Northern Technical University

Oil & Gas technologies engineering- Kirkuk

Department of Renewable Energy Techniques Engineering

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Level 1 (2023-2024)

First Semester

Mechanics

المادة: الميكانيك الهندسي
المستوى الاول
المحاضرة الاولى

مبادىء علم الهندسة الميكانيكية

د.أفراح تركي

Mechanics I- Static
1st level
1st Lecture

Outlines

1.1 Basic fundamentals of Mechanics.

1.2 Basic terms.

1.3 Laws of Mechanics.

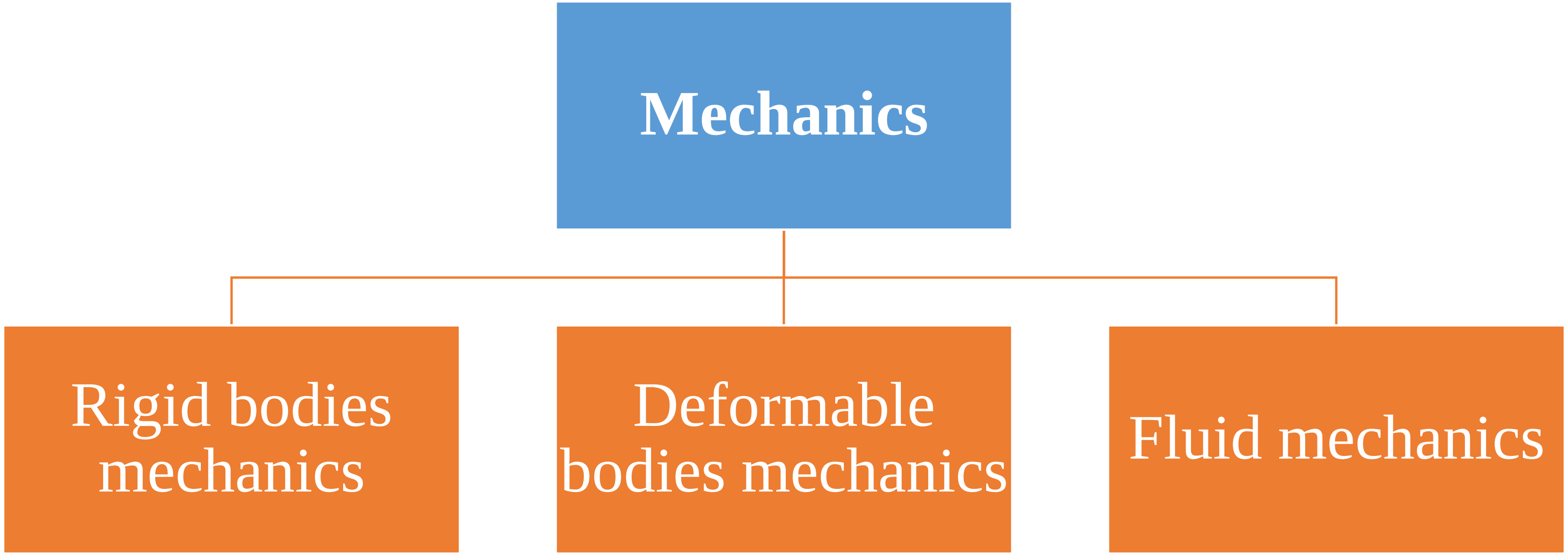
1.4 Units and dimensions of quantities.

1.5 Units and their relations.

1.6 Example.

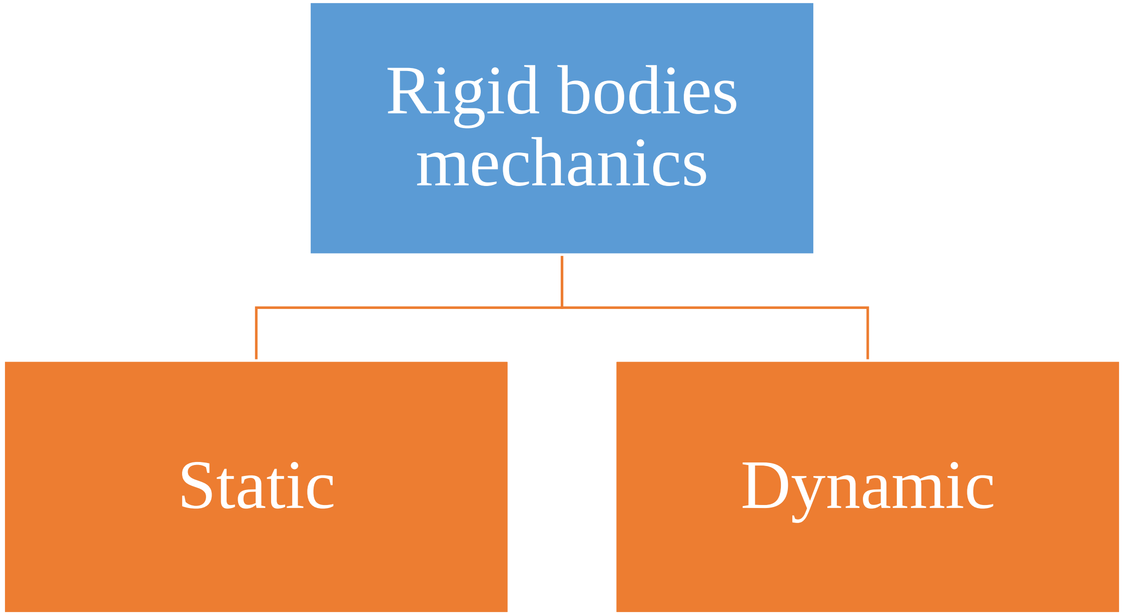
1.1 Basic fundamentals of Mechanics:

- **Mechanics:** is a physical science which concerned with the state of bodies that are influenced by forces.



1.1 Basic fundamentals of Mechanics:

- Deformable bodies mechanics can be considered when the shape of the body is important.
- Rigid bodies mechanics mean that the body keeps the same shape after applying force.



1.1 Basic fundamentals of Mechanics:

- **Statics**: investigates the equilibrium of bodies that are either at rest or move with constant velocity.

- **Dynamics:** is other branch of mechanics. In contrast to the static, it deals with accelerated motion of bodies under effect of external forces.

1.1 Basic fundamentals of Mechanics:

- **Vector quantity**: is the quantity that has magnitude and direction, for example: velocity, acceleration, displacement, distance, weight, force.
- **Scalar quantity**: is the quantity that has only magnitude, for instance: time, size, density, volume.
- **Force**: is the action that changes or tends to change the state of motion of the body.

1.2 Basic terms

- **Mass:** is the quantity of the matter owned by body. It cannot be changed unless the body damages and lost part of it.
- **Length:** is used to measure the linear distances.
- **Time:** is the measurements of the succession of events.

1.2 Basic terms

- **Displacement:** is the distance moved by the body in a specified direction.
- **Velocity:** can be defined as the rate of change of displacement with respect to time.
- **Acceleration:** is the rate of change of velocity with respect to time.

1.3 Laws of Mechanics

i. Newton’s first law:

1.3 Laws of Mechanics

ii. Newton's second law: if an external force acts on a particle, the particle will be accelerated in the direction of the force.

The magnitude of the acceleration will be directly proportional to the force and inversely proportional to the mass of the particle.

1.3 Laws of Mechanics

According to Newton's second law,

- Force = rate of change of momentum.
- momentum = mass × velocity, (mass do not change),
- Force = mass × rate of change of velocity
- i.e., Force = mass × acceleration

$F = m \times a$

1.3 Laws of Mechanics

iii. Newton's third law: states that for every action there is an equal reaction with opposite direction.

1.4 Units and dimensions of quantities

- There are four systems of units used for the measurement of physical quantities:

1. FPS (Foot – Pound – Second) system
2. CGS (Centimeter – Gram – Second) system
3. MKS (Meter - Kilogram – Second) system
4. SI (System International).

Table (1-1) shows the SI units

Quantity	Units in SI system
Length	m
Mass	kg
Area	m^2
Volume	m^3
Velocity	$m.\ sec^{-1}$
Acceleration	$m.\ sec^{-2}$
Momentum	$kg.\ m.\ sec^{-1}$
Stress	$kg.\ m^{-1}.\ sec.^{-2}$
Force	N ($kg.\ m.\ sec.^{-2}$)
Power	$kg.\ m^2.\ sec.^{-3}$
Density	$kg.\ m^{-3}$

1.5 Units and their relations

Table (1-2) shows the SI and U. S. units

Quantity	**Symbol**	**SI unit**		**U.S customary unit**	
		Unit name	Symbol	Unit name	Symbol
Mass	M	Kilogram	kg	Slug	slug
Length	L	Meter	m	Foot	ft
Time	T	Second	s	Second	Sec
Force	F	Newton	N	pound	Ib

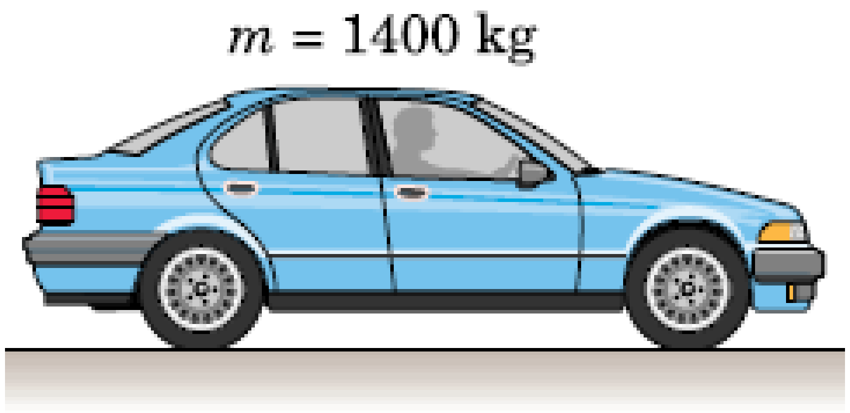
Table (1-3) shows the units conversions

1 m	100 cm
1 in	2.54 cm
1 m	1000 mm
1 ft	12 in
1 km	1000 m
1 mile	1609.1 m
1 yard	3 ft
1 kg	2.204 Ib (pound)
1 ton	1000 kg

1.6 Example:

Example 1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds. (Knows that 1 slug = 14.594 kg, and g= 9.81 m. sec^{-2}, and in British unit g= 32.2 ft.sec^{-2}).



1.6 Example:

Solution:

According to the Newton second law:

F= m g
F= 1400 * 9.81 = 13734 N

m= 1400 kg [1 slug/ 14.594 kg]

m=95.93 slugs

1.6 Example:

Solution:

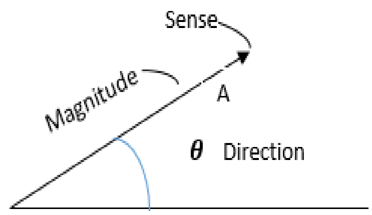
F= m g
F= 95.93* 32.2

F= 3088.9 Ib

The vectors (2nd lecture)

2.1 Vectors and force analysis:

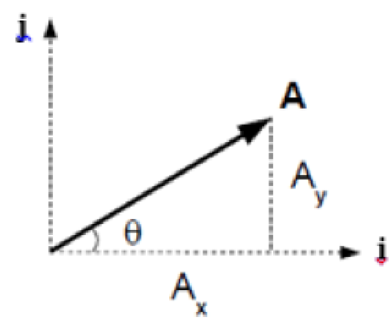
A vector is shown graphically by an arrow. The length of the arrow represents the magnitude of the vector, and the angle between the vector and a fixed axis defines the direction of its line of action. The head or tip of the arrow indicates the sense of direction of the vector.



2.2 The rectangular form of vector in 2-Dimensions:

The rectangular form of vector in 2-Dimensions can be written as follows:

$\vec{A}$ = Ax i + Ay j



The magnitude of the resultant vector can be found by:

A= $\sqrt{Ax^2 + Ay^2}$

The x and y components can be found by the following equations:

Ax= A cos θ

Ay= A sin θ

The direction of vector is found by:

tan $\theta = \frac{Ay}{Ax}$

$\theta = \tan^{-1} \frac{Ay}{Ax}$

The unit vector is:

n = $\frac{\vec{A}}{A}$

2.3 Vector Operations (Vectors' addition):

<table>
<tr><th>Parallelogram law</th><th>Triangular law</th></tr>
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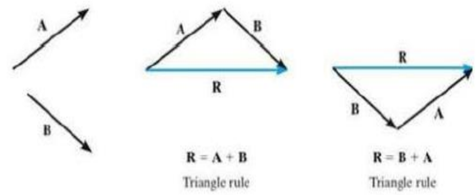

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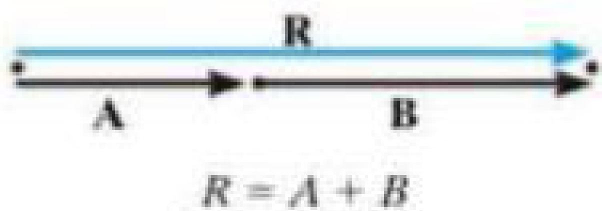
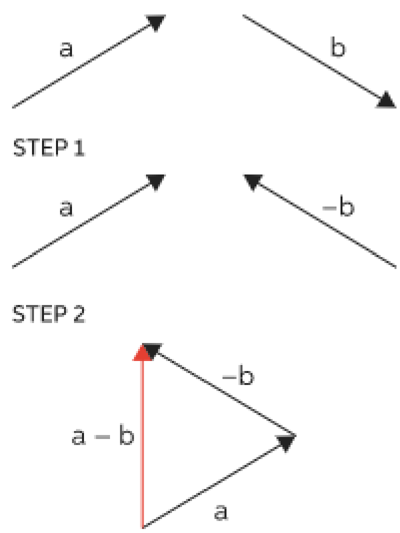
As a special case, if the two vectors A and B are collinear (both have the same line of action), the parallelogram law reduce to an algebraic or scalar addition:

R= A-B= A+ (-B)

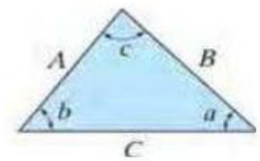




2.4 Cosine Law and Sine law:

The cosine law and sine law are applicable to compute angles and sides of a triangle.

- **Law of cosines:** $C = \sqrt{A^2 + B^2 - 2A.B\cos c}$ ...(1)
- **Law of sines:** $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$ ...(2)

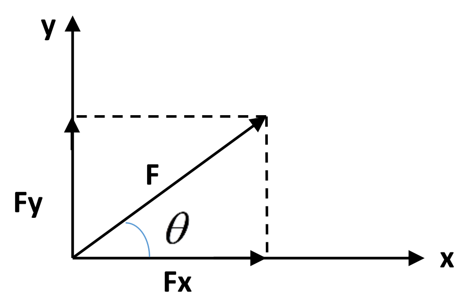


2.3 Resolution of force

The analysis of force F to its x and y components are:

Fx= F cos θ

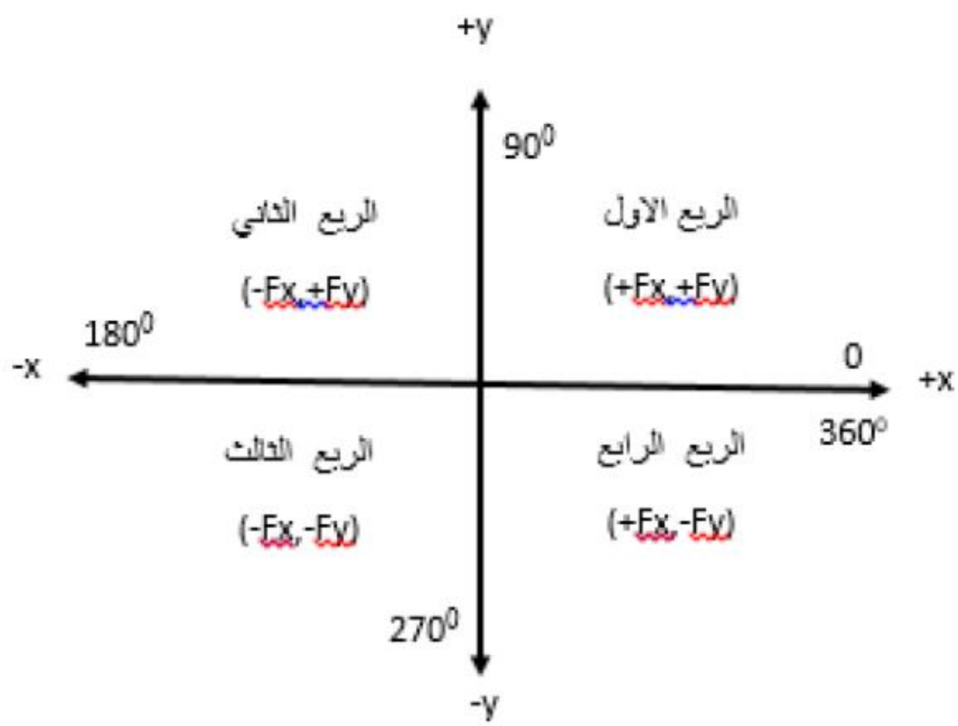
Fy= F sin θ



F= $\sqrt{Fx^2 + Fy^2}$

tan θ= $\frac{Fy}{Fx}$, θ=tan^{-1} ($\frac{Fy}{Fx}$)

- توزيع الزوايا على مخطط الاعداد xy axis



ملاحظة / إذا كان جواب الزاوية موجب (+) فترسم الزاوية ابتداءاً من المحور X وبعكس عقارب الساعة.

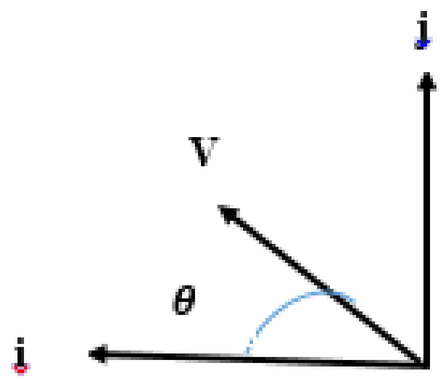
Examples

Example (1): Determine the angle made by the vector $\vec{V}$= -10 i+ 24 j with the positive x-axis. Write the unit vector (n) in the direction of V.

Solution:

V= $\sqrt{Ax^2 + Ay^2}$

V= $\sqrt{10^2 + 24^2} = 26$



tan θ = Ay/ Ax

tan θ = 24/ (-10)

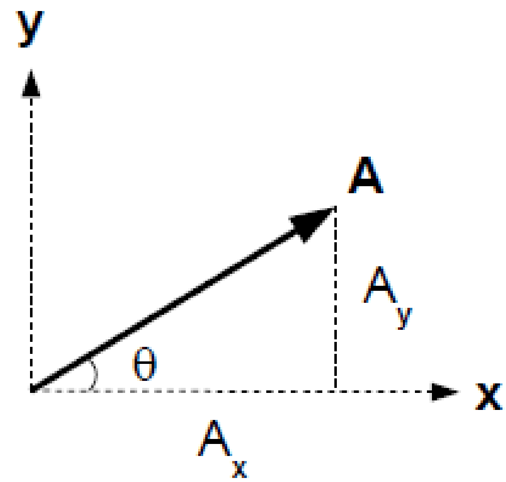
$\theta = 112.6^0$

n=$\frac{\vec{V}}{V}$

n=$\frac{-10\ i+24j}{26}$

n= -0.385 i + 0.923 j

Example (2): Write the vector in a rectangular form if A= 5 N and θ= 36.8^0



Solution:

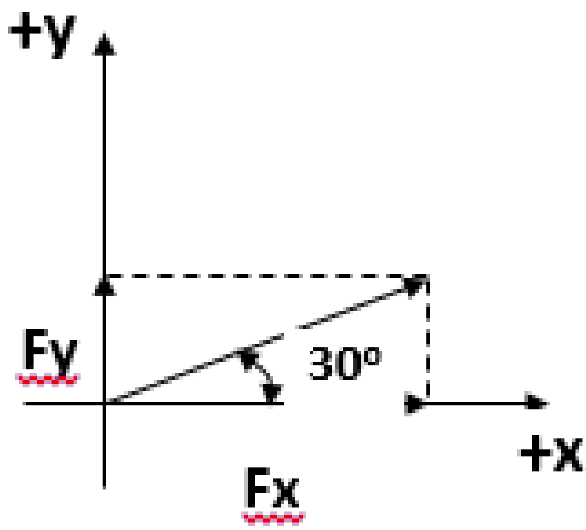
Ax= A cos θ
Ax= 5 cos 36.8^0
Ax= 4 N

Ay= A sin θ
Ay= 5 sin 36.8^0
Ay= 3 N

The vector in a rectangular form:
A= Ax i + Ay j

A= 4 i + 3 j

Example (3):- Determine the X (Fx) and Y (Fy) components of 4 N as shown in figure below.



Solution:-

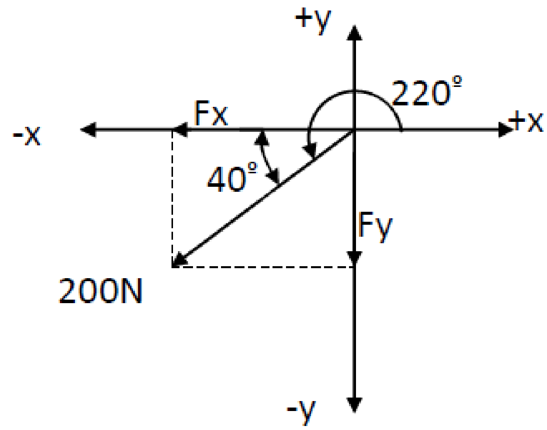
$$Fx = F \cos 30$$

$$\therefore Fx = 4 \cos 30 = +3.46 \text{ N} \quad \overset{+}{\longrightarrow}$$

$$Fy = F \sin 30$$

$$\therefore Fy = 4 \sin 30 = +2\text{N} \quad \uparrow$$

Example (4): Determine the X (Fx) and Y (Fy) components of 200 N force as shown in the below figure, when θ= 220º.



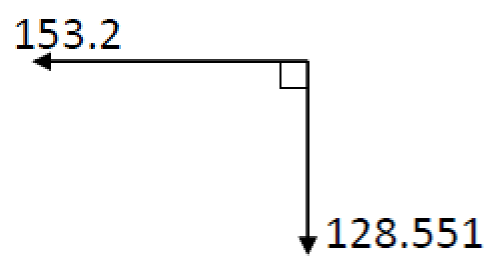
Solution:

$Fx = F \cos \theta$

$Fx = 200 \cos 220^{o}$

$Fx = -153.2$ N

$Fx = 153.2$ N ←



$Fy = F \sin \theta$

$Fy = 200 \sin 220^{o}$

$Fy = -128.55$ N

$Fy= 128.55$ N ↓

Other way for solution:

Angle= 220-180= 40^0

Fx = F cos θ

Fx = 200 cos 40^0

Fx = 153.2 N

Fy = 200 sin θ

Fy = 200 sin 40^0

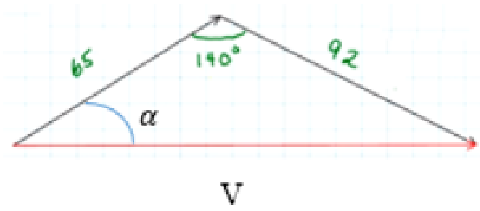
Fy = 128.55 N

The locations of the force components are in the 3rd quarter (angle more than 180^0 and less than 270^0)

Example (5) find the magnitude and direction of V where v1= 65 and v2=92 and θ is 140^0.

Solution:

Apply the cosine law to find the magnitude



V= $\sqrt{(V1)^2 + (V2)^2 - 2V1V2\cos\theta}$

V= $\sqrt{(65)^2 + (92)^2 - 2 * 65 * 92 * \cos 140}$

V= 148

Apply the sine law to find the direction:

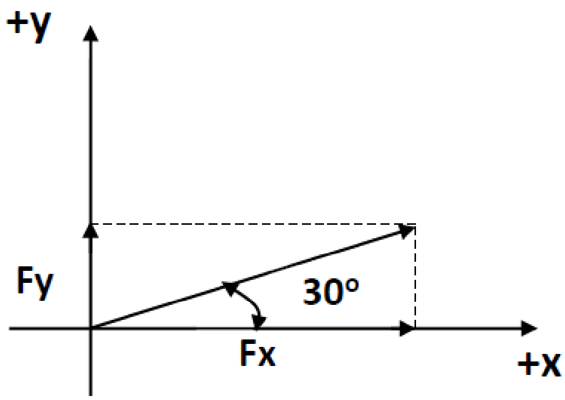
$$\frac{\sin\alpha}{92} = \frac{\sin 140}{V}$$

$\alpha = 24^0$

Example (6):- the direction of the force (F) is 30^0, find the horizontal component if the vertical component is 30 N?

Solution:

From the diagram shown:



Fy= 30 N

Fy= F sin θ

30 = F sin 30

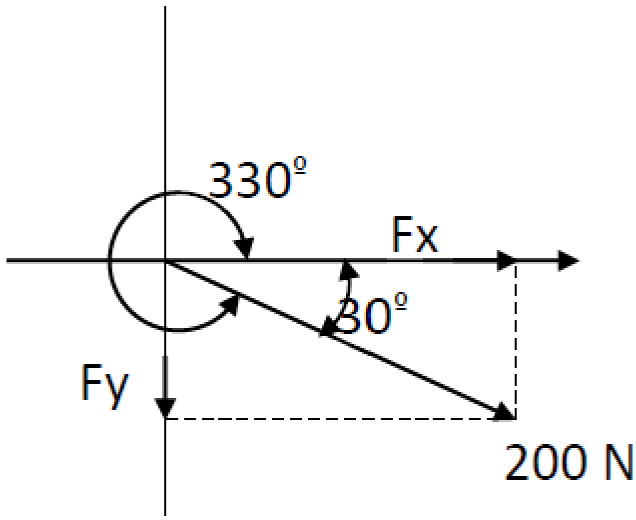
F= 60 N

Fx= F cos θ

Fx= 60* cos 30

Fx=51.96 N

Example (7): Determine the X and Y components of 200 N force with angle= 330º, as shown in the figure.



Solution:-

Angle= 360-330 =30^0

$Fx = F \cos \theta$

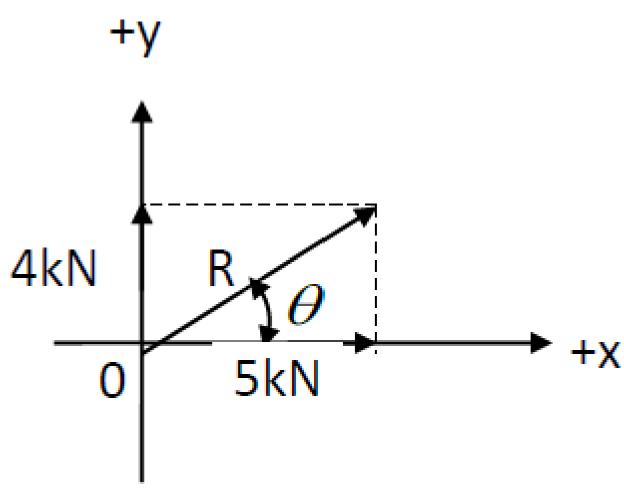
$Fx = 200 \cos 30^o$

Fx = 173.2 →

$Fy = F \sin \theta$

Fy = 100 N ↓

Example (8): Determine the (magnitude and direction) of the resultant (R) of two forces as shown in the below figure.

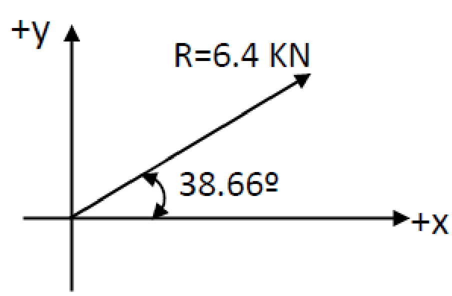


Solution:

$R = \sqrt{Fx^2 + Fy^2}$

$R = \sqrt{5^2 + 4^2} = 6.4$ kN

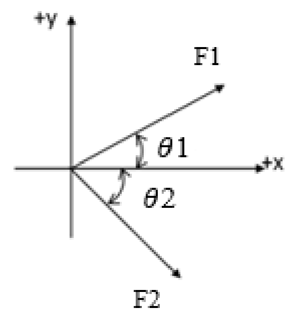
$\theta = \tan^{-1}(4/5) = 38.66^o$



The vectors (3rd lecture)

3.1 Resultant forces in 2-Dimensions

The resultant forces applied when there are more than one force affects in the x-y directions as shown in figure below:



$Fx= \sum Fxi$, $Fx= Fx1+Fx2$

$Fy= \sum Fyj$, $Fy= Fy1+Fy2$

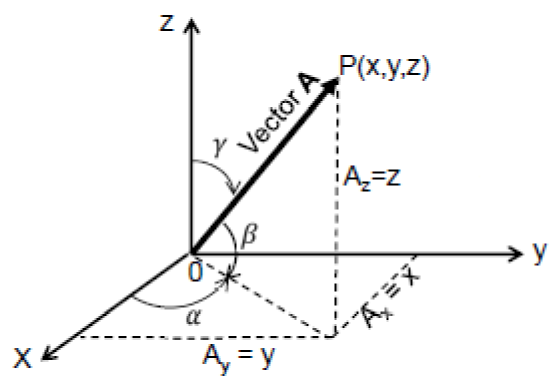
$R= \sqrt{Fx^2 + Fy^2}$

The body is in equilibrium, when $\sum F= 0$

3.2 Vector in 3 dimension

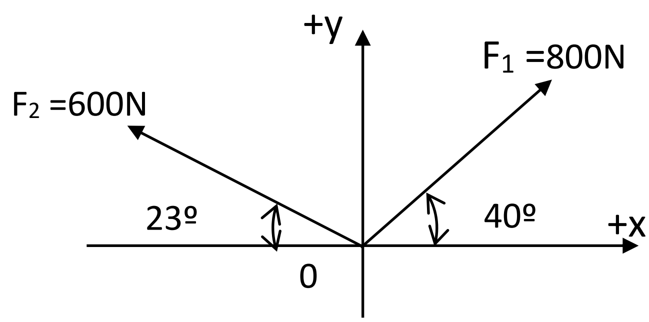
The magnitude of the resultant vector can be found by:

$A= \sqrt{(Ax)^2 \; + \; (Ay)^2 + (Az)^2}$

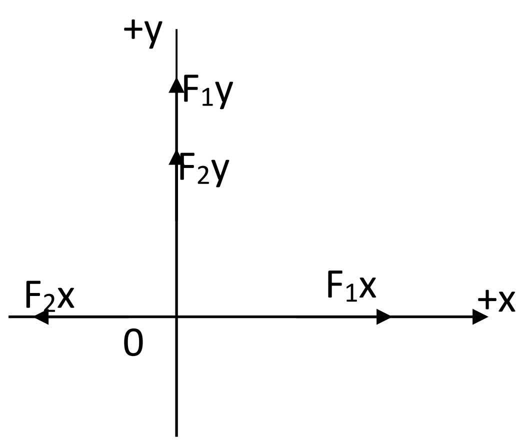


Examples

Example (9): Determine the (magnitude and direction) of the resultant (R) of two forces as shown in Figure below.



Solution:

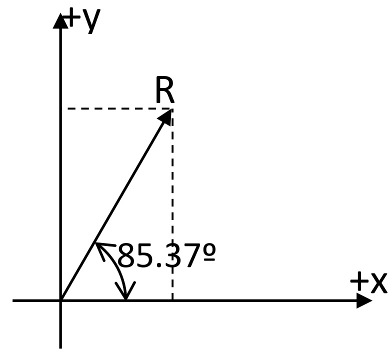


Force	Components in the X direction	Components in the Y direction
F_1	F_1x = 800 cos40º = 612.83 N	F_1y = 800 sin40 = 514.23 N
F_2	F_2x = -600 cos23º = -552.3 N	F_2y = 600 sin23º = 234.43 N
المجموع	Rx=∑Fx= Fx1+Fx2	Ry=∑Fy= Fy1+Fy2
المجموع	Rx = ∑Fx = 60.53 N ⟶	Ry = ∑Fy = 748.66 N ↑

R= $\sqrt{Rx^2 + Ry^2}$

R= $\sqrt{(60.53)^2 + (748.66)^2}$

R = 751.1 N



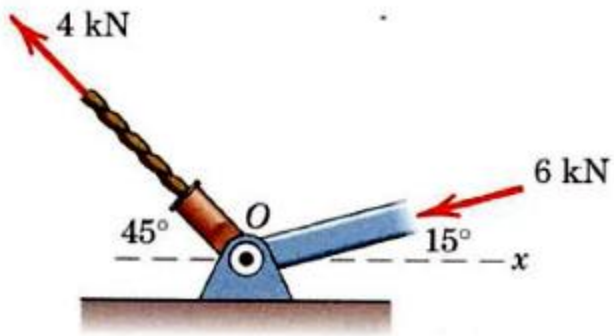
θ= $\tan^{-1}$ (Ry/Rx)

θ= $\tan^{-1}$ (748.66/60.53)

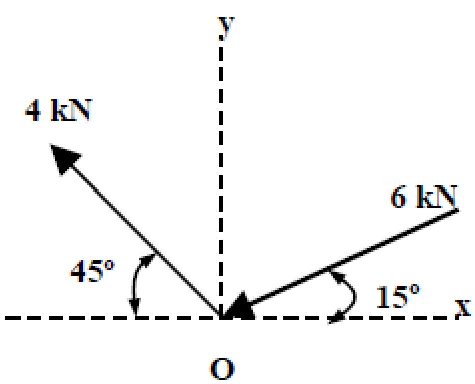
θ= 85.37^0

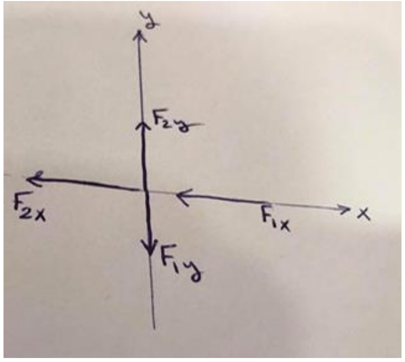
Example (10):

Determine the (magnitude and direction) of the resultant of the force system.



Solution:





Force	Components in the X direction	Components in the Y direction
F_1	$F_1x = -6 \cos 15^o$	$F_1y = -6 \sin 15$
F_2	$F_2x = -4 \cos 45^o$	$F_2y = 4 \sin 45^o$
المجموع	$Rx = \sum Fx = Fx1 + Fx2$	$Ry = \sum Fy = Fy1 + Fy2$
المجموع	$Rx = \sum Fx = -8.62$ kN ⟵	$Ry = \sum Fy = 1.276$ kN ↑

R= $\sqrt{Rx^2 + Ry^2}$

R= $\sqrt{(-8.62)^2 + (1.276)^2}$

R =8.72 kN

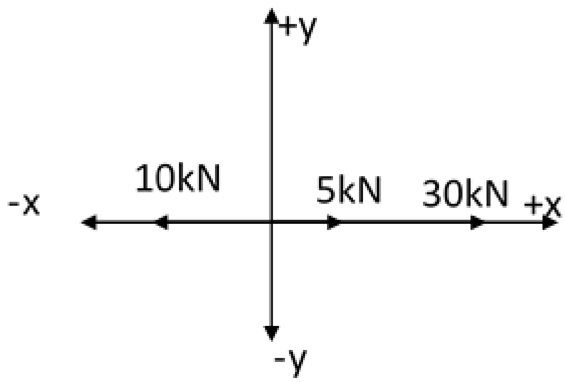
θ= $\tan^{-1}$ (Ry/Rx)

θ= $\tan^{-1}$ (1.276/-8.62)

θ= 171.6^0

Example (11):

Determine the (magnitude and direction) of the resultant of the force system as shown in Figure below.



Solution:

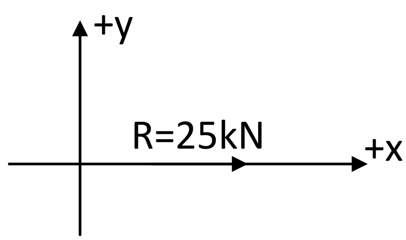
$R = F_1 + F_2 - F_3$

$R = 30 + 5 - 10$

$R = 25$ kN → تتجه دائما باتجاه الأكبر

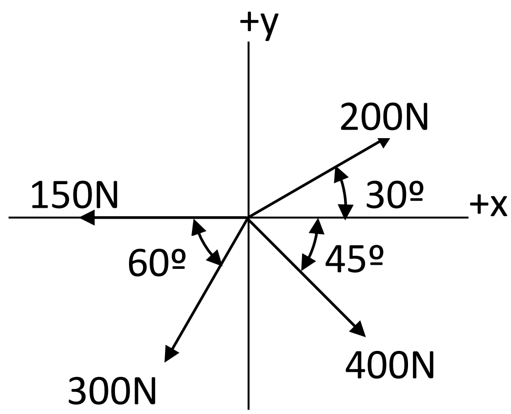
$\theta = 0$

$\theta = \tan^{-1}(0/25) = 0$



Example (12):

Determine the (magnitude and direction) of the resultant (R) of the forces system as shown in the figure.

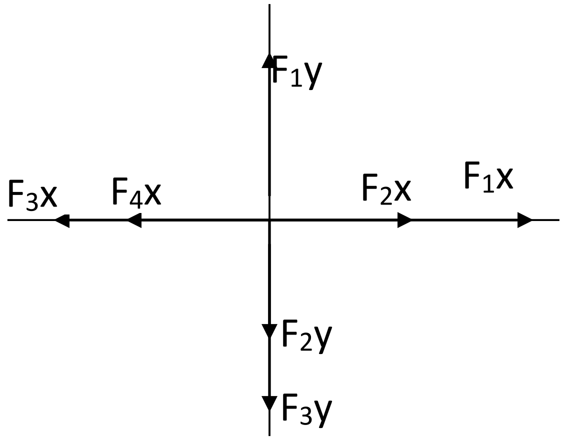


Solution:

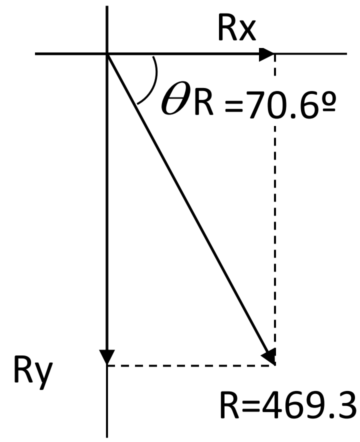
Rx = $\sum$ Fx

Ry = $\sum$ Fy

Force	Components in the X direction X – axis	Components in the Y direction Y - axis
F_1	+200 cos 30º = +173.2 N	200 sin 30º = +100 N
F_2	+400 cos 45º = +282.8 N	400 sin 45º = -282.8 N
F_3	- 300 cos 60º = -150 N	300 sin 60º = -259.8 N
F_4	= -150 N	
المجموع	Rx=∑Fx= Fx1+Fx2+Fx3+Fx4 = 173.2+282.8+(-150)+(-150)	Ry=∑Fy= Fy1+Fy2+Fy3 =100+(-282.8)+(-259.8)
المجموع	Rx =∑ Fx = +156 N	Ry =∑Fy = -442.6 N



$R = \sqrt{(156)^2 + (-442.6)^2} = 469.3N$



$\theta = \tan^{-1}(Ry/Rx)$

$\theta = \tan^{-1}(-442.6/156)$

$\theta = -70.6^o$

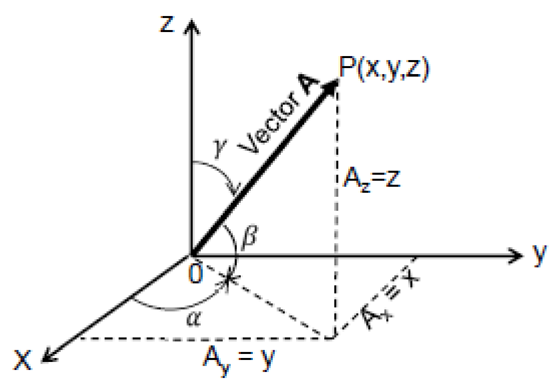
Example (13):

If vectors A = 2i +4k and B = 5j +6k, determine: (a) what planes do these two vectors exist, and (b) their respective magnitudes. (c) The summation of these two vectors. (d) The subtraction of these two vectors.

Solution:

(a) Vector A may be expressed as A = 2i +0j + 4k, so it is positioned in the x-z plane in the schematic Figure. Vector B on the other hand may be expressed as B = 0i + 5j +6k with no value along the x-coordinate. So, it is positioned in the y-z plane in a rectangular coordinate system.

(b) The magnitude of vector A is:



A= $\sqrt{(Ax)^2 \, + \, (Ay)^2 + (Az)^2}$

A= $\sqrt{(2)^2 \, + \, 0 + (4)^2}$

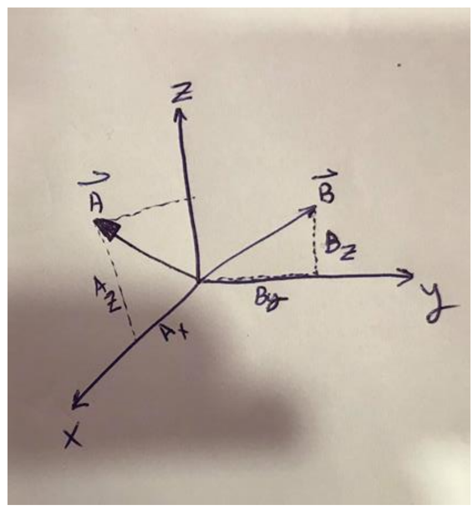
A= 4.47

and the magnitude of vector B is:

B= $\sqrt{(Bx)^2 \, + \, (By)^2 + (Bz)^2}$

B= $\sqrt{0 \, + \, (5)^2 + (6)^2}$

B= 7.81



(c) The addition of these two vectors is:

A+ B= (2+0) i+ (0+5) j + (4+6) k

= 2i +5j+10k

(d) The subtraction of these two vectors:

A-B= (2-0) i+ (0-5)j +(4-6)k

= 2i-5j-2k

3.3 The DOT product

Dot product of two vectors **A** and **B** is expressed as:

$$\mathbf{A.\ B= |A|\ |B|\ cos\ \theta = a\ scalor}$$

where θ is the angle between these two vectors.

- **We notice that the DOT product of two vectors results in a SCALAR.**

The algebraic definition of dot product of vectors can be shown as:

$$\mathbf{A.B= AxBx + AyBy + AzBz}$$

where Ax, Ay and Az = the magnitude of the components of vector **A** along the x-, y- and z-coordinate respectively,

And Bx, By and Bz = the magnitude of the components of vector **B** along the same rectangular coordinates.

The angle between the two vectors can be found by the equation below:

$$\cos \boldsymbol{\theta} = (\boldsymbol{A}.\boldsymbol{B})/(\boldsymbol{AB})$$

Example (14) Determine (a) the result of dot product of the two vectors: **A** = 2**i** + 7**j** + 15**k** and **B** = 21**i** + 31**j** + 41**k**, and (b) the angle between these two vectors

Solution:

(a) the result of the dot product of vectors:

A●B = (AxBx) + (AyBy) + (AzBz)

= (2*21)+(7*31)+ (15*41)

=42+217+615

=874

(b) In order to get the angle between these two vectors, we need to compute the magnitudes of both vectors:

A = $\sqrt{(2^2)+(7^2)+(15^2)}$

A= 16.67

B = $\sqrt{(21^2)+(31^2)+(41^2)}$

=55.52

Which lead to the angle θ between vectors **A** and **B** to be:

$\cos\theta = (A.B)/(AB)$

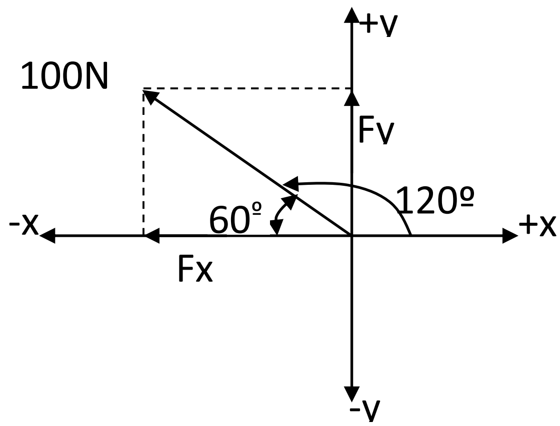
$\cos\theta = 874/(16.67 * 55.52)$

$\cos\theta = 0.94433$

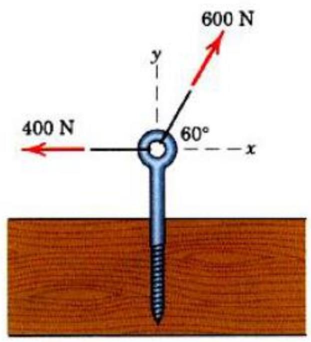
$\theta = 19.21^0$

HOWMWORKS

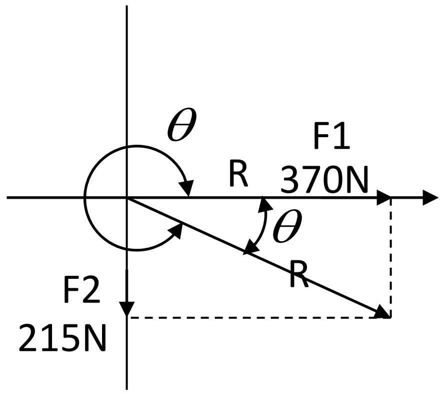
Q1: Determine the X and Y components of (100) N force as in figure below.



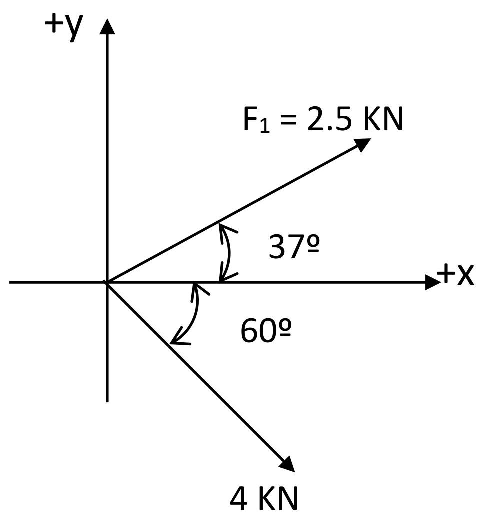
Q2: Determine the (magnitude and direction) of the resultant (R) of two forces by the use of (a) cosine and sine laws? (b) The resultant form?



Q3: Determine the (magnitude and direction) of the resultant (R) of two forces.



Q4: Determine the (magnitude and direction) of the resultant (R) of the forces system as shown in Figure below:



Moment

4.1 What is the moment?

By applying a force on the body, it will produce a tendency for the body to rotate about a point is not on the line of action of the force.

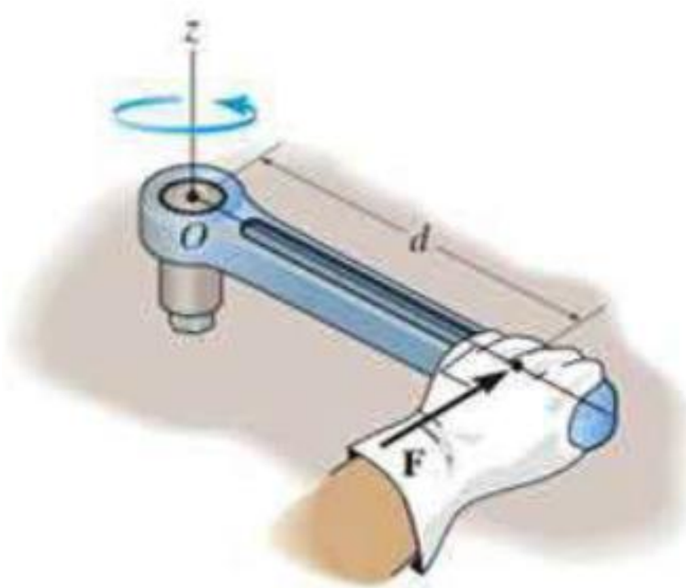
- This tendency to rotate the body represents the moment (may also called the moment of a force or sometimes a torque)

$$M = F\,d$$

M is the moment of a force (N.m)

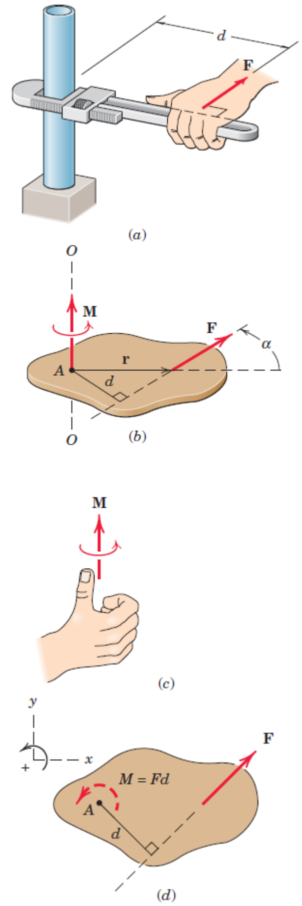
F the applied force (N)

d represents the perpendicular distance between the point of action of the force and moment center (O).



4.2 The direction of moment:

- (-) clockwise مع عقارب الساعة
- (+) counterclockwise عكس عقارب الساعة



4.3 The resultant moment:

$M_{Ro} = \sum M_o$

$M_R = \sum F\,d$

$F_R\,d = (F1\,d1) + (F2\,d2) + (F3\,d3)$

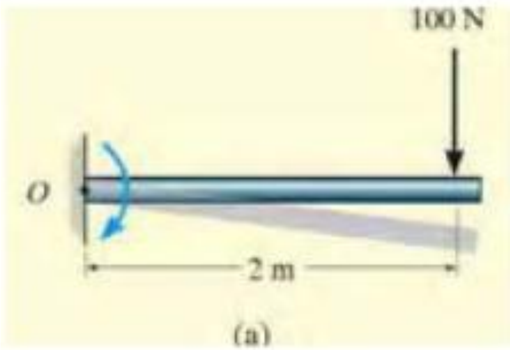
Where: M_{Ro} represent the resultant moment about point O (N.m), F_R is the resultant force (N), and d is the distance (m)

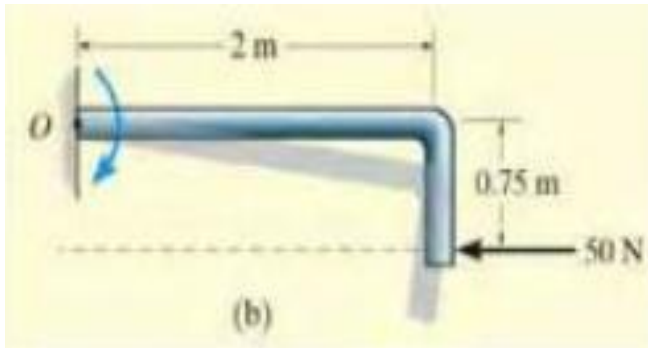
F1, F2, and F3 are the component forces (N).

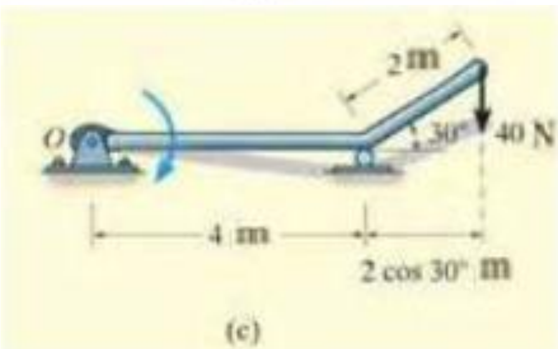
d1, d2, and d3 are the perpendicular distances for F1, F2, and F3 with point O (m)

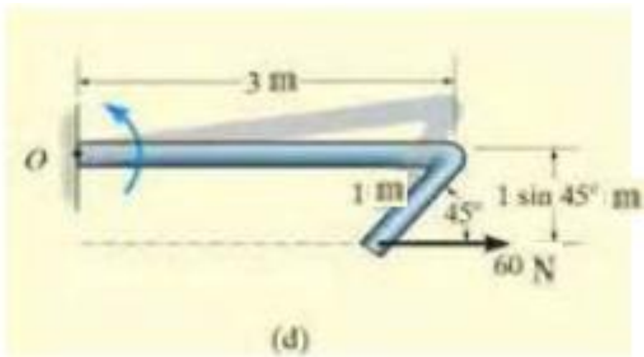
Examples

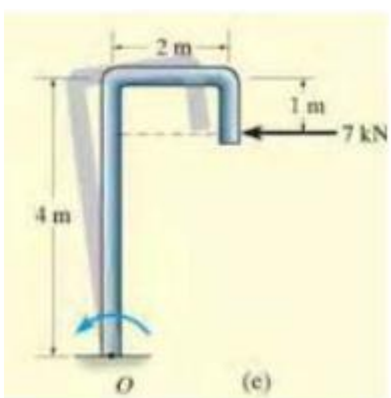
Example (15): determine the magnitude of the moment of the force about point O for the figures shown below:









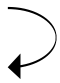


Solution:

(a) M= F d

M=- 100*2

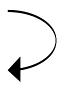
=-200 N.m

=200 N.m 

(b) M= F d

M= -50*0.75

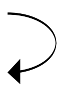
=-37.5 N.m

= 37.5 N.m 

(c) M= F d

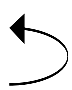
M=- 40* (4+ 2 cos 30)

=-229 N.m

=229.28 N.m 

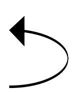
(d) M= F d

M= 60* 1 sin 45

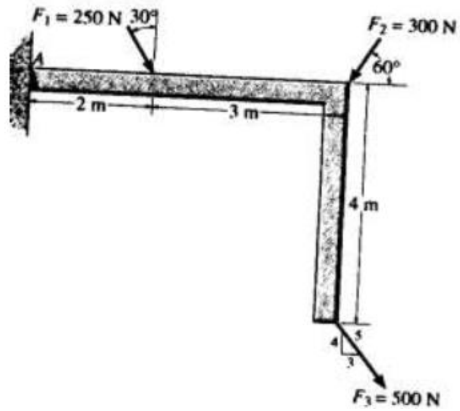
=42.4 N.m 

(e) M= F d

M= 7* (4-1)

=21 kN.m 

Example (16): determine the moment of each of the three forces about point A



Solution:

F1y= 250 cos 30, d1= 2 m

F2y= 300 sin 60, d2= 3+2= 5 m

F3y= 500 cos 36.87, d3x= 3+2= 5 m,,,,, tan φ=3/4, φ= 36.87^0

F3x= 500 sin 36.87, d3y= 4 m

$(M_{F1})_A$= F1 d1

$(M_{F1})_A$=- 250 cos 30 * 2 (clockwise)

$(M_{F1})_A$= -433 N.m

$(M_{F2})_A$= F2 d2

$(M_{F2})_A$=- 300 sin 60 * 5 (clockwise)

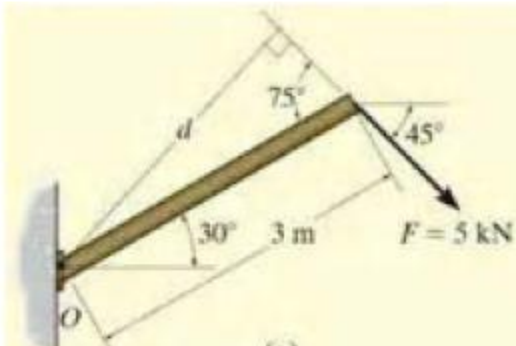
$(M_{F2})_A$= -1299 N.m

$(M_{F3})_A$= F3 d3

$(M_{F3})_A$= (500 sin 36.87* 4) – (500 cos 36.87* 5) (clockwise)

$(M_{F3})_A$= -800 N.m

Example (17): determine the moment of each of the force about point O



(a)

Solution:

The moment arm d in the above figure can be found from trigonometry

$$\frac{d}{\sin 75} = \frac{3}{\sin 90}$$

d= 3 sin 75/ sin 90

d= 2.898 m

Mo= F d

= -5*2.898

=- 14.5 kN.m (clockwise)

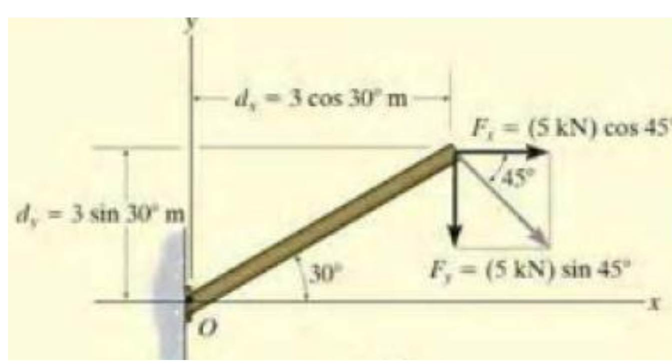
Another way for solution

Mo= F d

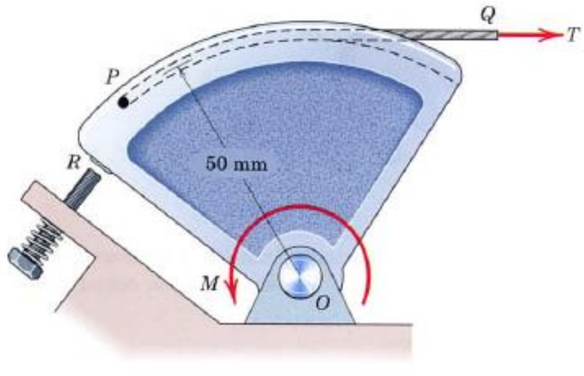
= -Fx dy –Fy dx

=- (5 cos 45) (3 sin 30) – (5 sin 45) (3 cos 30)

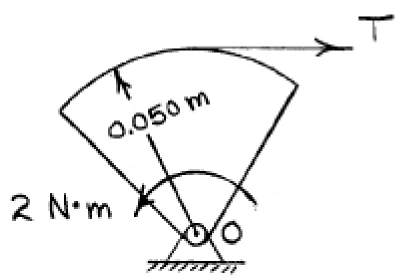
=- 14.5 kN. m (clockwise)



Example (18): the throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment M= 2 N.m on the sector when in the position shown, for design purposes determine the necessary throttle-cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.



Solution:



d= 50 mm *(1 m/1000 mm)

d= 0.05 m

F represented by T

∑M= 0

2 +(F*d)=0

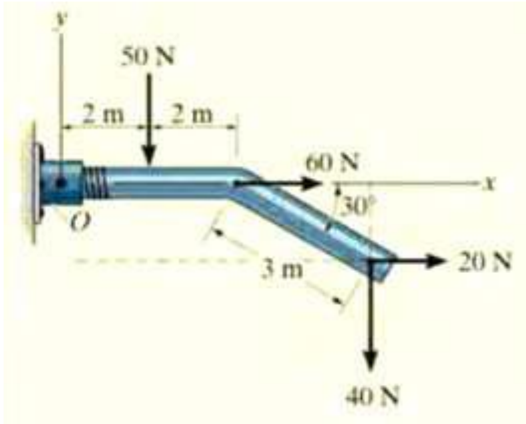
2+(-T*0.05)=0

0.05 T=2

T=2/0.05

T= 40 N

Example (19): Determine the resultant moment of the forces shown about point O



Solution:

F1=- 50 N, d1= 2 m

F2= 60 N, d2= 0

F3=+ 20 N, d3= (3 sin 30) m

F4=- 40 N, d4= 2+2+ (3 cos 30) m

$M_{Ro}= \sum M_o$

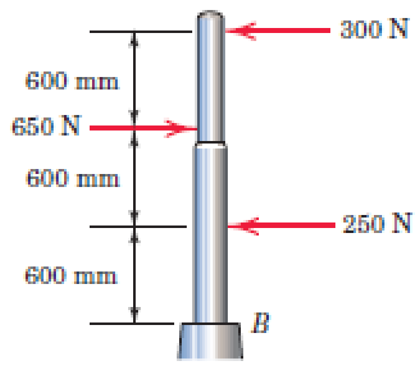
M_{Ro} = (F1 d1)+ (F2 d2) + (F3 d3) + (F4 d4)

= (-50*2)+ (60*0)+(20*3 sin 30)+(-40*(4+3 cos 30))

M_{Ro} = -334 N.m

M_{Ro} = 334 N.m ↻

Example (20): Determine the height h above the base B at which the resultant of the three forces acts.



Solution:

F1= -300 N, d1= 1800 mm* (1 m/1000 mm)= 1.8 m

F2= 650 N, d2= 1200 mm* (1 m/1000 mm)= 1.2 m

F3= -250 N, d3= 600 mm* (1 m/1000 mm)= 0.6 m

R= ∑F

= -300-250+650

=100 N

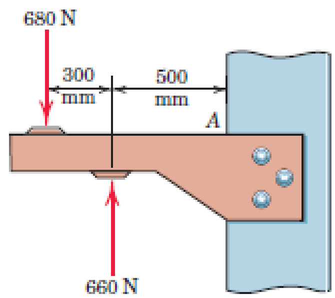
$M_{RB}= \sum M_B$

R*h= F1*d1 + F2*d2 + F3*d3

100 h= (-300*1.8) + (650*1.2) +(-250*0.6)

h=0.9 m

Example (21): Where does the resultant of the two forces act?



Solution:

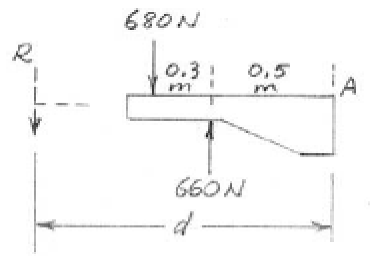
F1= -680 N, d1= 800 mm *(1 m/1000 mm)= 0.8 m

F1= 660 N, d1= 500 mm *(1 m/1000 mm)= 0.5 m

R= ∑F

= 660-680

= -20 N



M_{RA}= $\sum M_A$

R d= F1 d1 + F2 d2

M_{RA} = (-680*0.8)+ (660*0.5)

M_{RA}= -214 N.m

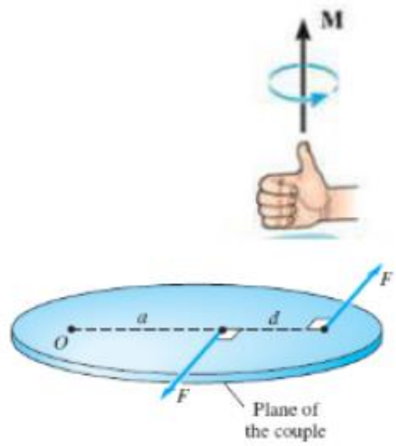
-20 d= -214

d= 10.7 m the location is to the left of A

Couple

5.1 Couple: consists of two parallel, noncollinear forces that are equal in magnitude and opposite in direction.

- A couple is a purely rotational effect, it has a moment but no resultant force (resultant equlas to zero therefore, couple has no tendency to translate the body in any direction).



The magnitude moment of the couple is:

Mo= F (a+d) –Fa

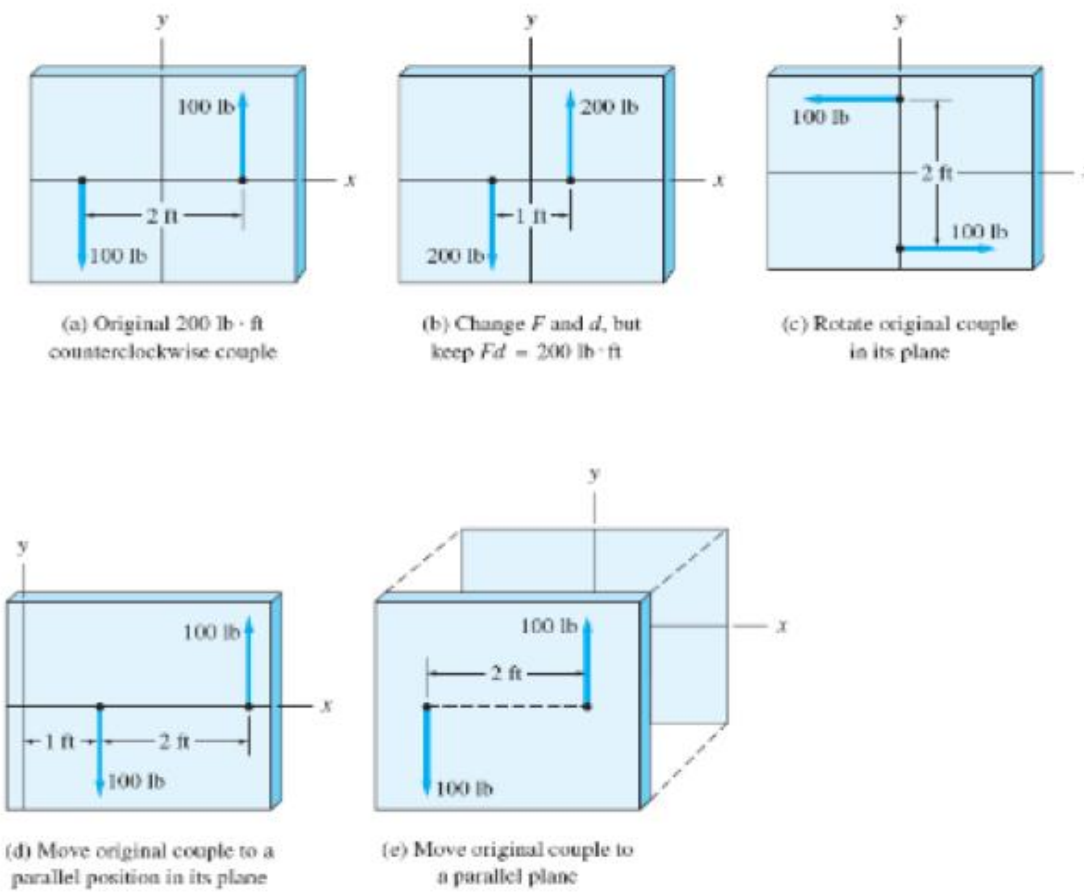
M= F d

Where F represent the magnitude of one forces (N) and d is the perpendicular distance or moment arm between the forces (m).

5.2 **Equivalent couples:** If two couples produce a moment with the *same magnitude and direction*, then these two couples are ***equivalent***.

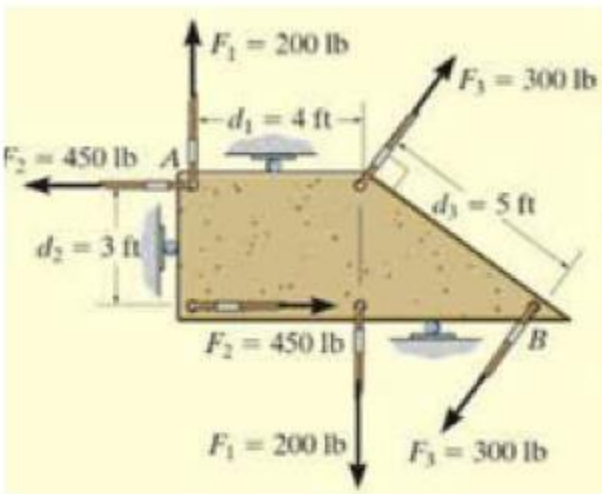
Figure below illustrates the four operations that may be performed on a couple without changing its moment; all couples shown in the figure are equivalent. The operations are

1. Changing the magnitude F of each force and the perpendicular distance d while keeping the product Fd constant,
2. Rotating the couple in its plane,
3. Moving the couple to a parallel position in its plane
4. Moving the couple to a parallel plane



Examples

Example (22): determine the resultant couple moment of three couples acting on the plate in the figure below:



Solution:

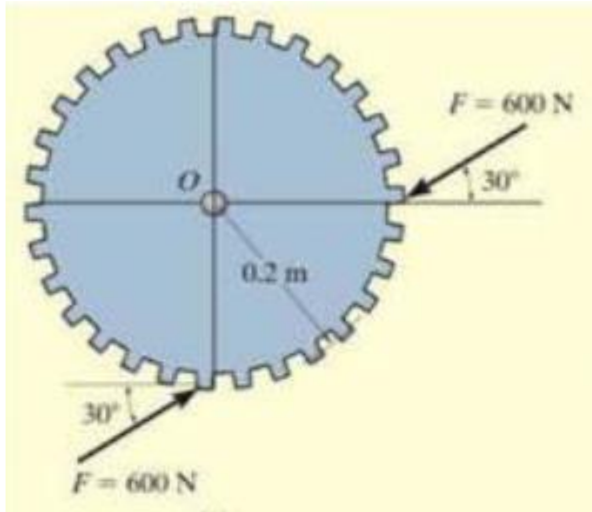
$\curvearrowleft + M_R = \Sigma M;$

$M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$

$= (-200)(4) + (450)(3) - (300)(5)$

$= -950 \text{ lb.ft} = 950 \text{ lb.ft} \curvearrowright$

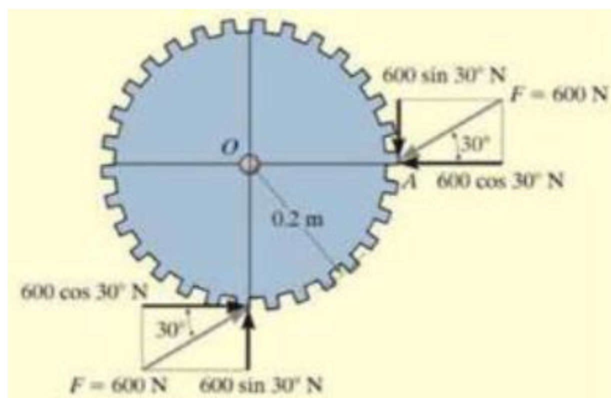
Example (23): determine the magnitude and direction of the couple moment acting on the gear in figure (a):



(a)

Solution:

The easiest solution by the resolving of the forces into its component as shown in figure (b).



(b)

F1= F cos 30, d1= 0.2 m

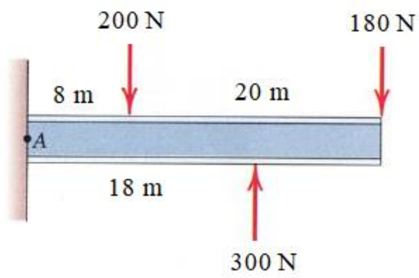
F2= F sin 30, d2= 0.2 m

$M_R = \sum M_O$

M= (600 cos 30)*0.2 – (600 sin 30)* 0.2

M= 43.9 N.m عكس عقارب الساعة

Example (24): Reduce the given loading system to a force-couple system at point A.



Solution:

F1= -200 N,   d1= 8 m

F2= 300 N,   d2= 18 m

F3= -180 N,   d3= 20 +8 = 28 m

R= ∑F

R= -200+300-180

R= -80 N

R= 80 N

M_R = ∑M_A

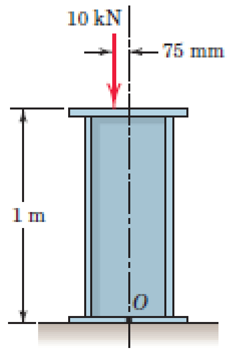
M_R = ∑F.d

M_R = (-200*8)+ (300*18)+(-180*28)

M_R =- 1240 N.m

M_R = 1240 N. m

Example (25): Replace the 10-kN force acting on the steel column by an equivalent force–couple system at point O. This replacement is frequently done in the design of structures.

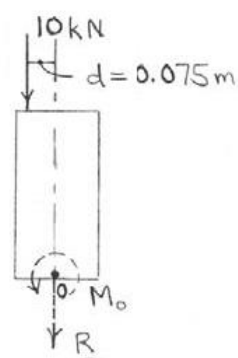


Solution:

F= 10 kN

d= 75 mm *(1 m/1000 mm)

d= 0.075 m

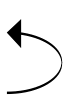


R=- 10 kN

M_O= F d

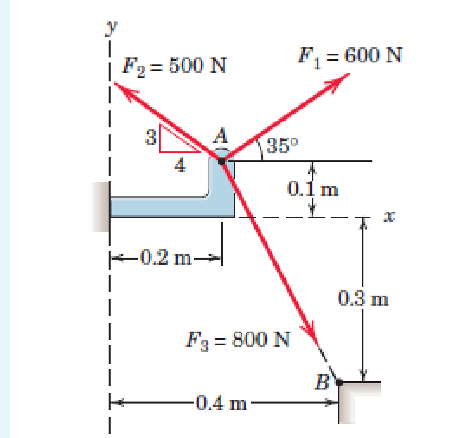
= 10*0.075

=0.75 kN. m

= 0.75 kN.m 

SAMPLE PROBLEM 2/1

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. a, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

The scalar components of $\mathbf{F}_2$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$

Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

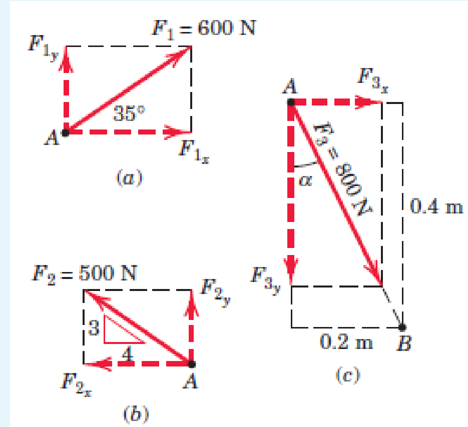
The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

1 Then,

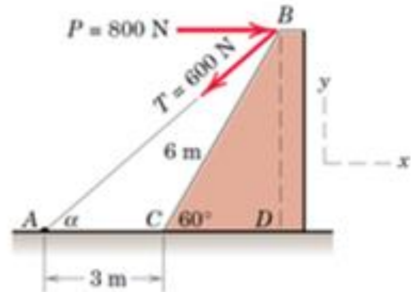
$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$



Sample Problem 2/2

Combine the two forces **P** and **T**, which act on the fixed structure at B, into a single equivalent force **R**.



Algebraic solution. By using the x-y coordinate system on the given figure, we may write

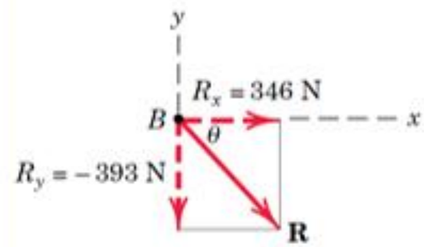
$$R_x = \Sigma F_x = 800 - 600\cos 40.9° = 346 \text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9° = -393 \text{ N}$$

The magnitude and dipection of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ N} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6° \qquad \textit{Ans.}$$



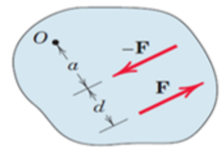
The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ N} \qquad \textit{Ans.}$$

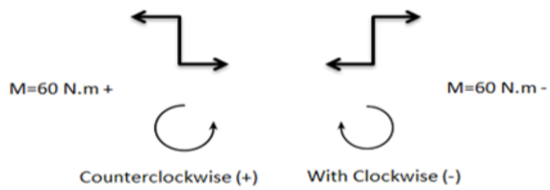
General information regarding couple

$M = F(a - d) - Fa$

$$\boxed{M = Fd}$$



(a)

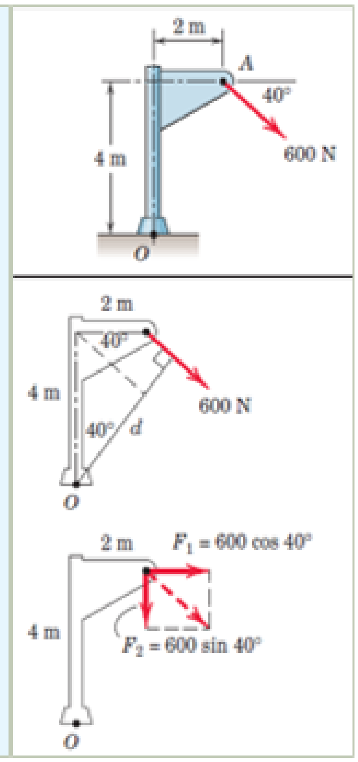


Moment

ملاحظة مهمة: ممكن حساب العزوم بطريقتين اما 1) ايجاد ذراع العزم 2) تحليل القوى كما موضح في المثال ادناه

SAMPLE PROBLEM 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.



Solution. *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

1 By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

2 $$M_O = 460(4) + 386(2) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

Equilibrium

6.1 Equilibrium: When a system of forces acting on a body has no resultant, the body is in equilibrium.

If system in equilibrium, both of the resultant force and resultant couple are zero.

$\sum Fx = 0$

$\sum Fy = 0$

$\sum Mo = 0$

6.2 Free Body Diagram: (F.B.D): is a sketch of a body or a portion of a body completely issolated (or free) from its surrondings.

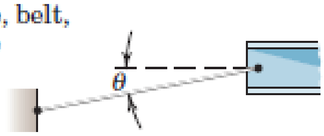
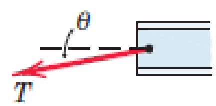
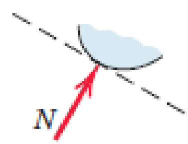
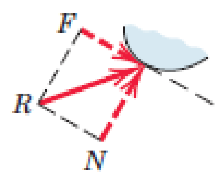
In this sketch, it is necessary to show all the forces and moments that the surroundings exert on the body. By using this diagram, the effect of all applied forces and moments acting on the body can be accounted by the equations of equilibrium.

Forces that act on a body can be divided into two general categories:

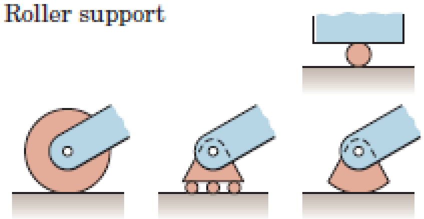
1) **Reactions** are the forces which are exerted on a body by the supports to which is attached.
2) **Applied forces** are the foces that act on a body which are not provided by the supports.

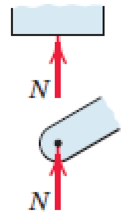
The general procedure for constructing a F.B.D is:

1) A skectch of the body is drawn assuming that all supports (surface of contact, supporting cables, etc.) have been removed.
2) All applied forces are drawn and labeled on the sketch. The weight of the body is considered to be applied force acting at the center of gravity.
3) The support reactions are drawn and labeled on the sketch.
4) All relevant angles and dimensions are shown on the skectch.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible 	 Force exerted by a flexible cable is always a tension away
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent *F* (frictional force) as well as a normal component *N* of the resultant

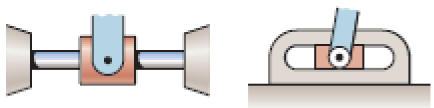
4. Roller support

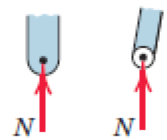




Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

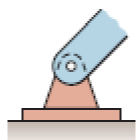
5. Freely sliding guide



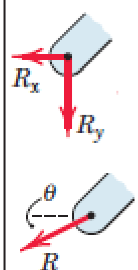


Collar or slider free to move along smooth guides; can support force normal to guide only.

6. Pin connection

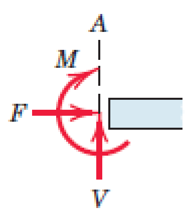


Pin free to turn



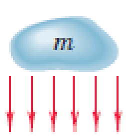
A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis.
We may show two components R_x and R_y or a magnitude R and direction θ.

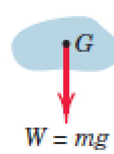
7. Built-in or fixed support



A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

8. Gravitational attraction





The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

Typical examples of actual supports are shown in the following sequence of photos. The numbers refer to the connection types in Table 5–1.

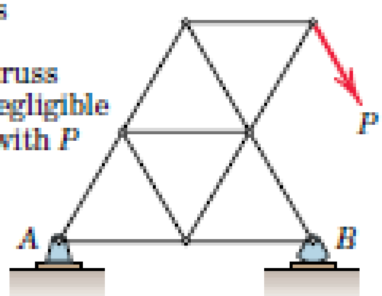
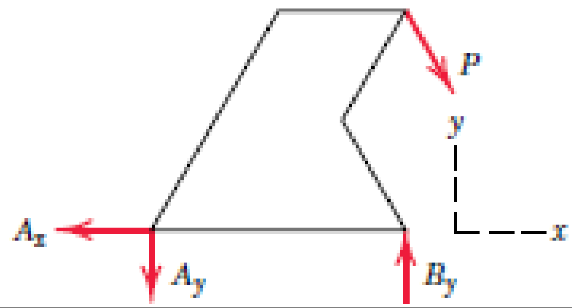
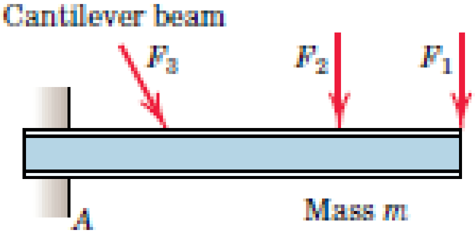
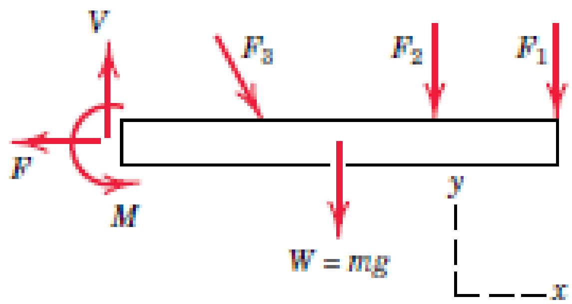
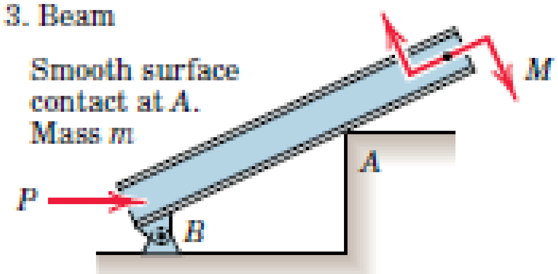
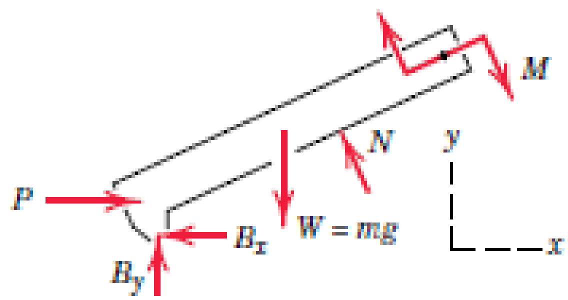
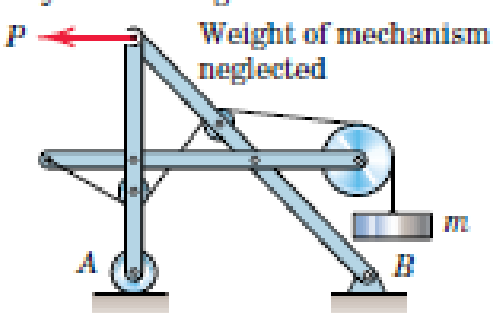
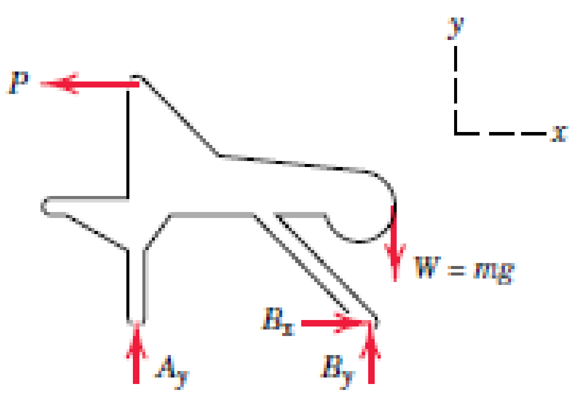
The cable exerts a force on the bracket in the direction of the cable. (1)

The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature. (5)

This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface. (6)

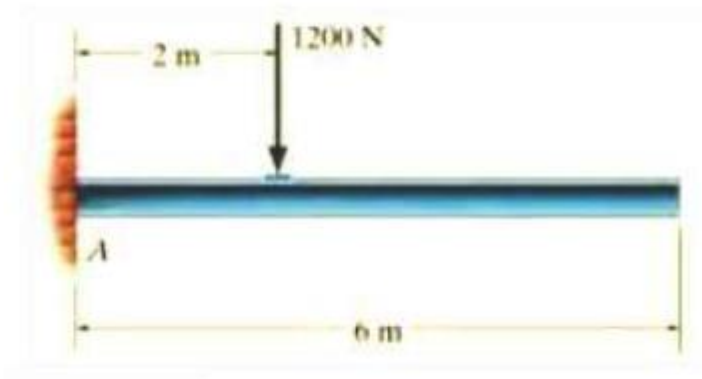
This utility building is pin supported at the top of the column. (8)

The floor beams of this building are welded together and thus form fixed connections. (10)

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	**Free-Body Diagram of Isolated Body**
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam 	
4. Rigid system of interconnected bodies analyzed as a single unit 	

Examples

Example (26): Draw the free-body diagram of the uniform beam shown below. The beam has a mass of 100kg.

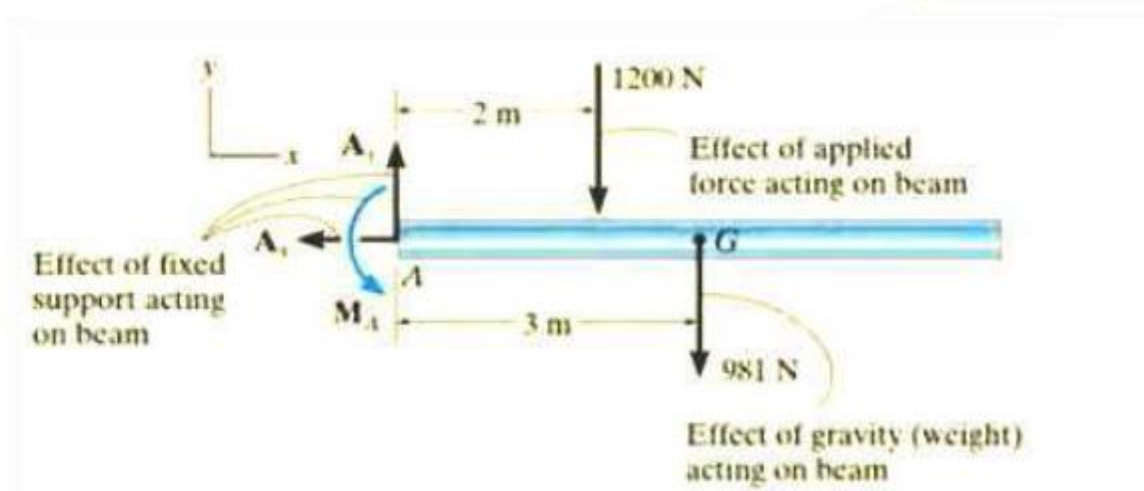


Solution:

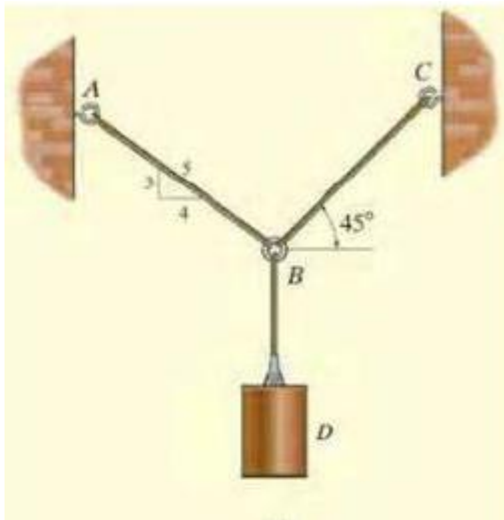
The beam weight= mass* g

The beam weight = 100*9.81

The beam weight = 981 N



Example (27): determine the tension in cables AB and BC necessary to support the 60 kg cylinder in figure (a).



(a)

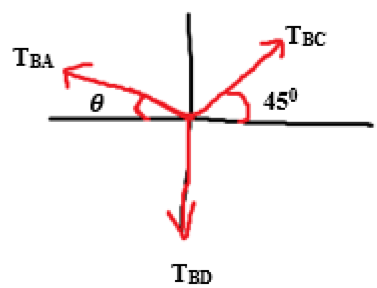
Solution

T_{BD}= the weight of the cylinder= mass * g

T_{BD}= 60*9.81

T_{BD}= 588.6 N

tan $\boldsymbol{\theta}$= ¾, θ= 36.87^0



Equations of equilibrium:

∑Fx=0

T_{BC} cos 45- T_{BA} cos 36.87= 0

0.707 T_{BC} – 0.8 T_{BA} =0

T_{BA} =0.884 T_{BC} **Equation (1)**

$\sum Fy=0$

T_{BC} sin 45+ T_{BA} sin 36.87 - T_{BD} = 0

0.707 T_{BC} + 0.6 T_{BA} – 588.6 = 0 **Equation (2)**

Substituting equation (1) into equation (2):

0.707 T_{BC} + 0.6 (0.884 T_{BC}) – 588.6 = 0

0.707 T_{BC} + 0.53 T_{BC} – 588.6 = 0

1.237 T_{BC} – 588.6 = 0

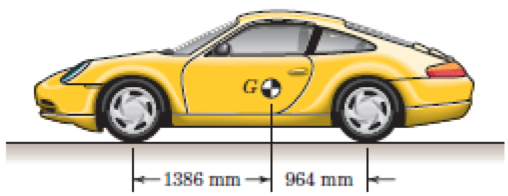
1.237 T_{BC} = 588.6

T_{BC} =475.829 N

Substituting (T_{BC} =475.829 N) into equation (1):

T_{BA}=420.63 N

Example (28): The mass center *G* of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.

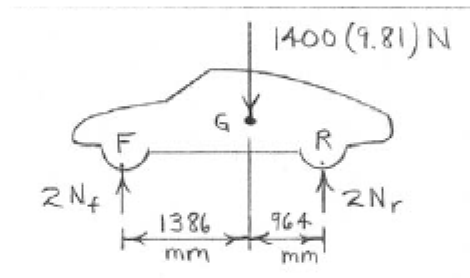


Solution:

The weight of the car= mass * g

= 1400*9.81

= 13734 N



Equations of equilibrium:

$\sum Fy=0$

$2N_f + 2N_r - 13734=0$

$2N_f= 13734-2N_r$

$N_f= 6867-N_r$ **Equation (1)**

Car weight = 13734 N, d=1386 mm * (1m/1000 mm) = 1.386 m

N_r, dN_r=1386+964= 2350 mm * (1m/1000 mm)= 2.350 m

$\sum M_F=0$

$(-13734*1.386)+ (2N_r * 2.350)= 0$

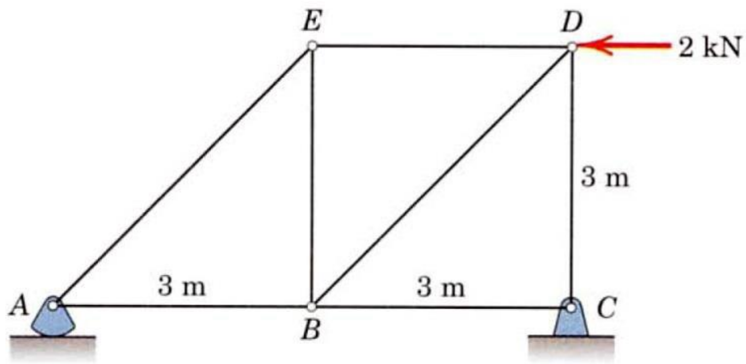
Nr= 4050 N

Substituting (N_r= 4050 N) into equation (1):

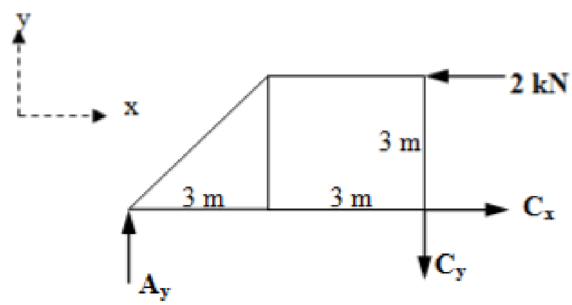
N_f= 6867-4050

N_f= 2816.93 N

Example (29): Determine the reaction forces on each cables.



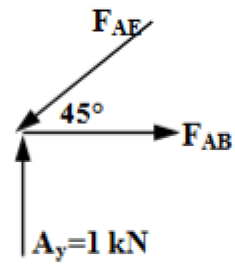
Solution:



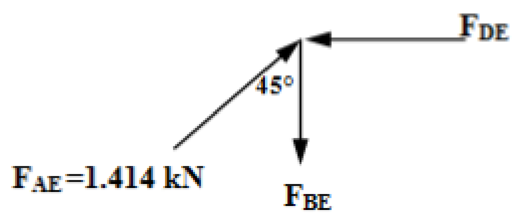
∑Mc=0	∑Fy=0	∑Fx=0
-Ay* 6+2*3=0 6 Ay= 6 Ay=1 kN	Ay-Cy=0 1-Cy=0 Cy=1 kN	Cx-2=0 Cx=2 kN

$\tan \alpha = 3/3, \quad \alpha = 45^0$

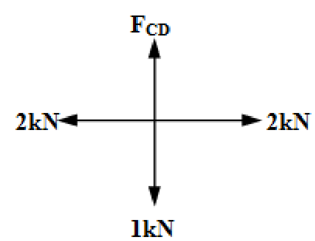
$\sum Fx=0$	$\sum Fy=0$
$F_{AB} - F_{AE} \cos 45 = 0$ $F_{AB} - 1.414 * \cos 45 = 0$ $F_{AB} = 1$ kN	$Ay - F_{AE} \sin 45 = 0$ $1 - F_{AE} \sin 45 = 0$ $F_{AE} = 1.414$ kN



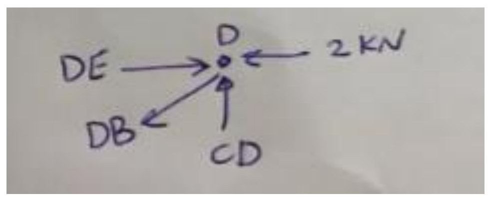
$\sum Fx=0$	$\sum Fy=0$
$F_{AE} \sin 45 - F_{DE} = 0$ $1.414 * \sin 45 - F_{DE} = 0$ $F_{DE} = 1$ kN	$F_{AE} \cos 45 - F_{BE} = 0$ $1.414 * \cos 45 - F_{BE} = 0$ $F_{BE} = 1$ kN



$\sum Fx=0$	$\sum Fy=0$
$F_{BC} - 2 = 0$ $F_{BC} = 2$ kN	$F_{CD} - 1 = 0$ $F_{CD} = 1$ kN



$\sum Fx=0$	$\sum Fy=0$
$F_{DE} - 2 - F_{BD} \cos 45 = 0$ $F_{DE} - 2 - 1.414 * \cos 45 = 0$ $F_{BC} = 3$ kN	$F_{CD} - F_{BD} \sin 45 = 0$ $1 - F_{BD} * \sin 45 = 0$ $F_{BD} = 1.414$ kN



Lecture 6:Equilibrium

لغرض تسهيل خطوات الحل في موضوع الاتزان:

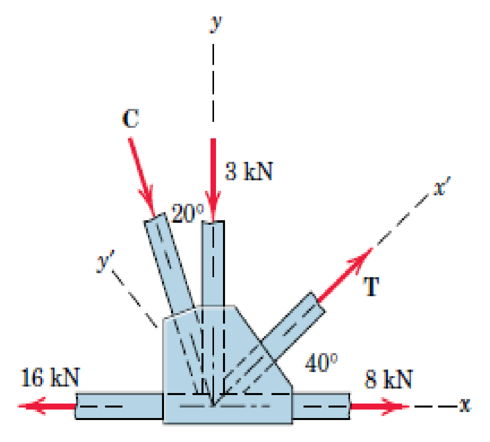
* يفضل اخذ العزوم عند النقاط التي يكون فيها اكثر عدد من المجاهيل مثلا عند المسمار (Pin).

* يتم افتراض اتجاهات ردود الافعال وبحل السؤال ان كانت نتيجة القوه رقم سالب يتم عكس الاتجاه المفترض.

* الاجسام المنتظمة (uniform) تكون القوه المعبره عن الوزن فيها في وسط الجسم,اما اذا لم يتم ذكر اي معلومات عن الكتلة او الوزن فيتم اهمال الوزن.

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution. The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium. ①

Solution I (scalar algebra). For the *x*-*y* axes as shown we have

$[\Sigma F_x = 0]$ $\quad 8 + T \cos 40° + C \sin 20° - 16 = 0$

$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $\quad T \sin 40° - C \cos 20° - 3 = 0$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (*a*) and (*b*) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

Solution II (scalar algebra). To avoid a simultaneous solution, we may use axes ② x'-y' with the first summation in the y'-direction to eliminate reference to T. Thus,

$[\Sigma F_{y'} = 0]$ $\quad -C \cos 20° - 3 \cos 40° - 8 \sin 40° + 16 \sin 40° = 0$

$$C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

$[\Sigma F_{x'} = 0]$ $\quad T + 8 \cos 40° - 16 \cos 40° - 3 \sin 40° - 3.03 \sin 20° = 0$

$$T = 9.09 \text{ kN} \qquad \textit{Ans.}$$

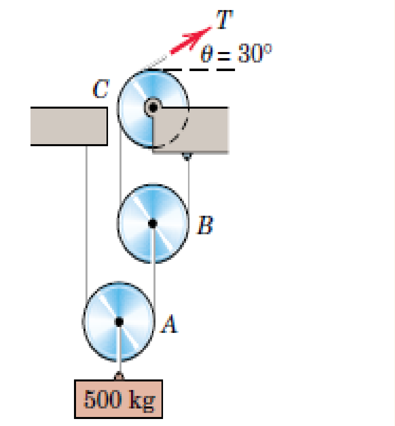
Helpful Hints

① Since this is a problem of concurrent forces, no moment equation is necessary.

② The selection of reference axes to facilitate computation is always an important consideration. Alternatively in this example we could take a set of axes along and normal to the direction of **C** and employ a force summation normal to **C** to eliminate it.

Sample Problem 3/2

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force. With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

① $[\Sigma M_O = 0]$ $\quad T_1 r - T_2 r = 0 \quad T_1 = T_2$

$[\Sigma F_y = 0]$ $\quad T_1 + T_2 - 500(9.81) = 0 \quad 2T_1 = 500(9.81) \quad T_1 = T_2 = 2450 \text{ N}$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

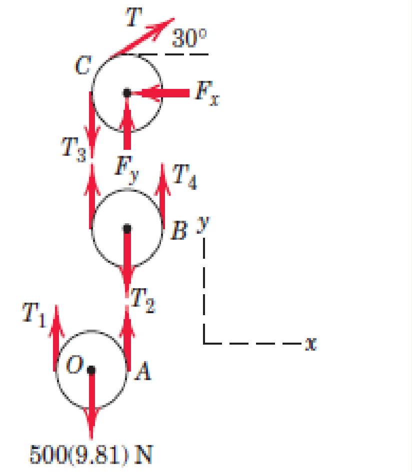
$$T = T_3 \quad \text{or} \quad T = 1226 \text{ N} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the x- and y-directions requires

$[\Sigma F_x = 0]$ $\quad 1226 \cos 30° - F_x = 0 \quad F_x = 1062 \text{ N}$

$[\Sigma F_y = 0]$ $\quad F_y + 1226 \sin 30° - 1226 = 0 \quad F_y = 613 \text{ N}$

$[F = \sqrt{F_x^2 + F_y^2}]$ $\quad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N} \qquad \textit{Ans.}$

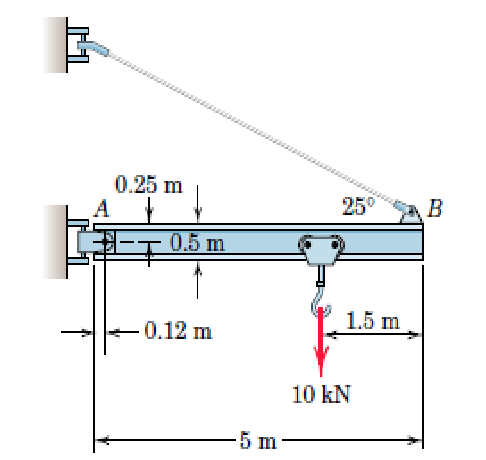


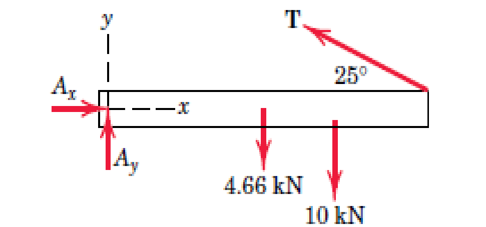
Helpful Hint

① Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.





Free-body diagram

Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T, which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equa-
① tion. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of $\mathbf{T}$ than it is to compute the perpendicular distance from $\mathbf{T}$ to A. Hence, with the counterclockwise sense as positive we write

② $[\Sigma M_A = 0]$ $$(T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $T = 19.61$ kN *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

$[\Sigma F_x = 0]$ $A_x - 19.61 \cos 25° = 0$ $A_x = 17.77$ kN

$[\Sigma F_y = 0]$ $A_y + 19.61 \sin 25° - 4.66 - 10 = 0$ $A_y = 6.37$ kN

③ $[A = \sqrt{A_x^2 + A_y^2}]$ $A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88$ kN *Ans.*

Helpful Hints

① The justification for this step is Varignon's theorem, explained in Art. 2/4. Be prepared to take full advantage of this principle frequently.

② The calculation of moments in two-dimensional problems is generally handled more simply by scalar algebra than by the vector cross product $\mathbf{r} \times \mathbf{F}$. In three dimensions, as we will see later, the reverse is often the case.

Friction

7.1 Friction: Friction may be defined as the contact resistance exerted by one body upon a second body when the second body move or tends to move past the first body.

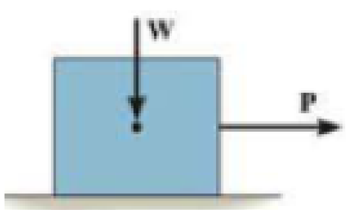
From the definition, it should be observed that friction is a retarding force always acting opposite to the motion or the tendency to move.

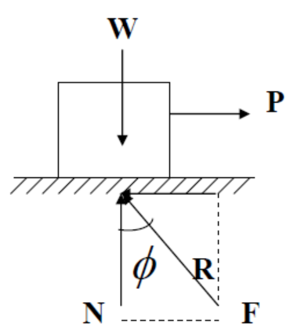
As we shall see friction exists primarily because of the roughness of contact surface.

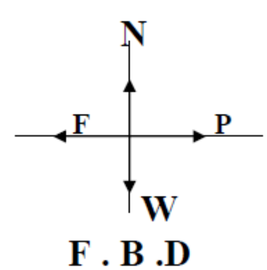
7.2 Solving the friction problems:

The solution of friction problem is following this procedure:

1. Draw the F.B.D.







(a) (b) (c)

Where:

W =weight of body وزن الجسم
N = Normal force القوة العمودية
P = Pull force قوة السحب
F = Friction force قوة الاحتكاك

R = Resultant of normal force (N) and friction force (F) القوة المحصلة

2. Apply the equilibrium equations.

   Friction Force is **directly proportional** to the normal force.

وحسب نظرية الاحتكاك فأن فوة الاحتكاك تتناسب طرديا مع القوة العمودية:

$$F \propto N$$

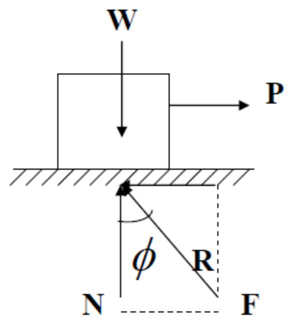
F = (كمية ثابتة) * N

$$F = f * N$$

f = coefficient of friction معامل الاحتكاك

3. Angle of friction (Ø)

tan Ø = F/N



F= μ* N

tan Ø = μ *N/N

tan Ø = μ

Table 6-1: Typical Values for μ_s

Contact materials	Coefficient of static friction (μ_s)
Metal on ice	0.03-0.05
Wood on wood	0.3-0.7
Leather on wood	0.2-0.5
Leather on metal	0.3-0.6
Aluminum on aluminum	1.1-1.7

Key Concepts:

(a) To determine the value of friction force (*F*) and the reaction force (*N*) we can used the equilibrium conditions.

[ΣFx = 0] ; [ΣFy = 0]

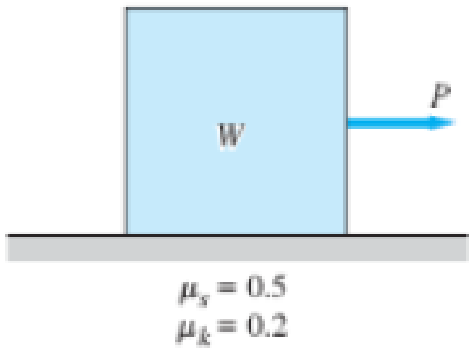
(b) F < ($F_{max} = \mu_s N$): Here the friction force necessary for equilibrium can be supported, and therefore the body is in static equilibrium as assumed. We confirm that the actual friction force *F* is less than the limiting value *Fmax*, and that *F* is determined solely by the equations of equilibrium.

(c) F = (Fmax = μs N): Since the friction force *F* is at its maximum value *Fmax*, impending motion, the assumption of static equilibrium is valid.

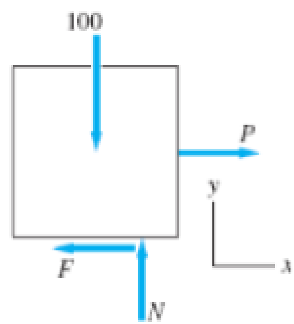
(d) F > (Fmax = μs N): Clearly this condition is impossible, because the surfaces cannot support more force than the maximum *μs N*. The assumption of equilibrium is therefore invalid, and motion occurs. The friction force *F* is equal to *μk N*.

Examples

Example (30): The 100 N block in the figure below is at rest on a rough horizontal plane before the force P is applied. Determine the magnitude of P that would cause impending sliding to the right.



Solution:



$\sum Fy=0$

N-100=0

N= 100 N

F= μ N

= 0.5*100

F= 50 N

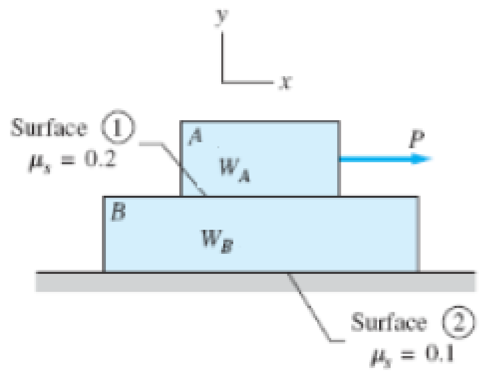
$\sum Fx=0$

P-F=0

P-50=0

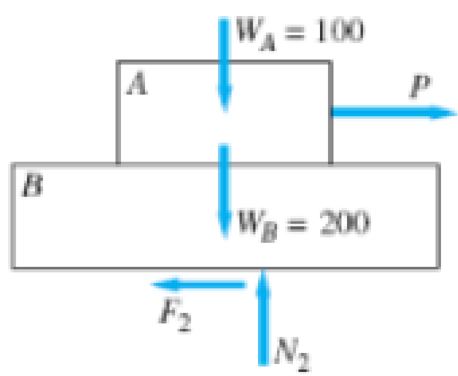
P=50 N

Example (31): Determine the maximum force P that can be applied to block A in the shown figure without causing either block to move. W_A= 100 N and W_B= 200 N.

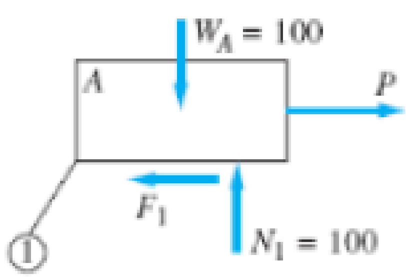


Solution:

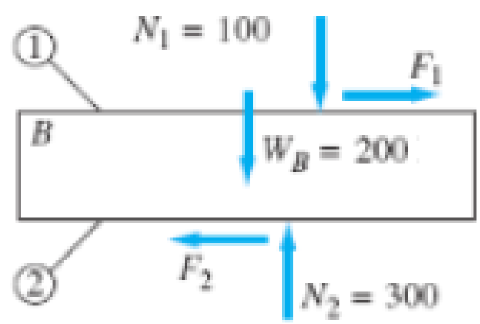
في بداية الحل نرسم ال .F.B.D :



(1)



(2)



(3)

From the F.B.D. in figure (1):

$\sum Fy=0$

$N2-W_A-W_B=0$

$N2-100-200=0$

$N2= 300$ N

From the F.B.D. in figure (2):

$\sum Fy=0$

$N1-W_A=0$

$N1-100=0$

$N1=100$ N

From the F.B.D. in Figure (2), we can calculate the friction force (F1) as following:

$F1= (\mu_s)_1*N1$

$= 0.2*100$

$= 20$ N

$\sum Fx=0$

$P-F1=0$

$P-20=0$

$P=20$ N

From the F.B.D. in Figure (1), we can calculate the friction force (F2) as following:

$F2= (\mu_s)_2 * N2$

$= 0.1*300$

$=30$ N

$\sum Fx=0$

$P-F2 =0$

$P-30=0$

$P=30$ N

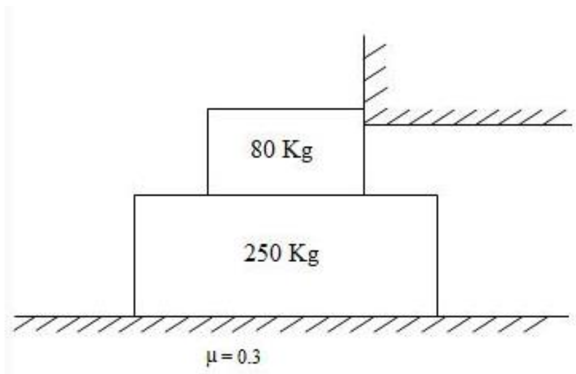
Therefore, the largest force that can be applied without causing either block to move is,

$$P=20 \text{ N}$$

with sliding impending at surface 1.

Be sure you understand that the largest force that can be applied is the smaller of the two values determined in the preceding calculations. If sliding impends when $P=20$ N, then the system would not be at rest when $P=30$ N

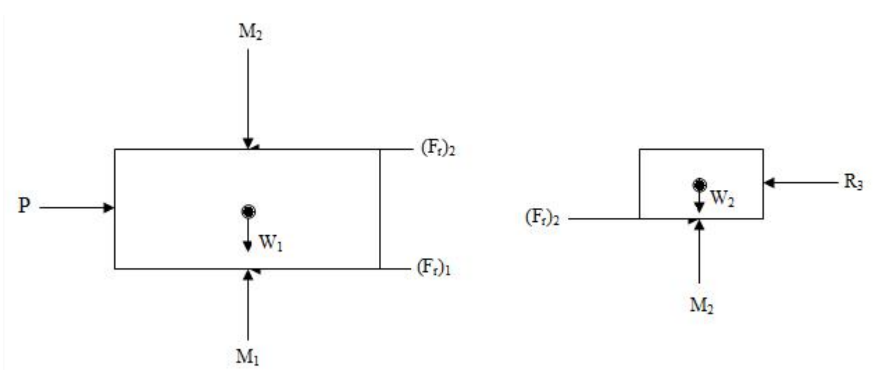
Example (32): If coefficient of friction between all surfaces shown in Figure below is 0.30. What is the horizontal force required to get 250 kg block moving to the right?



Solution:

نلاحظ ان الجسم ذو ال 80 kg مقيد وغير متحرك وبالتالي عند تسليط قوة لتحريك الجسم ذو ال 250 kg, فانه لاتوجد قوى عمودية عند سطح التلامس مابين العائق والجسم ذو الكتلة 80 kg. وعلى ضوئة فان قوة الاحتكاك تؤثر فقط على السطح السفلي للجسم ذو الكتلة 80 kg بينما توجد قوى احتكاك عند الجسم العلوي والسفلي للجسم ذو الكتلة 250 kg .

In this problem 80 kg block is completely restrained against motion and as we apply force P on 250 Kg block as shown in Fig.3, there is no force acting vertically at the contact surfaces between the obstacle and 80 kg block. Hence frictional force acts only at bottom and top surfaces of 250 kg block while only at lower surface of 80 kg block.



(a) Lower block (b) Upper block

Note that $\sum F_y = 0$ for upper block gives M2 = W2

Therefore, M2 = 80 × 9.81 = 784.8 N

For lower block, $\sum F_y = 0$ gives M1 = W1 + M2

Therefore, M1 = (250 × 9.81) + 784.8 = 3237.3 N

Also (Fr)1 = μ M1 = (0.3) (3237.3) = 971.19 N

and (Fr)2 = μ M2 = (0.3) (784.8) = 235.44 N

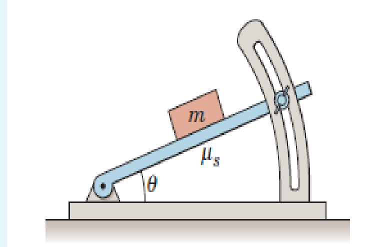
$\sum F_x = 0$ for lower block gives P = (Fr)1 + (Fr)2

or P = 1206.63 N

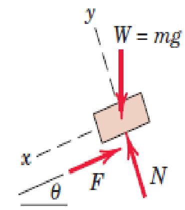
Note - $\sum F_x = 0$ is not necessary for upper block in this problem.

SAMPLE PROBLEM 6/1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



Solution. The free-body diagram of the block shows its weight $W = mg$, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present.



① Equilibrium in the x- and y-directions requires

$[\Sigma F_x = 0]$ $\quad mg \sin\theta - F = 0 \qquad F = mg \sin\theta$

$[\Sigma F_y = 0]$ $\quad -mg \cos\theta + N = 0 \qquad N = mg \cos\theta$

Dividing the first equation by the second gives $F/N = \tan\theta$. Since the maximum angle occurs when $F = F_{max} = \mu_s N$, for impending motion we have

② $$\mu_s = \tan\theta_{max} \quad \text{or} \quad \theta_{max} = \tan^{-1}\mu_s \qquad \textit{Ans.}$$

Helpful Hints

① We choose reference axes along and normal to the direction of F to avoid resolving both F and N into components.

② This problem describes a very simple way to determine a static coefficient of friction. The maximum value of θ is known as the *angle of repose*.