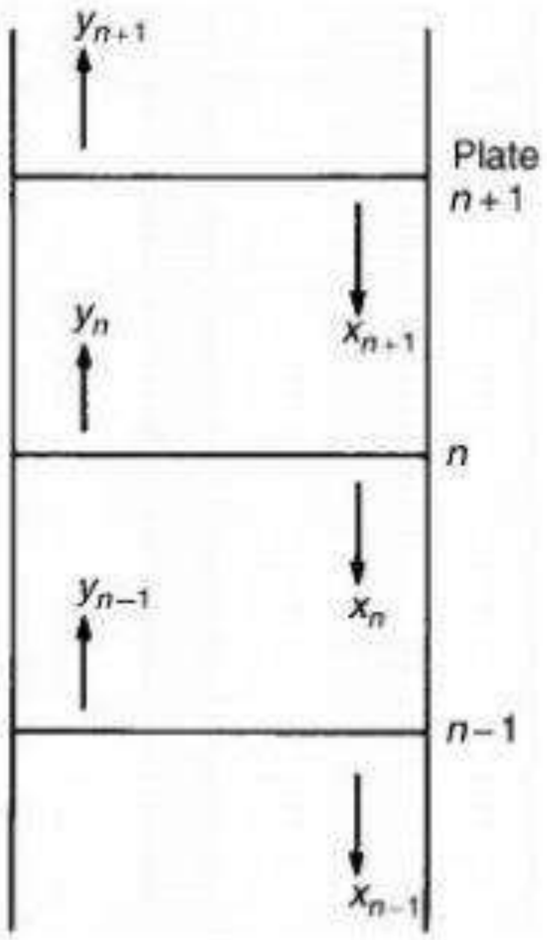
$$G_s\,Y_{n-1} + L_s\,X_{n+1} = G_s\,Y_n + L_s\,X_n \qquad \text{..........(1)}$$

The equilibrium relation is:

$$Y = m\,X \qquad \text{... ... ... (2)}$$

Substitute Eq.(2) in to Eq.(1) to get:

$$G_s\,Y_{n-1} + \frac{L_s}{m}\,Y_{n+1} = G_s\,Y_n + \frac{L_s}{m}\,Y_n$$



Absorption

$$G_s\, Y_{n-1} + \frac{L_s}{m}\, Y_{n+1} = \left(G_s + \frac{L_s}{m}\right) Y_n$$

$$\mathbf{Y_{n+1}} - \left(\frac{m\, G_s}{L_s} + 1\right) \mathbf{Y_n} + \frac{m\, G_s}{L_s}\, \mathbf{Y_{n-1}} = 0$$

Where:

$$\frac{m\, G_s}{L_s} = \boldsymbol{\phi}$$

$$\mathbf{Y_{n+1}} - (\mathbf{1} + \boldsymbol{\phi})\, \mathbf{Y_n} + \boldsymbol{\phi}\, \mathbf{Y_{n-1}} = 0$$

By using E-operator:

$$E\, \mathbf{Y_n} - (\mathbf{1} + \boldsymbol{\phi})\, \mathbf{Y_n} + \boldsymbol{\phi}\, E^{-1}\, \mathbf{Y_n} = 0 \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

multiply by (E)

$$(E^2 - (\mathbf{1} + \boldsymbol{\phi})\, E + \boldsymbol{\phi})\, \mathbf{Y_n} = 0$$

Change (E) symbol by (ρ):

$$\rho^2 - (\mathbf{1} + \boldsymbol{\phi})\, \rho + \boldsymbol{\phi} = 0$$

$$(\rho - 1)(\rho - \phi) = 0$$

Absorption

The equation roots are:

$\rho_1 = 1$ and $\rho_2 = \phi$

The general solution is:

$$\mathbf{Y_n} = c_1\, \rho_1^n + c_2\, \rho_2^n$$

Substitute the equation roots in to the general solution to get:

$$\mathbf{Y_n} = \mathbf{c_1} + \mathbf{c_2}\, \boldsymbol{\phi}^n$$

$$\mathbf{n} = \frac{\ln\left[\frac{\mathbf{Y_n} - \mathbf{c_1}}{\mathbf{c_2}}\right]}{\ln \boldsymbol{\phi}}$$

Absorption

To find the total number of trays, we substitute (n) by (N) to get:

$$N = \frac{\ln\left[\frac{Y_N - c_1}{c_2}\right]}{\ln \phi}$$

To find the equation constants C_1 and C_2 we substitute the boundary conditions:

B. C. 1: at $n = 0 \rightarrow Y_n = Y_0$

B. C. 2: at $n = 1 \rightarrow Y_n = Y_1 \rightarrow Y_1 = m X_1$

$$Y_n = c_1 + c_2 \phi^n$$

B. C. 1: $Y_0 = c_1 + c_2 \phi^0 \rightarrow Y_0 = c_1 + c_2 \quad \dots\dots(1)$

B. C. 2: $Y_1 = c_1 + c_2 \phi^1 \rightarrow m X_1 = c_1 + c_2 \phi \quad \dots\dots(2)$

From Eq.(1) and Eq.(2) we get:

$$c_2 = \frac{Y_0 - m X_1}{1 - \phi}$$

$$c_1 = Y_0 - c_2$$

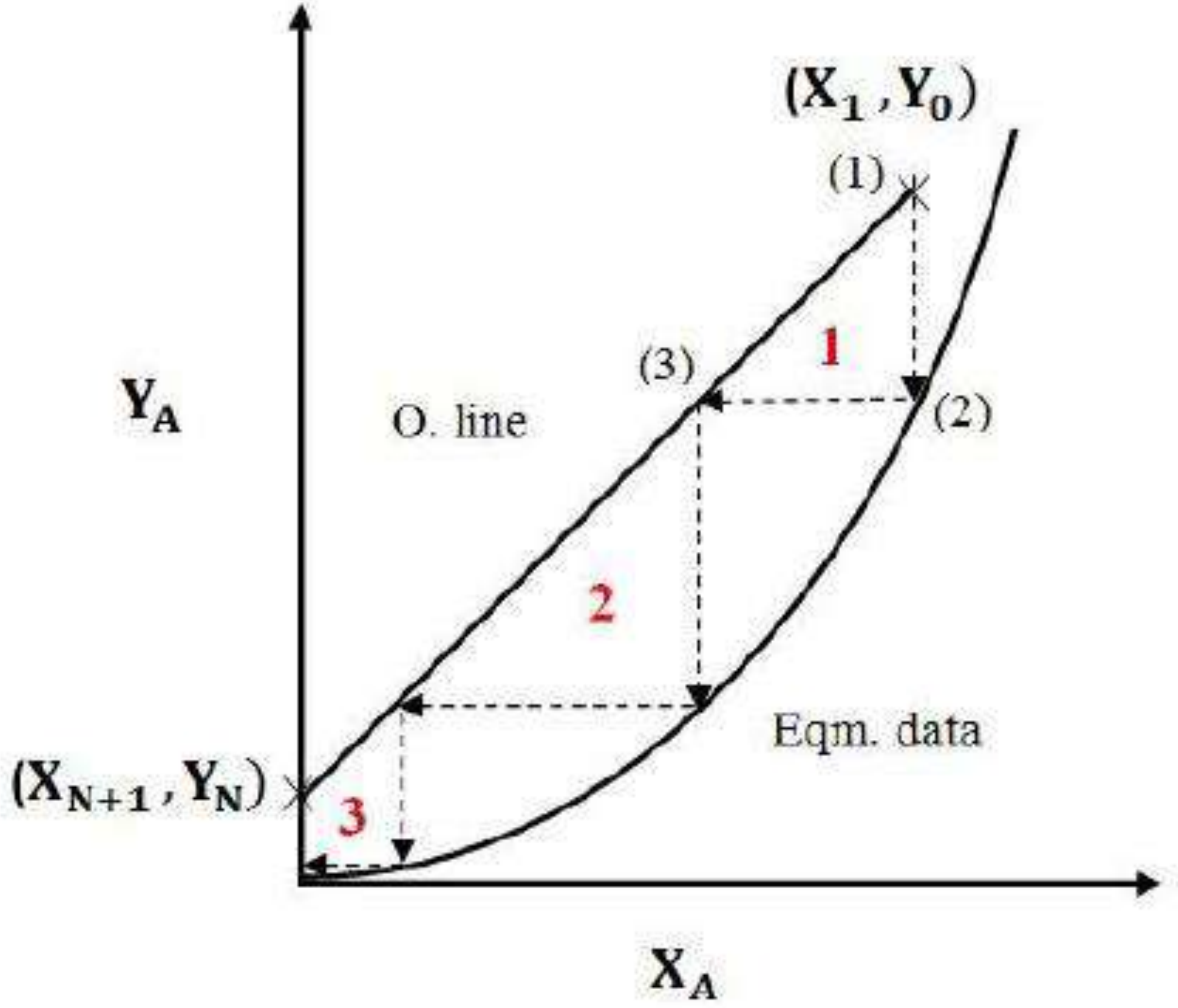
Absorption

B. For Non-linear Equilibrium Relationship (Graphical method):

In this case the number of theoretical plates will be calculated using graphical method following steps below:

1. Complete the material balance to calculate all the unknowns (all compositions and flow rates of the inlet and the outlet streams must be known).
2. Draw the equilibrium curve (or line) either from given data or from the equilibrium equation: $Y = m X$.
3. Draw the operating line, from two points (X_1, Y_0) and (X_{N+1}, Y_N) or one point and slope of $(\frac{L_s}{G_s})$ according to the condition of the process.
4. Draw a vertical line from **point 1** which represents the point (X_1, Y_0) {as shown in the figure} to **point 2** which will intersect the equilibrium line (Curve). Then draw a horizontal line from **point 2** to **point 3**, intersecting the operating line. The triangular formed will represent the plate number one.
5. Continue drawing the vertical lines and horizontal lines as in step 4 (shown in the fig.) until we reach to the point (X_{N+1}, Y_N) or pass it.
6. Count the triangles constructed, this number represents the number of theoretical plates.

Absorption



Column efficiency:

The number of ideal stages required for a desired separation may be calculated by one of the methods discussed previously, although in practice more trays are required than ideal stages. There are two types of efficiency usually used:

1. **Overall column efficiency (E_c):**

$$\text{overall column efficiency } (E_c) = \frac{\text{The theoritical number plates}}{\text{The actual number plates}} = \frac{N_{th}}{N_{act}}$$

Absorption

$$E_c = \frac{N_{th}}{N_{act}}$$

Where: $N_{act} > N_{th}$

and: $N_{act} = N_{th}$ if $E_c = 100\%$

$$\mathbf{Z = N_{act} * tray\ spacing}$$

Where: $Z_{act} > Z_{th}$

2. Plate efficiency (Em):

The proportion of liquid and vapour, and the physical properties of the mixtures on the trays, will vary up the column, and conditions on individual trays must be examined, as suggested by ***Murphree*** (1925). For a single ideal tray, the vapour leaving is in equilibrium with the liquid leaving, and the ratio of the actual change in composition achieved to that which would occur if equilibrium between **Yn** and **Xn** were attained is known as the **Murphree plate efficiency (Em)**. The plate efficiency can be expressed in terms of gas and liquid as given below:

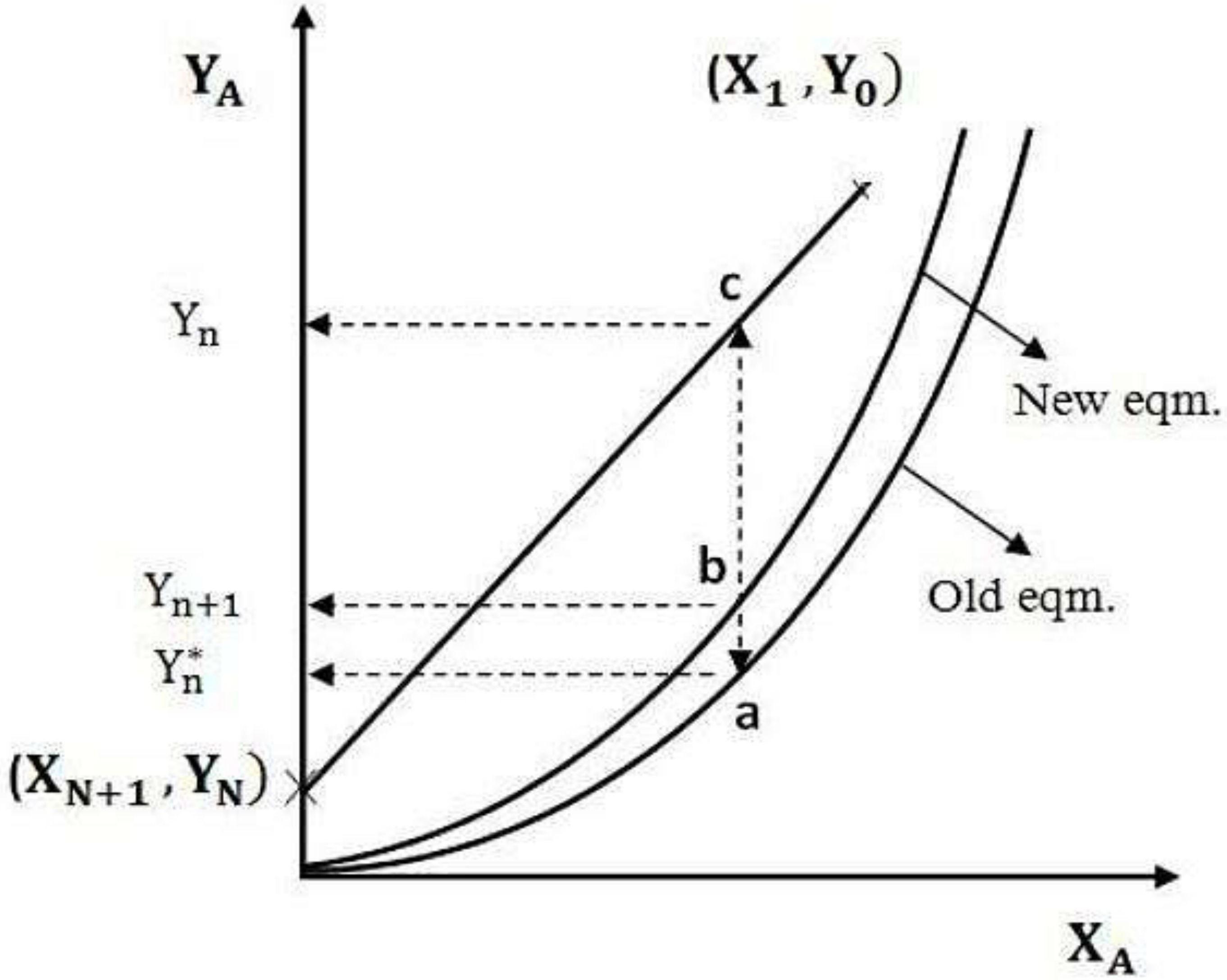
a. Plate efficiency based on gas phase (E_{mv}):

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*} = \frac{\overline{bc}}{\overline{ac}}$$

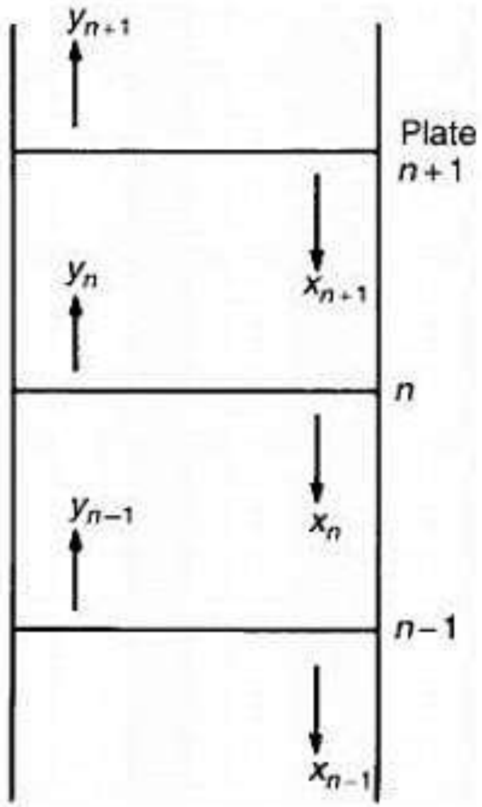
Where:

Y_n^* : is the composition of the gas that would be in equilibrium with the liquid of composition $\mathbf{X_n}$ actually leaving the plate.

Absorption



Y_A
(X_1, Y_0)
c
Y_n
New eqm.
b
Y_{n+1}
Y_n^*
Old eqm.
a
(X_{N+1}, Y_N)
X_A



y_{n+1}
Plate
$n+1$
y_n
x_{n+1}
n
y_{n-1}
x_n
$n-1$
x_{n-1}

Absorption

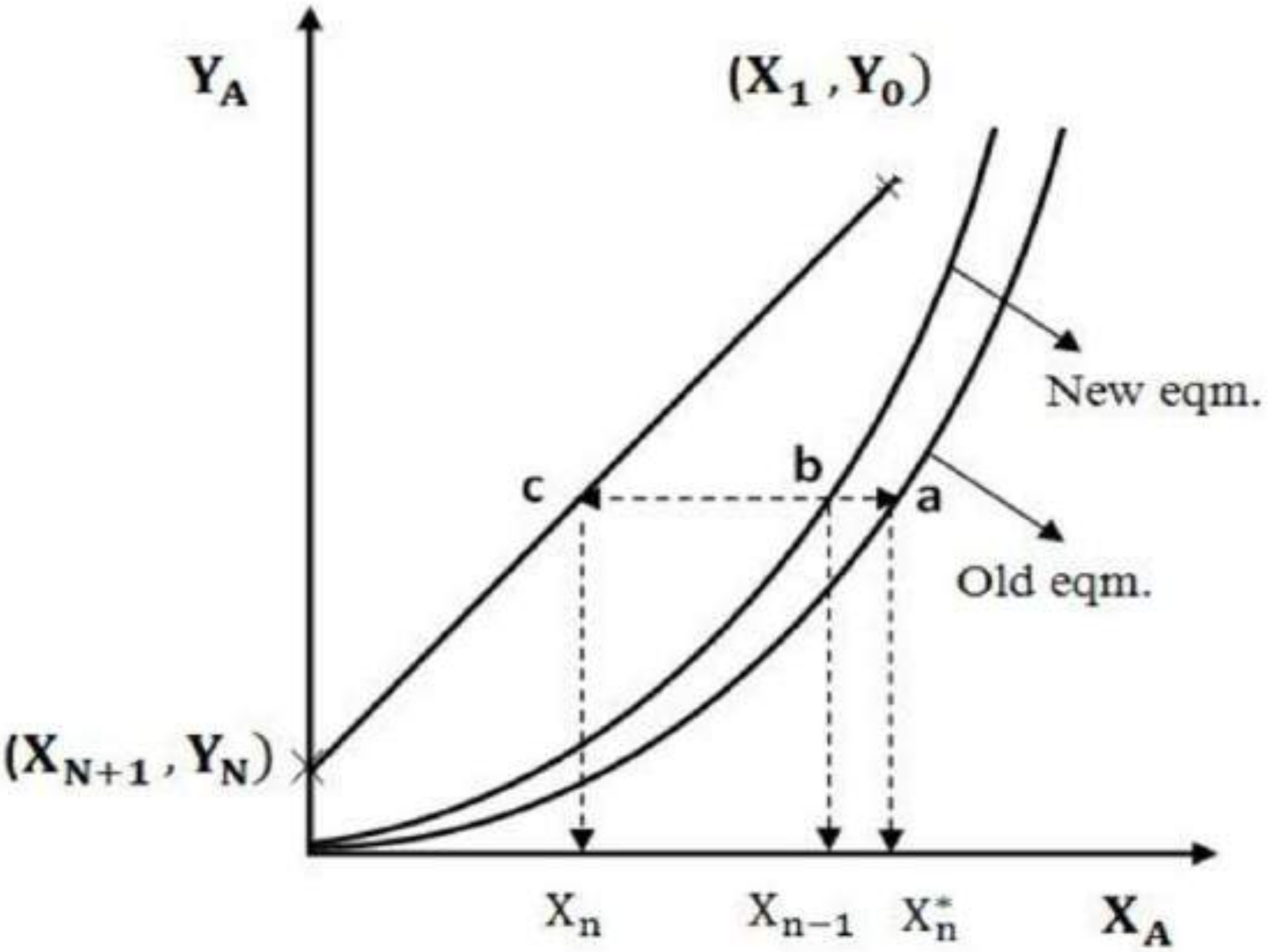
b. Plate efficiency based on liquid phase (E_{ml}):

$$E_{ml} = \frac{X_n - X_{n-1}}{X_n - X_n^*} = \frac{\overline{bc}}{\overline{ac}}$$

Where:

X_n^* : is the composition of the liquid that would be in equilibrium with the gas of composition $\mathbf{Y_n}$ actually leaving the plate.

Absorption



Absorption

لايجاد عدد الصواني الحقيقي (N_{act}) فيما لو اعطيت كفاءة الصينية المعتمدة على البخار (E_{mv}) او السائل (E_{ml})
نتبع الخطوات التالية:

1. يتم القياس بالمسطرة المسافة بين منحني التعادل (a) وخط التشغيل (c) وتمثل هذه المسافة (ac).
2. نجد المسافة (bc) باستخدام معادلة كفاء الصينية (E_{mv}) او (E_{ml}).
3. تعاد هذه العملية لخمس نقاط لكي يتم ايجاد منجني التعادل الجديد.
4. عدد الصواني الحقيقي (N_{act}) يتم ايجاده بالتسقيط بين خط التشغيل ومنحني التعادل الجديد، في حين عدد الصواني النظري (N_{th}) يتم ايجاده بالتسقيط بين خط التشفيل ومنحني التعادل القديم.

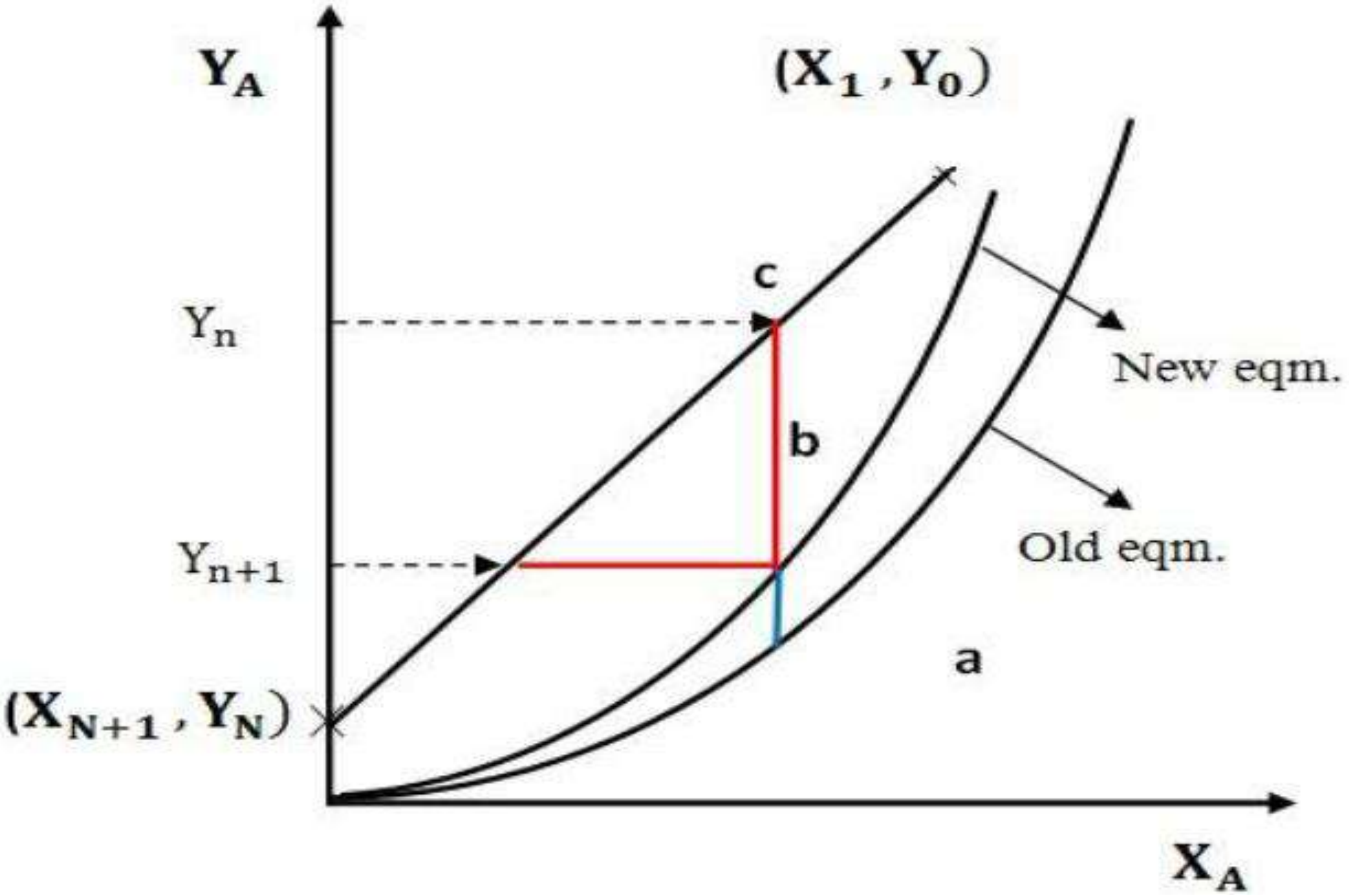
Absorption

ملاحظات مهمة جدا:

1. اذا اعطي في السؤال معلومات التعادل وكذلك تركيز المذاب على صينيتين متجلورتين فلكي يتم حساب كفاءة الصينية نتبع الطريقة التالية:

نرسم معلومات التعادل وكذلك خط التشغيل ثم نحدد تركيز المذاب المعطى على الصينيتين المتجاورتين على محور (Y-axis) (اذا كان التركيز المعطى بالنسبة للغاز). بعد ذلك نسقط هذه التراكيز على خط التشغيل ونرسم المثلث كما موضح ادناه فالنقطة التي تمثل راس المثلث هي نقطة على منخني التعادل الجديد وعند ذاك نطبق علاقة كفاءة الصينية بقياس المسافات.

Absorption



Absorption

2. اذا لم تعطى معلومات التعادل في السؤال واعطيت كفاءة الصينية (E_{mv}) فلكي يتم حساب ارتفاع برج الامتصاص نستخدم غلاقة كفاءة الصينية التالية:

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*}$$

وبما أن معلومات التعادل لم تعطى في السؤال هذا يعني السائل المستخدم شديد الامتصاص أي أن ميل علاقة التعادل = صفر، أي لا يوجد منحني تعادل وهذا يعني أن $(Y_n^* = 0)$ فيتم التعويض بالمعادلة السابقة وتبسيط المعادلة باستخدا م (E-operator) لإيجاد العلاقة بين (Y_n) و (n) وبالتعويض عن التركيز النهائي نجد عدد الصواني الحقيقية ثم بعد ذلك نجد ارتفاع البرج.

3. أذا لم تعطى كفاءة الصينية في السؤال فيتم فرضها على إنها (E_{mv}).

Absorption

Calculation of the Height Equivalent of a Theoretical Plate (HETP):

The height of a theoretical plate (HETP), also called the height of an equivalent equilibrium stage, is the ***height of packing*** that will give the same separation as an equilibrium stage. The relationship between transfer units (**HOG**) and the height of an equivalent theoretical plate (**HETP**) is given by:

$$\mathbf{HETP = tray\ spacing = HOG * \frac{\ln \phi}{1 - \phi}}$$

Where: $\phi = \frac{mG_s}{L_s}$

$$\mathbf{Z = N * HETP}$$

Where: N = number of plates

* اذا لم يعطى في السؤال المسافة بين الصواني وكان المطلوب حساب ارتفاع البرج ذو الصواني ولم تعطى المسافة بين الصواني و هناك معلومات متوفرة يمكن من خلالها حساب (HOG) فيتم حساب (HETP) لاستخدامه في حساب ارتفاع البرج.

Absorption

$$Z_{OG} = HOG * NOG = \frac{\bar{G}_s}{KoG.a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z_{OL} = HOL * NOL = \frac{\bar{L}_s}{KoL.a} \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}$$

$$Z_g = Hg * Ng = \frac{\bar{G}_s}{Kg.a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y_i)}$$

$$Z_L = H_L * N_L = \frac{\bar{L}_s}{K_L.a} \int_{X_2}^{X_1} \frac{dY}{(X_i - X)}$$

Where: $Z = Z_{OG} = Z_{OL} = Z_g = Z_L$

Absorption

Example (1): Calculate the height of plate column with tray spacing of 0.51 m and plate efficiency based on gas phase 40% to reduce the concentration of NH_3 from 5.5 mol% to 0.1 mol% in an NH_3-Air mixture using fresh water. The gas and liquid flow rates are 300 and 400 Kg/ m^2.hr, respectively, and the equilibrium relationship is such that the vapor pressure of NH_3 over the liquid is negligible.

Solution:

For plate tower: **Z = N * tray spacing**

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*}$$

Absorption

Since the vapor pressure of NH_3 over the liquid is negligible, then: **m = 0**

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n} \quad \rightarrow \quad 0.4 = \frac{Y_n - Y_{n+1}}{Y_n}$$

$$Y_{n+1} - 0.6\, Y_n = 0$$

By using E-operator:

$$(E - 0.6)Y_n = 0 \quad \rightarrow \quad \rho - 0.6 = 0$$

$$\rho = 0.6$$

$$Y_n = C\,(\rho)^n = C\,(0.6)^n$$

Absorption

Using the boundry condition:

$n = 0 \quad \rightarrow \quad Y_n = Y_0 = 0.055$

$0.055 = C\,(0.6)^0$

$\rightarrow \quad C = 0.055$

$Y_n = 0.055\,(0.6)^n$

$0.001 = 0.055\,(0.6)^N$

$N = 7.83 \approx 8$

Z = N * tray spacing

Z = 7.83 * 0.51 = 4 m

Absorption

Example (2): A mixture of ammonia and air is scrubbed in a plate column with fresh water. If the ammonia concentration is reduced from 5% to 0.5% .Given that: $Y = 2X$.

a. Calculate the No. of theoretical plate and the tower height. Given that: $L = 0.65\ Kg/m^2.s$ and $G = 0.4\ Kg/m^2.s$, $K_{OG}.a = 0.0008\ Kmol/m^3.s.kPa$

b. Calculate the No. of theoretical plate, given that: $\left(\frac{L}{G}\right) = 2\left(\frac{L}{G}\right)_{min}$.

c. Calculate $\left(\frac{L}{G}\right)$ if the actual No. of plates $= 12$, and the column efficiency $= 0.5$.

d. Calculate the theoretical and actual No. of plates, give that:

$\left(\frac{L}{G}\right) = 1.5\left(\frac{L}{G}\right)_{min}$ and $E_{mv} = 0.7$

e. Given the concentration of a gas in the two adjacent plates are 4% and 3.3%. Calculate E_{mv} and E_{ml} if $L = 0.65\ Kg/m^2.s$ and $G = 0.4\ Kg/m^2.s$.

Absorption

Solution:

Since the inlet gas concentration is 5% then no need to convert the mole fraction to mole ratio:

$$\bar{L}_s = \frac{0.65}{18} = 0.0361 \quad \frac{kmol}{m^2.s}$$

$$\bar{G}_s = \frac{0.4}{29} = 0.01379 \quad \frac{kmol}{m^2.s}$$

Overall solute material balance on the tower:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$0.01379 (0.05 - 0.005) = 0.0361 (X_1 - 0)$$

$$X_1 = 0.01718$$

Absorption

To find the number of theoretical plates:

1. **Plot the operating line:**

$$(X_1, Y_1) = (0.01718, 0.05)$$

$$(X_2, Y_2) = (0, 0.005)$$

2. **Plot the equilibriam relation (Y = 2 X):**

From the figure below we can find the theroritical No. of plates by stepping off:

$N = 5$ Plates

The height of plate tower is:

$$Z = N * HETP$$

$$HOG = \frac{\overline{G}_s}{KOG.\ a.\ P_T} = \frac{0.01379}{(0.0008)(101.3)} = 0.17\ \text{m}$$

$$\phi = \frac{mG_s}{L_s} = \frac{2(0.01379)}{0.0361} = 0.7639$$

$$HETP = HOG * \left(\frac{\ln \phi}{1 - \phi}\right) = (0.17) * \left(\frac{\ln 0.7639}{1 - 0.7639}\right) = 0.19\ \text{m}$$

$$Z = (5)(0.19) = 0.97\ \text{m}$$

Absorption

b. $\left(\frac{L}{G}\right)_{act} = 2\left(\frac{L}{G}\right)_{min}$

For linear equilibrium relation:

$$\left(\frac{L}{G}\right)_{min} = m\left(1 - \frac{Y_2}{Y_1}\right) = 2\left(1 - \frac{0.005}{0.05}\right) = 1.8$$

$$\left(\frac{L}{G}\right)_{act} = 2\left(\frac{L}{G}\right)_{min} = 2\,(1.8) = 3.6$$

Overall solute material balance on the tower:

$$\overline{G}_s\,(Y_1 - Y_2) = \overline{L}_s\,(X_1 - X_2)$$

$$X_1 = \frac{\overline{G}_s}{\overline{L}_s}\,(Y_1 - Y_2) = \frac{1}{3.6}\,(0.05 - 0.005)$$

$$X_1 = 0.0125$$

Absorption

To find the number of theoretical plates:

1. Plot the operating line:

$(X_1, Y_1) = (0.0125, 0.05)$

$(X_2, Y_2) = (0, 0.005)$

2. Plot the equilibriam relation (Y = 2 X):

From the figure below we can find the theroritical No. of plates by stepping off:

N = 3 Plates

c. Theoritical No. of plates = Actual No. of plates * overall column efficiency

$N_{th} = N_{act} * E_c$

$N_{th} = 12 * 0.5 = 6$ plates

لمعلوم في هذا السؤال هو عدد الصواني (N) والمطلوب هو ($\frac{L}{G}$) أي ميل خط التشغيل فسوف نقوم بالمحاولة والخطا برسم خط التشغيل من النقطة :

$(X_2, Y_2) = (0, 0.005)$

Absorption

بحيث يحقق عدد الصواني (6) عند $(Y_1 = 0.05)$. عند ايجاد خط التشغيل الصحيح نقوم بحساب ميله والذي يساوي $(\frac{L}{G})$:

From plot:

$$\frac{\bar{L}_s}{\bar{G}_s} = 2.23$$

d. $$\left(\frac{\mathbf{L_s}}{\mathbf{G_s}}\right)_{\mathbf{min}} = \frac{\mathbf{Y_1 - Y_2}}{\mathbf{X_1^* - X_2}}$$

From plot at $Y_1 = 0.05 \quad \rightarrow \quad \mathbf{X_1^*} = 0.025$

$$\left(\frac{L_s}{G_s}\right)_{min} = \frac{0.05 - 0.005}{0.025 - 0} = 1.8$$

$$\left(\frac{L_s}{G_s}\right)_{act} = 1.5\left(\frac{L_s}{G_s}\right)_{min} = 1.5(1.8) = 2.7$$

Absorption

e.

$$\bar{L}_s = \frac{0.65}{18} = 0.0361 \quad \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

$$\bar{G}_s = \frac{0.4}{29} = 0.01379 \quad \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

Overall solute material balance on the tower:

$$\bar{G}_s\,(Y_1 - Y_2) = L_s\,(X_1 - X_2)$$

$$0.01379\,(0.05 - 0.005) = 0.0361\,(X_1 - 0)$$

$$X_1 = 0.01718$$

To find the number of theoretical plates:

1. **Plot the operating line:**

$$(X_1\,,Y_1) = (0.01718\,, 0.05)$$

$$(X_2\,,Y_2) = (0\,, 0.005)$$

2. **Plot the equilibriam relation ($Y = 2\,X$):**
3. **From the plot we find (Emv) and (EmL) at:**

$Y_N = 4\%$ and $Y_{N+1} = 3\%$

$$E_{mv} = \frac{ab}{ac} = \frac{19\text{ mm}}{25\text{ mm}} = 0.67$$

$$E_{ml} = \frac{ab}{ac} = \frac{6\text{ mm}}{12\text{ mm}} = 0.5$$

Extraction

Introduction:-

1- An extraction process makes use of the partitioning of a solute between two immiscible or partially miscible phases.

2- When the extraction takes place from one Liquid medium to another, the process is referred to as Liq-Liq. extraction

3- When a liquid is used to extract solutes from a solid material, the process is referred to as solid-Liquid extraction or Leaching.

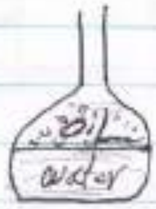
4- When a supercritical fluid is used as an extracting solvent, the process is referred to as Supercritical fluid extraction (SFE).

Liquid-Liquid extraction:- is a useful method to separate components (Compounds) of a mixture.

Examples:-
Suppose that you have a mixture of Sugar in Vegetable oil (it testes Sweet!) and you want to Separate the Sugar from oil. You observe that the Sugar particles are too tiny to filter and you suspect that the Sugar is partially dissolved in the Vegetable oil.

By shaking the mixture (sugar + vegetable oil) with water, it can be seen the sugar is much more soluble in water than in vegetable oil and as you know, water is immiscible (not soluble) with oil.

Also it can be seen the water phase is the bottom layer and the oil phase is the top layer, because the water is denser than oil.



The Concept of Liquid - Liquid extraction :

Liquid-Liquid extraction is based on the transfer of a solute substance from one liquid phase into another liquid phase according to the solubility.

Extraction becomes a very useful tool if you choose a suitable extraction solvent. You can use extraction to separate a substance selectively from a mixture, or to remove unwanted impurities from a solution. In the practical use, usually one phase is water or water based (aqueous) solution and the other an organic solvent which is immiscible with water.

Mechanisum of extraction :

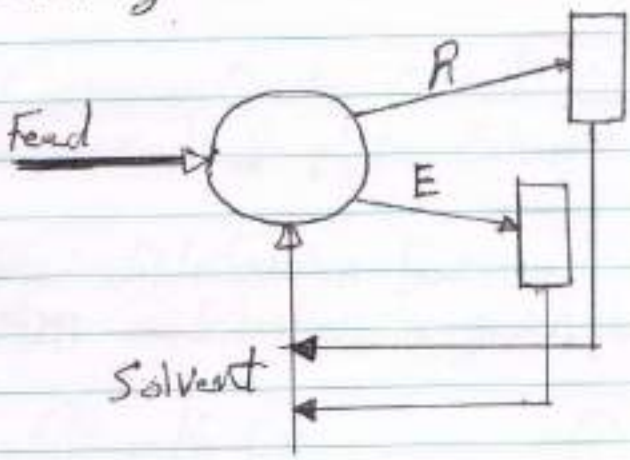
1. mixing of feed and solvent

2. Separation of phase by settling

R = (original solution + remainder solute)

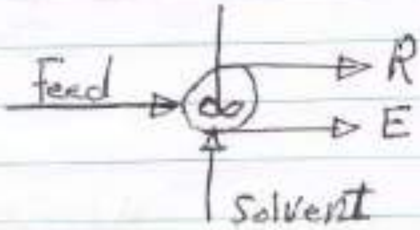
E = (extract solute + solvent)

3. Recovery of solute and solvent.

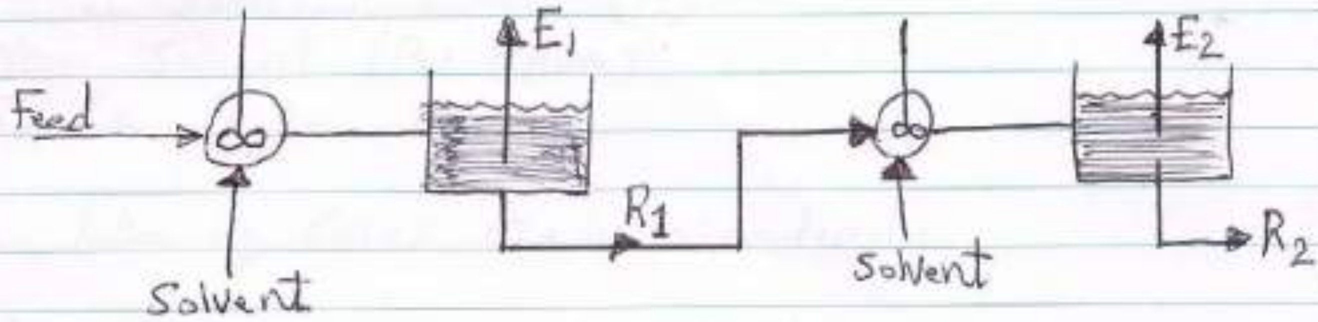


Types of Contact :

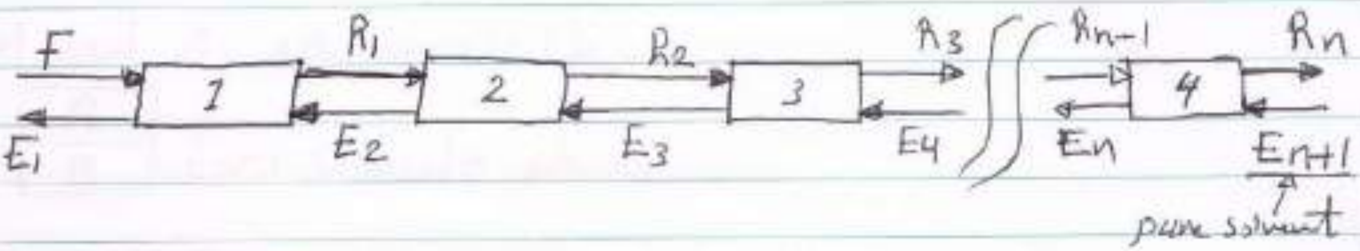
1- Single Contact



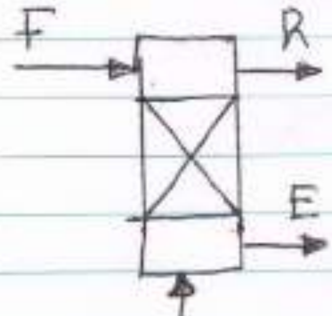
2- Multi Concurrent contact



3- Multiple Counter Current Contact



4- Continuous Counter Current Contact :-
a packed of plate Column is used



The distribution law
Em distribution is given by :

$$C_E = K\, C_R \quad \text{——— (1)}$$

C_E = Concentration of Solute in extract phase
C_R = Concentration of Solute in Raffinate phase
K = Distribution Coefficient

eq (1) holds if :-
1) The two solvent are immicible.
2) No association or dissociation of Solute ,
In terms of mole ratio ($Y = mX$)

Ternary Systems:

- Initial solution Solvent (A).
- New Solvent (B) (pure)
- Solute (C).

The following Cases can be considered:

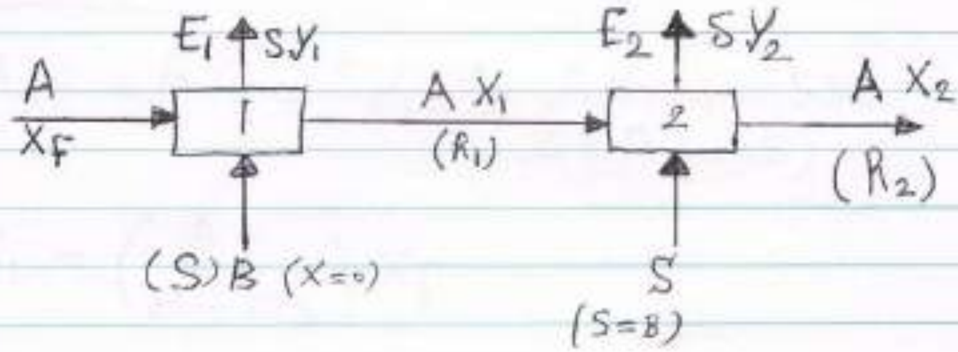
① A and B are completely miscible.
[A+B] Homogeneous solution is formed.

② A and B are immiscible.
[A / B] two Separate phases.

③ A and B partially miscible.
[A+B / B+A] Two partially miscible Liquid phase.

Number of theoretical Stages:

1- Immiscible Solvents.
ⓐ- Co-Current contact



Define :

A = mass of Solvent A.
S = mass of fresh Solvent S.
X_f = Mass ratio of C in A $\left(\frac{\text{mass of C in feed}}{\text{mass of A in feed}}\right)$
X = Mass ratio of C in Raffinate $\left(\frac{\text{mass of C in R}}{\text{mass of A in feed}}\right)$
Y = Mass ratio of C in Extract $\left(\frac{\text{mass of C in S'}}{\text{mass of S in E}}\right)$

M.B on Solute gives

$$AX_F = AX_1 + S Y_1$$

$$\therefore \frac{Y_1}{X_1 - X_f} = -\left(\frac{A}{S}\right)$$

$$Y_1 = -\left(\frac{A}{S}\right)(X_1 - X_f) \longrightarrow \text{Slope} = -\frac{A}{S'}$$

If distribution law is avaliable (Applicable)

$$Y = mX$$

M.B on Solute
- First stage

$$AX_f = AX_1 + SY_1 = AX_1 + mSX_1$$
$$= X_1(A + mS)$$

$$\therefore X_1 = \left(\frac{A}{A+mS}\right) X_f$$

- Second Stage :-

Fresh solvent S

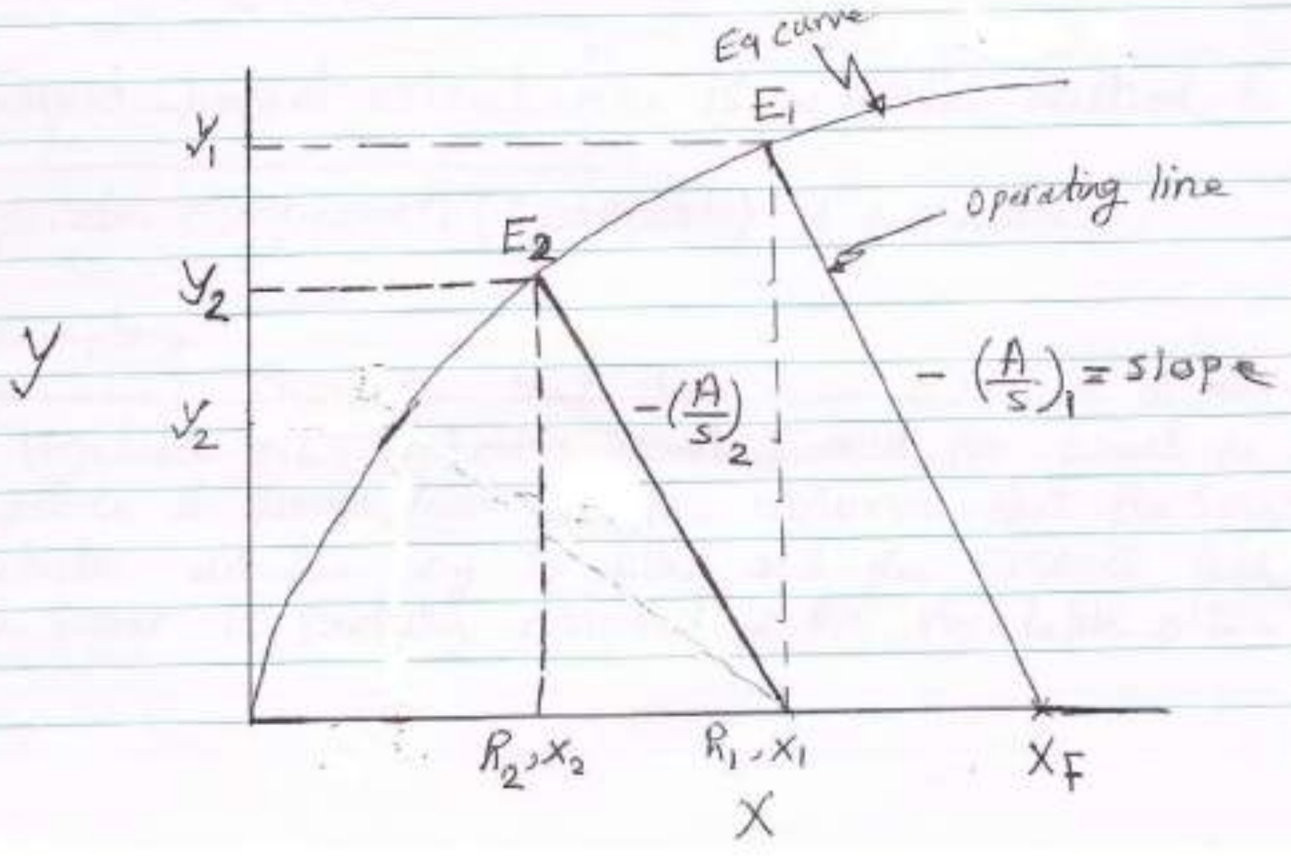
$$A x_1 = A x_2 + S y_2 = A x_2 + m S x_2$$
$$= x_2 (A + mS)$$

$$\therefore x_2 = \left(\frac{A}{A + mS}\right) x_1 = \left(\frac{A}{A+mS}\right)^2 x_F$$

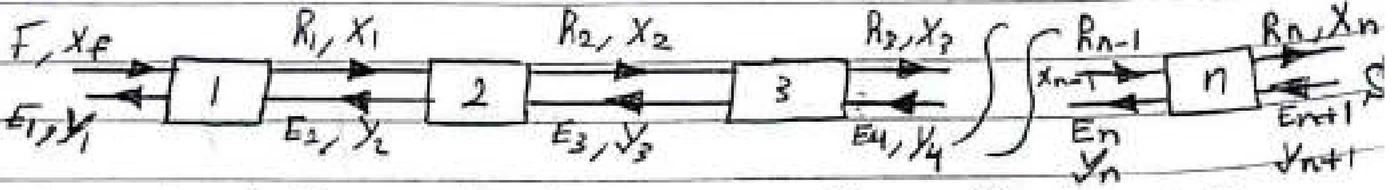
General $\quad x_N = \left(\frac{A}{A + mS}\right)^N x_F$

$$N = \frac{\log\left(\frac{x_N}{x_F}\right)}{\log\left(\frac{A}{A+mS}\right)}$$

N = Number of stages.
m = Slope of distribution line.
S is the same in all stages.



(B) - Counter Current Contact



F = Feed ; E = Extract phase , R = Raffinate phase

Since the two solvents are immiscible

$$F = R_1 = R_2 = R_3 \ldots\ldots R_n = A$$

$$E_1 = E_2 = E_3 \ldots\ldots E_{n+1} = S$$

M.B on Solute

(a) First stage :-

$$A x_F + S y_2 = A x_1 + S y_1$$

(b) N of stage :-

$$A x_{n-1} + S y_{n+1} = A x_n + S y_n$$

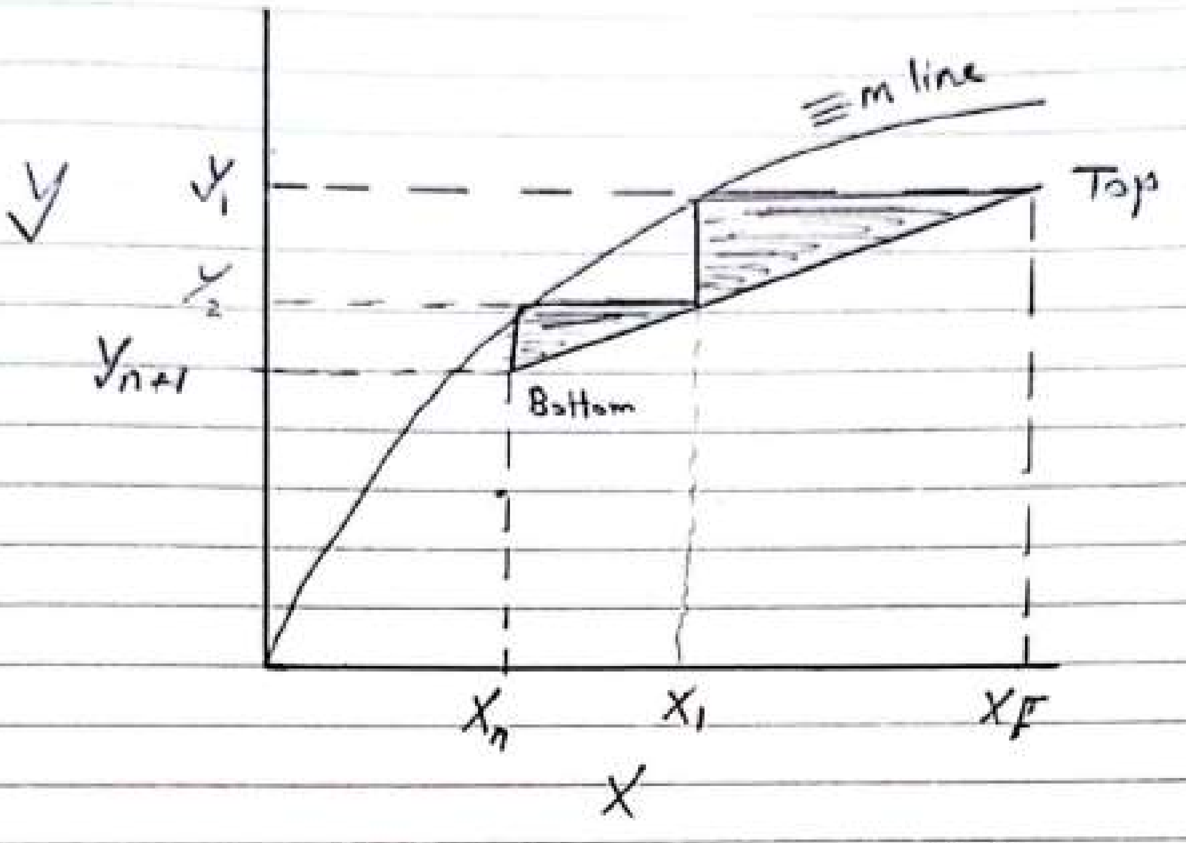
Overall balance

$$\therefore A x_F + S y_{n+1} = A x_n + S y_1$$

This is the eq of a straight line (O.L)

$\therefore$ O.L. passes through points (x_F, y_1) and (x_n, y_{n+1})

Graphical Method for N :

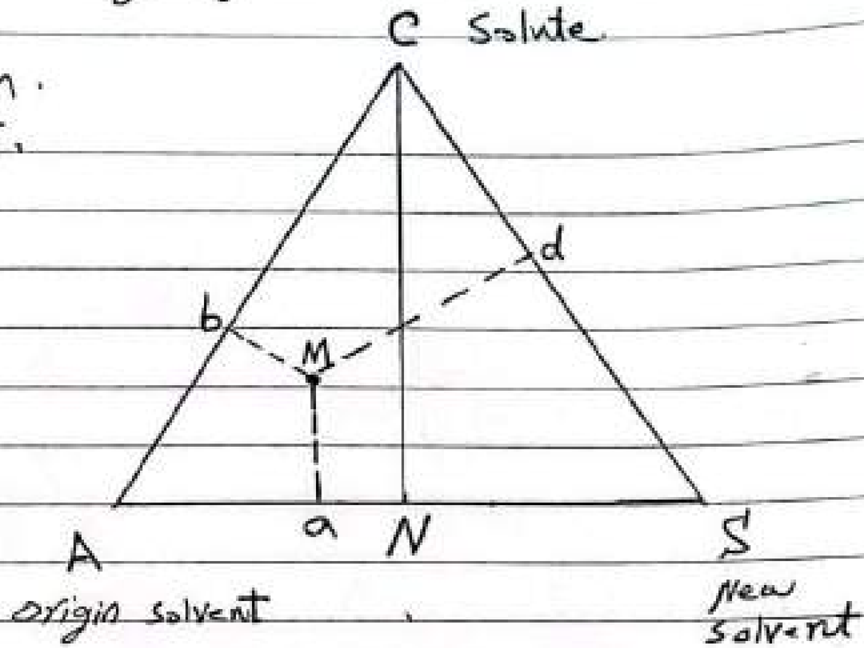


Number of Stage N = Number of Steps

2 - System with partially miscible Solvents:

- Triangular phase diagrams are used
- Consider the ternary system A.C.S.

A = Feed Solution.
S = Fresh Solvent.
C = Solute.



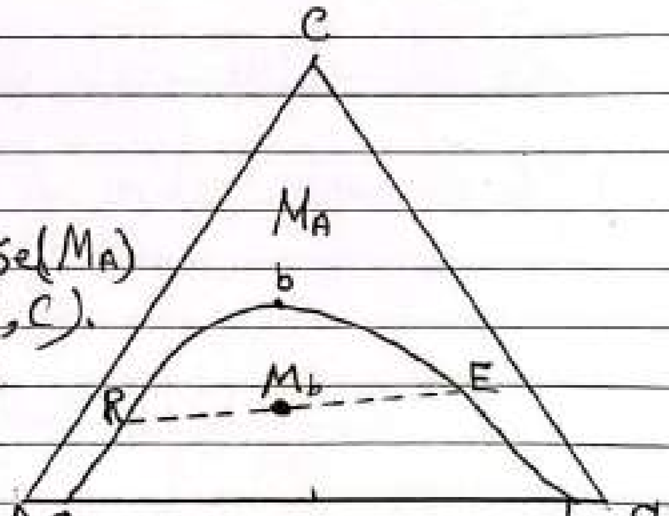
- For Point M:

$$X_{MC} = \frac{Ma}{NC}$$

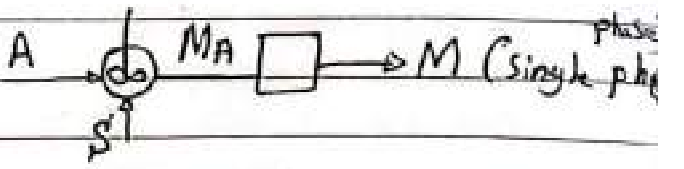
$$X_{MA} = \frac{Md}{NC}$$

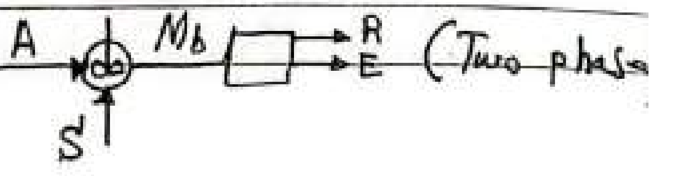
$$X_{MS} = \frac{Mb}{NC}$$

$$Ma + Mb + Md = Nc$$



- abd is equilibrium curve.
- outside abd, a single phase (M_A) exists with component (A, S, C).
- Inside the equilibrium curve abd, two phases exists.
- A mixture represented by M_b separates into equilibrium phase R and E.
- RME is a tie line.





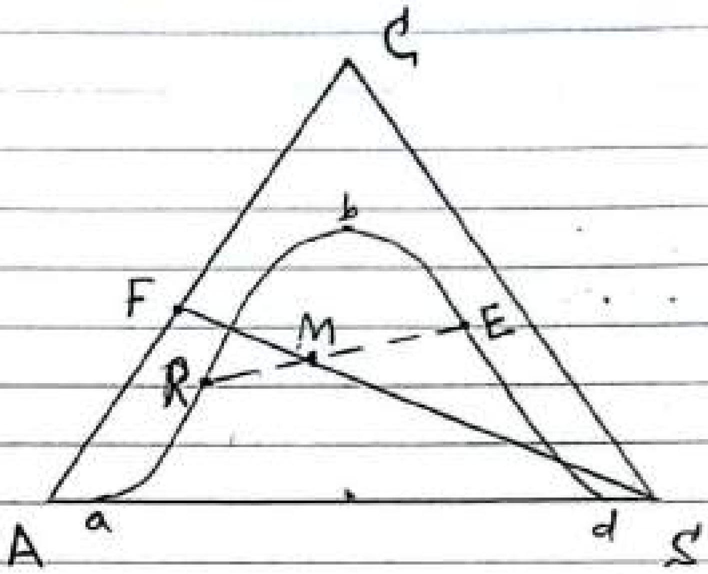
Mixture rule:

R = mass of Raffinate
E = mass of Extract
M = mass of mixture

$$\frac{R}{E} = \frac{\overline{ME}}{\overline{MR}}$$

also

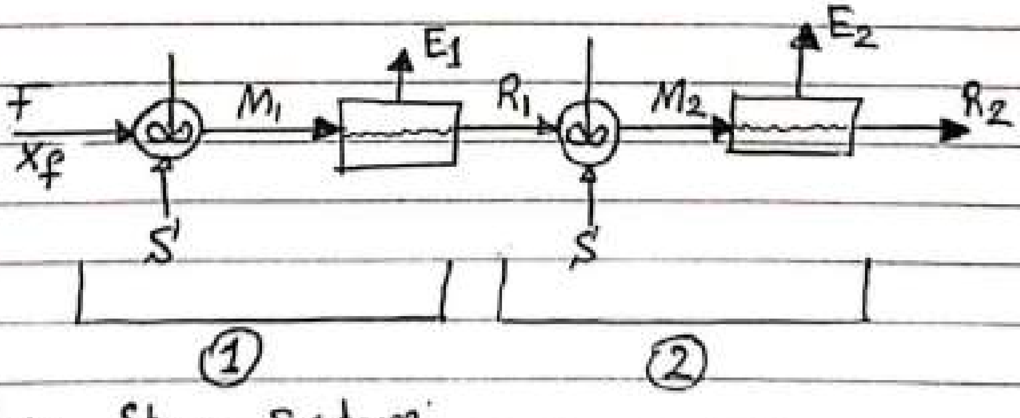
$$\frac{R}{E} = \frac{X_E - X_M}{X_M - X_R}$$



$$\frac{F}{S} = \frac{\overline{MS}}{\overline{MF}} \quad (1)$$

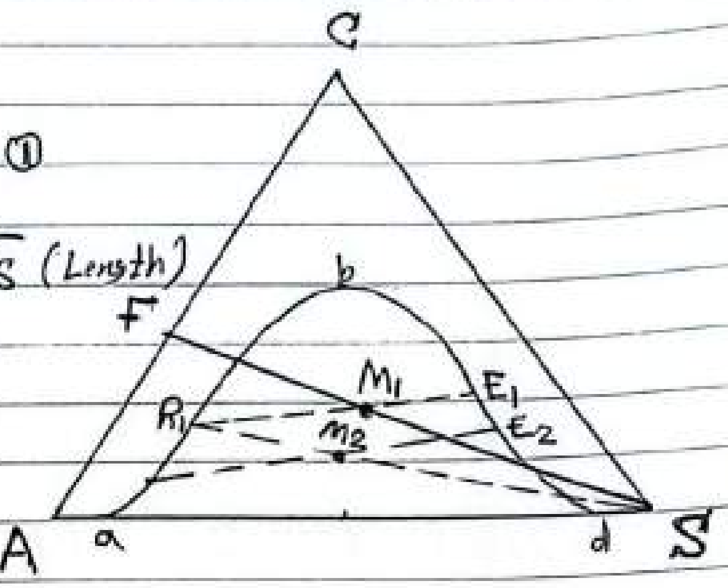
$$\overline{FS} = \overline{MS} + \overline{MF} \quad (2) \text{ (Length)}$$

(a) Co-Current System:



Consider a two Stage System
F = Feed with Component X_f.
S = Solute free Fresh Solvent.

M.B. on first Stage:



$$F + S = M_1 = R_1 + E_1 \quad \text{——} \; (1)$$

$$\frac{F}{S} = \frac{\overline{M_1S}}{\overline{M_1F}} \quad \text{and} \quad \overline{M_1S} + \overline{M_1F} = \overline{FS} \; \text{(Length)}$$

$$\frac{R_1}{E_1} = \frac{\overline{M_1E_1}}{\overline{M_1R_1}} \quad \text{——} \; (2)$$

R_1 and E_1 Can be obtained by Solving eq (1) and eq (2)

M.B. on Second Stage:

$$R_1 + S = M_2 = R_2 + E_2$$

$$\overline{M_2S} + \overline{M_2R_1} = \overline{SR_1}$$

$$\frac{R_1}{S} = \frac{\overline{M_2S}}{\overline{M_2R_1}}$$

Procedure:

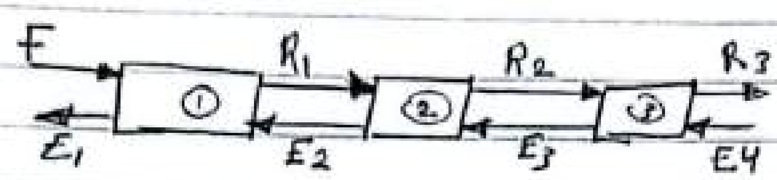
1) Locate M_1 using eq (1)

2) Locate R_1 and E_1 following a given tie line which nearest to M_1.

3) Locate point M_2 such that

$$\frac{R_1}{S} = \frac{\overline{M_2S}}{\overline{M_2R_1}}$$

(b) Counter Current Contact :-



M.B on First stage :-

$F + E_2 = R_1 + E_1 \Longrightarrow F - E_1 = R_1 - E_2$

M.B over two Stages (1-2) :-

$F + E_3 = R_2 + E_1 \Longrightarrow F - E_1 = R_2 - E_3$

M.B over n Stage (1-n) :-

$F + E_{n+1} = R_n + E_1 \Longrightarrow F - E_1 = R_n - E_{n+1}$

$R_1 - E_2 = R_2 - E_3 = R_n - E_{n+1} = P$

P = Common pole

all operating lines pass through P.

P is located outside phase triangular

To Final number of Stages :-

1. Join point F and S
2. Locate point M Such that $\frac{F}{S} = \frac{\overline{MS}}{\overline{MF}}$ and $\overline{FS} = \overline{MS} + \overline{MF}$

③ Draw a tie line through R_n to give E_1 (R_n is given)

④ Draw line E_1F_1 and R_nS to meet at pole point P.

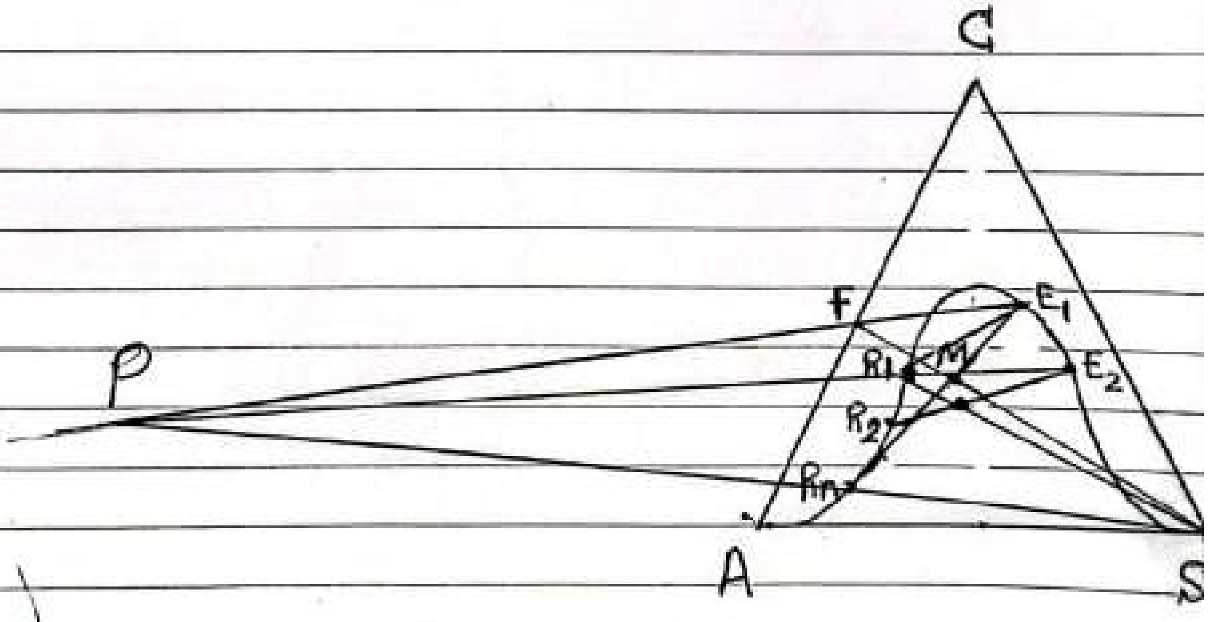
⑤ Locate R_1 by drawing tie line R_1E_1.

⑥ Locate E_2 by drawing O.L. PR_1 to cut equilibrium curve at E_2

⑦ Draw tie line R_2E_2 to Locate R_2.

⑧ Draw O.L. PR_2 to locate E_3.

⑨ Continue until Raffinate Composition < R_n, So number of Stages = number of tie lines.



Number of theoretical stages for counter current contact of immiscible solvent :

$$N = \frac{\log\left(\frac{X_N - A}{B}\right)}{\log \alpha}$$

$$\alpha = \frac{F}{mS} , \quad A = \frac{\frac{Y_1}{m} - \alpha X_f}{1-\alpha} , \quad B = \frac{X_f - \frac{Y_1}{m}}{1-\alpha}$$

Example (1) : A multiple three stages co-current system is used to extract benzoic acid from toluene using pure water as solvent. If the mass ratio of acid in the inlet feed and exit raffinate is 1.2 and 0.024 respectively. Calculate the concentration of raffinate exit from stage (1) and stage (2).

Sol :

$$N = \frac{\log \frac{X_n}{X_f}}{\log\left(\frac{\alpha}{\alpha+1}\right)}$$

$$3 = \frac{\log\left(\frac{0.024}{1.2}\right)}{\log\left(\frac{\alpha}{\alpha+1}\right)} \Rightarrow \log\left(\frac{\alpha}{\alpha+1}\right) = -0.566$$

on stage (1) :

$$1 = \frac{\log\left(\frac{X_1}{1.2}\right)}{\log\left(\frac{\alpha}{\alpha+1}\right)} = \frac{\log\left(\frac{X_1}{1.2}\right)}{-0.566}$$

$$X_1 = 0.325$$

On Stage (2) :

$$2 = \frac{\log\left(\frac{X_2}{1.2}\right)}{\log\left(\frac{\alpha}{\alpha+1}\right)} = \frac{\log\left(\frac{X_2}{1.2}\right)}{-0.566}$$

$$X_2 = 0.088$$

Example (2) : A multiple three stages Co-Current system is used to extract benzoic acid from toluene using pure water as solvent. If the mass ratio of acid in the inlet feed and exit raffinate is 1.2 and 0.024 respectively. The ratio of solvent to feed is 1.787. If this process (with other variables constant, X_f, X_n, $\frac{S}{F}$) is carried out in counter-current multistage system. Calculate:

(1) The number of stages (N).
(2) The concentration of raffinate exit from stage No (2).

Sol :

(1) Co-Current

$$N = \frac{\log\left(\frac{X_n}{X_f}\right)}{\log\left(\frac{\alpha}{1+\alpha}\right)} \Rightarrow 3 = \frac{\log\left(\frac{0.024}{1.2}\right)}{\log\left(\frac{\alpha}{1+\alpha}\right)}$$

$$\alpha = 0.372$$

$$\alpha = \frac{F}{mS} \Rightarrow 0.372 = \frac{F}{mS} \quad \text{---} (1)$$

$$\therefore \frac{S}{F} = 1.787 \quad \text{---} (2) \text{ sub in eq } (1)$$

$$\therefore \boxed{m = 1.5}$$

Counter-Current

$$N = \frac{\log\left[\frac{X_n - A}{B}\right]}{\log \alpha} \quad (*) \quad , \quad \alpha = \frac{F}{mS} = 0.372$$

M.B on N stages:

$$F(X_f - X_n) = S(Y_1 - Y_0)$$

$$Y_1 = \frac{F}{S}(X_f - X_n) = \frac{1}{1.787}(1.2 - 0.024) = 0.658$$

$$A = \frac{\frac{Y_1}{m} - \alpha X_f}{1-\alpha} = \frac{\frac{0.658}{1.5} - 0.372*(1.2)}{1-0.372}$$

$$A = -0.0123$$

$$B = \frac{X_f - \frac{Y_1}{m}}{1-\alpha} = \frac{1.2 - \frac{0.658}{1.5}}{1-0.372} = 1.212$$

$$\therefore N = \frac{\log\left[\frac{0.024 + 0.0123}{1.212}\right]}{\log 0.372} = 3.54 \simeq 4 \text{ Stages}$$

(2)

$$N = \frac{\log\left(\frac{X_2 - A}{B}\right)}{\log \alpha} \Rightarrow 2 = \frac{\log\left(\frac{X_2 - A}{B}\right)}{\log 0.372}$$

$$2(\log 0.372) = \log\left(\frac{X_2 - A}{B}\right)$$

$$\frac{X_2 - A}{B} = 0.138 \Rightarrow X_2 = 0.138(1.212) - 0.0123$$

$$X_2 = 0.155$$

Example (3) :- 160 cm^3/s of a solvent S used to treat 400 cm^3/s of a 10 per cent by mass solution of A in B, in a three-stage countercurrent multiple-contact liquid-liquid extraction plant. what is the composition of the final raffinate?

Using the same total amount of solvent, evenly distributed between the three stages, what would be the composition of the final raffinate if the equipment were used in a simple multiple-contact arrangement?

Equilibrium data:

Kg A/Kg B :	0.05	0.10	0.15
Kg A/Kg S :	0.069	0.159	0.258

Densities (kg/m³): $\rho_A = 1200$, $\rho_B = 1000$, $\rho_S = 800$

Solution

(a) *Countercurrent operation*

Considering the solvent **S**, $160 \text{cm}^3/\text{s} = 1.6 \times 10^{-4} \text{m}^3/\text{s}$

and: $$\text{mass flowrate} = (1.6 \times 10^{-4} \times 800) = 0.128 \text{ kg/s}$$

Considering the solution, $400 \text{cm}^3/\text{s} = 4 \times 10^{-4} \text{ m}^3/\text{s}$ containing, say, a m³/s **A** and $(4 \times 10^{-4} - a)$ m³/s **B**.

Thus: $$\text{mass flowrate of } \mathbf{A} = 1200a \text{ kg/s}$$

and: $$\text{mass flowrate of } \mathbf{B} = (4 \times 10^{-4} - a)1000 = (0.4 - 1000a) \text{ kg/s}$$

a total of: $$(0.4 + 200a) \text{ kg/s}$$

The concentration of the solution is:

$$0.10 = 1200a/(0.4 + 200a)$$

Thus: $$a = 3.39 \times 10^{-5} \text{m}^3/\text{s}$$

$$\text{mass flowrate of } \mathbf{A} = 0.041 \text{ kg/s, mass flowrate of } \mathbf{B} = 0.366 \text{ kg/s}$$

and: $$\text{ratio of } \mathbf{A/B} \text{ in the feed, } X_f = (0.041/0.366) = 0.112 \text{ kg/kg}$$

The equilibrium data are plotted in Figure 13.15 and the value of $X_f = 0.112$ kg/kg is marked in. The slope of the equilibrium line is:

$$(\text{mass flowrate of } \mathbf{B})/(\text{mass flowrate of } \mathbf{S}) = (0.366/0.128) = 2.86$$

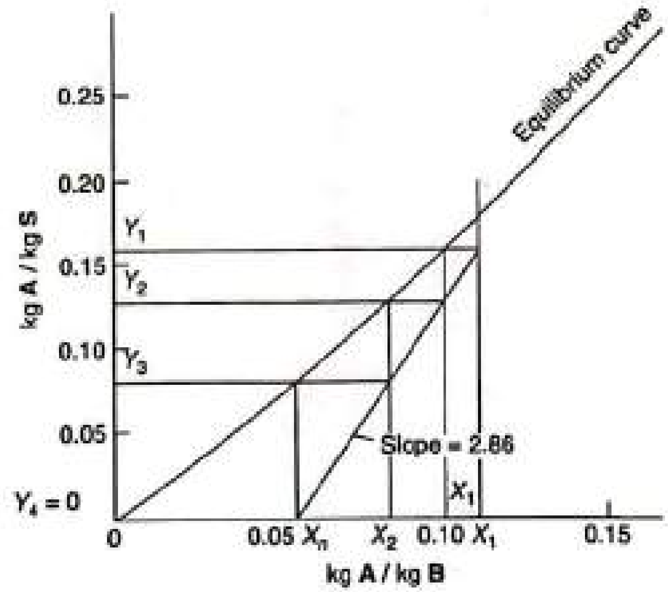


Figure 13.15. Construction for Example 13.1

Since pure solvent is added, $Y_{n+1} = Y_4 = 0$ and a line of slope 2.86 is drawn in such that stepping off from $X_f = 0.112$ kg/kg to $Y_4 = 0$ gives exactly three stages.

When $Y_4 = 0$, $X_n = X_3 = 0.057$ kg/kg.

Thus: the composition of final raffinate is **0.057 kg A/kg B**

(b) *Multiple contact*

In this case, (0.128/3) = 0.0427 kg/s of pure solvent **S** is fed to each stage.

Stage 1

$$X_f = (0.041/0.366) = 0.112\text{kg/kg}$$

and from the equilibrium curve, the extract contains 0.18 **A**/kg **S** and $(0.18 \times 0.0427) =$ 0.0077 kg/s **A**.

Thus: raffinate from stage 1 contains $(0.041 - 0.0077) = 0.0333$ kg/s **A** and 0.366 kg/s **B**

and: $$X_1 = (0.0333/0.366) = 0.091\text{kg/kg}$$

Stage 2

$$X_1 = 0.091\text{kg/kg}$$

and from Figure 13.15 the extract contains 0.14 kg **A**/kg **S**

or: $$(0.14 \times 0.0427) = 0.0060 \text{ kg/s } \mathbf{A}$$

Thus: the raffinate from stage 2 contains $(0.0333 - 0.0060) = 0.0273$ kg/s **A** and 0.366 kg/s **B**

Thus: $$X_2 = (0.0273/0.366) = 0.075 \text{ kg/kg}$$

Stage 3

$$X_2 = 0.075 \text{ kg/kg}$$

and from Figure 13.15, the extract contains 0.114 kg **A**/kg **S**

or: $$(0.114 \times 0.0427) = 0.0049 \text{ kg/s } \mathbf{A}.$$

Thus: the raffinate from stage 3 contains $(0.0273 - 0.0049) = 0.0224$ kg/s **A** and 0.366 kg/s **B**

and: $$X_3 = (0.0224/0.366) = 0.061 \text{ kg/kg}$$

Thus: the composition of final raffinate = **0.061kg A/kg B**

Example(4):- In a counter current extraction system 100 kg/hr of 40% acetone-water solution is to be reduced not to greater than 10% acetone by extraction with pure trichloroethane at 25°C.

(a) If the solvent rate is 32.5 kg/hr, find the number of stages required.

(b) For condition of part (a) find the weights of all streams.

(C) Repead (a) and (b) for case of co-current.

Equilibrium data in wt%					
Water layer			Trichloroethane layer		
T.C.E	water	acetone	T.C.E	water	acetone
0.52	93.52	5.96	90.93	0.32	8.75
0.73	82.23	17.04	73.76	1.1	25.14
1.02	72.06	26.92	59.21	2.27	38.52
1.17	67.95	30.88	53.92	3.11	42.97
1.6	62.67	35.73	47.53	4.26	48.21
2.1	57	40.9	40	6.05	53.95
3.75	50.2	46.05	33.7	8.9	57.4
6.52	46.7	51.78	26.2	13.4	60.34

Sol: (a) ① locate point M

$$F + S = M = 100 + 32.5 = 132.5$$

$$F x_f + S y_o = M x_m$$

$$x_M = \frac{F x_f + S y_o}{M} = \frac{100(0.4) + 0}{132.5} = 0.3$$

OR

$$\overline{FS} = \overline{MS} + \overline{MF} \quad \text{—— ①}$$

$$11\,cm = \overline{MS} + \overline{MF}$$

$$\frac{F}{S} = \frac{\overline{MS}}{\overline{MF}} \Rightarrow \frac{100}{32.5} = \frac{\overline{MS}}{\overline{MF}}$$

$$\overline{MS} = 3.076\,\overline{MF} \quad \text{—— ②} \quad \text{sub in eq ①}$$

$$11 = 3.076\,\overline{MF} + \overline{MF}$$

$$11 = 4.076\,\overline{MF} \Rightarrow \overline{MF} = 2.7\,cm$$

* from plot measure ($\overline{MF} = 2.7\,cm$) and locate point M.

* from plot ($E_1 = 0.48$) (read from X-axis)
* From plot ($R_1 = 0.34$) (read from Y-axis)
* From plot ($E_2 = 0.56$) (read from X-axis)
* From plot ($R_2 = 0.29$) (read from Y-axis)
* From plot ($E_3 = 0.65$) (read from X-axis)
* From plot ($R_3 = 0.22$) (read from Y-axis)
* From plot ($E_4 = 0.76$) (read from X-axis)
* From plot ($R_4 = 0.13$) (read from Y-axis)

∴ No. of Stage = 4 Stages [R_4 is very close to $R_N = 0.1$]

(b) * overall mass balance

$$F + S = R_N + E_1 \quad \text{—} ①$$

$$F x_f + S y_0 = R_N X_N + E_1 y_1 \quad \text{—} ②$$

* We can read the concentration from plot :

- From E_1 at X-axis ⟶ $y_1 = 0.48$
- From R_1 at Y-axis ⟶ $X_1 = 0.34$
- From E_2 at X-axis ⟶ $y_2 = 0.56$
- From R_2 at Y-axis ⟶ $X_2 = 0.29$
- From E_3 at X-axis ⟶ $y_3 = 0.65$
- From R_3 at Y-axis ⟶ $X_3 = 0.22$
- From E_4 at X-axis ⟶ $y_4 = 0.76$
- From R_4 at Y-axis ⟶ $X_4 = 0.13$

∴ $100 + 32.5 = R_n + E_1$ — ①

$100(0.4) + 0 = R_n(0.1) + E_1(0.48)$ — ②

∴ $E_1 = 70.39$ kg/hr

$R_n = 62.1$ kg/hr

* Material balance on first stage :

$F + E_2 = R_1 + E_1$ — ①

$F x_f + E_2 y_2 = R_1 X_1 + E_1 y_1$ — ②

∴ $100 + E_2 = R_1 + 70.39$ — ①

$100(0.4) + E_2(0.56) = R_1(0.34) + 70.39(0.48)$ — ②

from eq ① and eq ②: $E_2 = 17.48$ kg/hr
$R_1 = 47.1$ kg/hr

* Material balance on Second Stage:

$$R_1 + E_3 = R_2 + E_2 \quad \text{①}$$
$$R_1 X_1 + E_3 y_3 = R_2 X_2 + E_2 y_2 \quad \text{②}$$

$$\therefore 47.1 + E_3 = R_2 + 17.48 \quad \text{①}$$
$$47.1(0.34) + E_3(0.65) = R_2(0.29) + 17.48(0.56) \quad \text{②}$$

from eq ① and eq ②: $E_3 = 6.575$ kg/hr
$R_2 = 36.195$ kg/hr

Thus, Continue to find E_4, R_3 using material balance on the third stage.

(C) for Co-current

(a) ① Locate point M_1

$$F + S = M_1 \quad \text{①}$$
$$100 + 32.5 = M_1$$
$$M_1 = 132.5$$
$$F x_f + S y_0 = M_1 X_{M_1} \quad \text{②}$$
$$100(0.4) + 0 = 132.5 * X_{M_1} \quad \text{②}$$
$$X_{M_1} = 0.3$$

OR $\overline{FS} = \overline{M_1 S} + \overline{M_1 F} \quad \text{①}$

$$11\ cm = \overline{M_1 S} + \overline{M_1 F}$$

$$\frac{F}{S} = \frac{\overline{M_1 S}}{\overline{M_1 F}} \Rightarrow \frac{100}{32.5} = \frac{\overline{M_1 S}}{\overline{M_1 F}} \Rightarrow \overline{M_1 S} = 3.076\,\overline{M_1 F} \quad \text{②}$$

sub in eq ①

∴ $\overline{M_1F} = 2.7$ cm

* From plot measure ($\overline{M_1F} = 2.7$cm) and locate point M_1.

* Passes a line from M_1 and parallel to close tie line; then Find E_1, R_1.

$E_1 = 0.61 = y_1$, $R_1 = 0.25 = X_1$

$E_1 + R_1 = M_1$ — ①
$E_1 + R_1 = 132.5$ — ①
$E_1 y_1 + R_1 X_1 = M_1 X_{m_1}$ — ②
$E_1 (0.61) + R_1 (0.25) = 132.5 (0.3)$

$R_1 = 114.1$ kg/hr.
$E_1 = 18.4$ kg/hr.

* Join a line of $R_1 - S$.

② locate point M_2.

$R_1 + S = R_2 + E_2 = M_2$ — ①
$R_1 X_1 + S(y_0) = M_2 X_{m_2}$ — ②

$M_2 = 114.1 + 32.5 = 146.6$ — ① Sub in eq ②

$114.1 (0.25) + 0 = 146.6 \times (X_{M_2})$ — ②

$X_{M_2} = 0.194$

* Passes a line from M_2 and parallel to close tie line then find E_2, R_2

$E_2 = 0.72 = y_2$, $R_2 = 0.17 = X_2$

$E_2 + R_2 = M_2$ —①
$E_2 + R_2 = 146.6$ —①

$E_2 y_2 + R_2 X_2 = M_2 X_{m_2}$ —②
$E_2 (0.72) + R_2 (0.17) = 146.6 (0.194)$ —②

$R_2 = 140.2 \frac{kg}{hr}$
$E_2 = 6.4 \frac{kg}{hr}$

Thus, continue untill to get $R_n = 0.1$

H.W :- Benzoic acid in a solution with toluene is to be extracted using pure water as solvent in a counter current multistage extraction unit. The benzoic acid conc. in the toluene feed is 22.6×10^{-3} wt fraction. The weight of water to toluene is 14.5. 98 percent of benzoic acid is to be recovered in the water phase, Assumme that the two phases are completely immiscible solvents. Calculate the no. of theortical stages required in the following cases :-

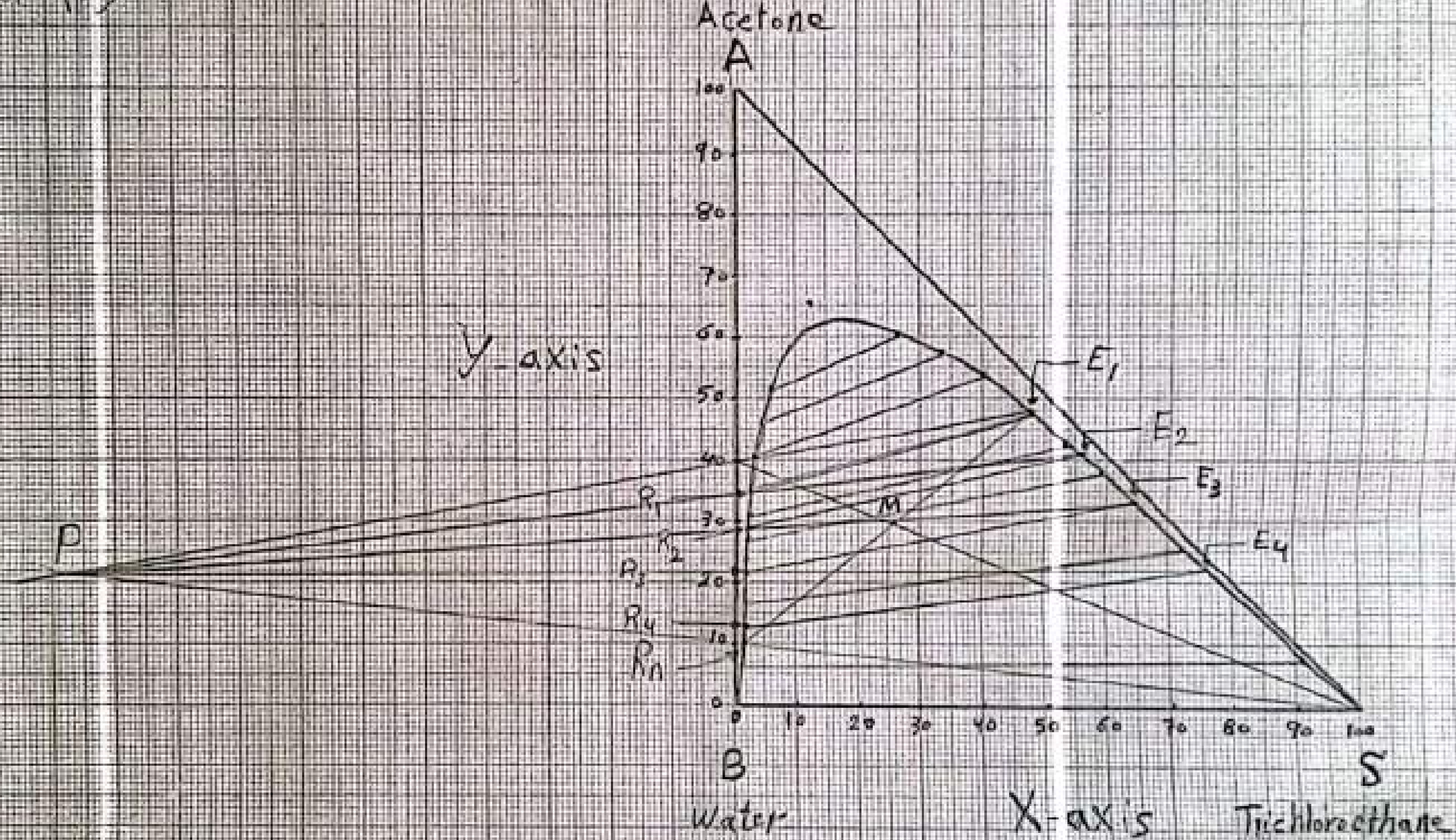
(a) If the equilibrium data is given as :

$X (\times 10^{-3})$	2	6	10	14	18	22	26
$Y (\times 10^{-4})$	2.9	7.2	10.1	12.4	14.2	15.6	16.8

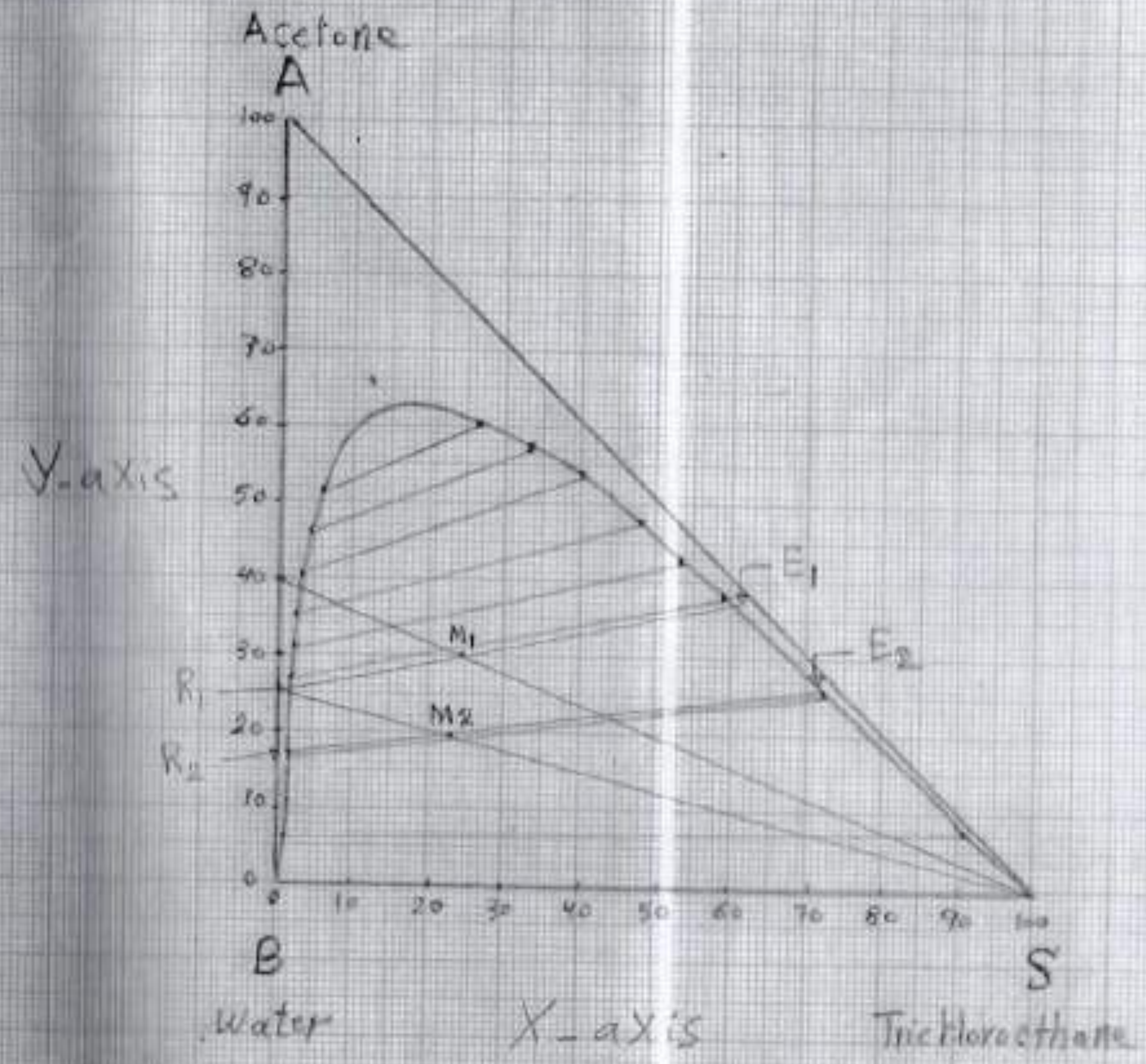
where X is wt of benzoic acid / wt of toluene
Y is wt of benzoic acid / wt of water

(b) If the equilibrium data is linear $Y = 0.074 X$
(c) Repeat (a) if the unit is co-current.
(d) Repeat (b) if the unit is co-current.

Example (14)
(a) and (b)



Example (4)
(C)


Acetone
A
100
90
80
70
60
50
40
30
20
10
0
Y-axis
R1
R2
M1
M2
E1
E2
0
10
20
30
40
50
60
70
80
90
100
B
Water
X-axis
S
Trichloroethane