Heat transfer

Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference.

The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange will take place under certain specified conditions. Supplements the first and second principles of thermodynamics by providing additional experimental rules that may be used to establish energy-transfer rates. As in the science of thermodynamics, the experimental rules used as basis of the subject of heat transfer are rather simple and easily expanded to encompass a variety of practical situations.

Heat transfer modes:

1. Conduction
2. Convection
3. Radiation

Conduction in Heat Transfer

Whenever a temperature gradient exists in a solid medium heat will flow from the higher-temperature to the lower-temperature region. The rate at which heat is transferred by conduction.

<u>Fourier's law</u>

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region.

$$\frac{q}{A} \alpha \frac{dT}{dx}$$

The constant of proportionality is inserted is the thermal conductivity of solid material (k)

$$\frac{q}{A} = -k\frac{dT}{dx} \quad or \quad q = -kA\frac{dT}{dx}$$

This is called Fourier's law of heat

conduction. q is the heat-transfer rate

dT/dx is the temperature gradient in the direction of the heat flow. The minus sign is inserted to make clear that the heat must flow in a direction of temperature decrease.

A area

K thermal conductivity w/m.C

Thermal Conductivity

Thermal conductivity, (k), is the property of a material's ability to conduct heat. It appears primarily in Fourier's Law for heat conduction.

- Experimental measurements may be made to determine the thermal conductivity of different materials.

- In general, the thermal conductivity is strongly temperature- dependent.
- The numerical value of the thermal conductivity indicates how fast heat will flow in a given material.

Some values of thermal conductivity of various materials are shown below:

Gases	Liquids	Solids
H_2 = 0.175 W/m.°C	H_2O = 0.556 W/m.°C	Ag = 410 W/m.°C
He = 0.141 W/m.°C	Hg = 8.21 W/m.°C	Cu = 385 W/m.°C
Air = 0.024 W/m.°C	NH_3 = 0.540 W/m.°C	AL = 202 W/m.°C
CO_2 = 0.0146 W/m.°C	Freon = 0.073 W/m.°C	Ni = 93 W/m.°C

Example (1)

One face of a copper plate 3 cm thick is maintained at 400◦C, and the other face is maintained at100◦C. How much heat is transferred through the plate? (K=370W/m· ◦C)

Solution

$$\frac{q}{A} = -k\frac{dT}{dx}$$

$\frac{q}{A}$ = - *370* $(100-400)/0.03$ =3,700 KW/ m^2

Example (2)

A plane wall is 150 mm thick and its wall area is 4.5 m^2. Its thermal conductivity is 9. 35 W/m. ℃ and surface temperatures are steady at 150 ℃ and 45 ℃ . Determine:

A. Heat flow across the plane wall

B. Temperature gradient in the flow direction

Solution:

Wall thickness, dx = 150mm = 0.15m

Area, A = 4.5m^2

Temperature difference,

dt = 45 – 150 = -105 ℃

Thermal conductivity, k = 9.35 W/m. ℃

A. Applying the Fourier's Law of heat conduction.

B. Temperature gradient.

Solution:

A)

$$\frac{q}{A} = -k\frac{dT}{dx} \rightarrow \quad q = -(9.35 \times 4.5 \times -105/0.15¿ \quad = 29452.5 \text{ W}$$

B. Temperature gradient:

$$dt/dx = -(29452.5/(9.35*4.5)) = 700 \text{ ℃ /m}$$

Convection in Heat Transfer

In general, convection heat transfer deals with thermal interaction between a surface and an adjacent moving fluid. Examples include the flow of fluid over a cylinder, inside a tube and between parallel plates. Convection also includes the study of thermal interaction between fluids. An example is a jet issuing into a medium of the same or a different fluid.

Convection was considered as it related to the boundary conditions of a conduction problem.

$$q = hA\,(T_w - T_\infty)$$

h = convection heat transfer coeff.(W/m^2.°C).

Tw= The temperature of the plate

T∞= the temperature of the

fluid

A= surface area

Example (1)

Air at 20°C blows over a hot plate 50 by 75 cm maintained at 250◦C. The convection heat-transfer coefficient is 25 W/m2 · °C. Calculate the heat transfer.

Solution:

q = hA (Tw −T∞)

= (25)(0.50)(0.75)(250−20)

= 2156 W

Example(2):

Awire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure an electric current is passed through the wire and is increased until the water boils at 100℃ find how much electric power must be supplied to the wire to maintain the wire surface at 120℃?

Solution:

Diameter of the wire =1.5mm=0.0015m

Length of the wire=150mm=0.15m

Surface area (A)=$\pi dl = \pi$×0.0015×0.15=7.068$\times 10^{-4} m^2$

Wire surface temp=120℃

Water temp=100℃

h=4500 $\frac{W}{m^2 * ℃}$

electric power(q)=h A$(T_w - T_\infty)$

=4500×7.068×10^{-4}× (120-100)

=63.6 W

Radiation in Heat Transfer:

In contrast to the mechanisms of conduction and convection, where energy transfer through a material medium is involved, heat may also be transferred through regions where a perfect vacuum exists. The mechanism in this case is electromagnetic radiation. We shall limit our discussion to electromagnetic radiation that is propagated as a result of a temperature difference; this is called thermal radiation.

- In conduction and convection, the energy transfer through a material medium.
- Radiation: the energy can be transferred through vacuum by propagation of electromagnetic radiation.
- Black body (ideal radiation): it's a body emit energy at a rate proportional to the fourth power of the absolute temperature (in Kelvin) of the body and directly proportional to its surface area. Thus

Stefan-Boltzmann law of thermal radiation is

$$q = \sigma A\epsilon\ (T_1^4 - T_2^4)$$

T_1 is the temperature of radiates body (K)
T_2 is the temperature of receiving body (K)
σ Planks constant $5.667*10^{-8}$
$\in$ Emissivity for black body =1

Example (3)

Two infinite black plates at 800°C and 300°C exchange heat by radiation. Calculate the heat transfer per unit area.

Solution:

$q/A = \sigma\ \epsilon\ (T_1^4 - T_2^4)$

$= (5.667\times10^{-8})(1073^4 - 573^4)$.

$= 74041.398\ W/m^2$

Example(4):
A vertical square plate, 30 cm on a side, is maintained at 50℃ and exposed to room air at 20℃. The surface emissivity is 0.8 and the lost from the plate surface by radiation equal to 28.7W if the convection coefficient is $4.5 W/m^2.℃$. Calculate the total heat lost by conduction of the plate?

$$q = q_{conv} + q_{rad}$$

$$q_{conv} = hA(T_w - T_\infty)$$

$$h = 4.5 \ \frac{W}{m^2 \cdot {}^\circ C}$$

$$q_{conv} = (4.5)(0.3)^2(50-20)$$

$$= 12.15W$$

$$q_{rad} = \sigma \varepsilon A_1(T_1^4 - T_2^4)$$

$$= 28.7 \ W$$

$$q_{total} : \quad 40.85W$$

Steady-State Conduction One Dimension

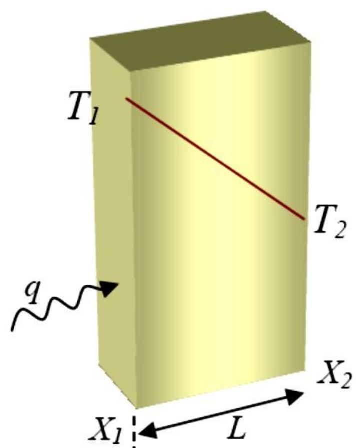
To examine the applications of Fourier's law of heat conduction to calculation of heat flow in some simple one-dimensional systems, we may take the following different cases:

1- The plane wall

A) One material

Using Fourier's law

$$\frac{dT}{dx} = \frac{\Delta T}{\Delta \chi} = \frac{T_2 - T_1}{\chi_2 - \chi_1}$$



$T_1 > T_2$

$$q = -\frac{kA}{\Delta x}(T_2 - T_1)$$

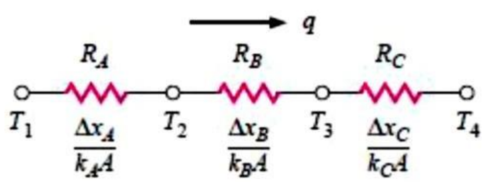
x
1
x
2

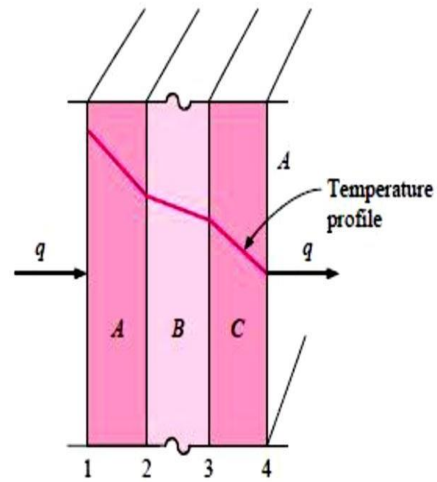
q=$\frac{(T_2 - T_1)}{\text{Thermal resistance}}$

Thermal resistance(R)=$\frac{\Delta X}{KA}$

B)More than one material (Composite wall)

If more than one material is present, as in the multilayer wall shown in Figure the analysis would proceed as follows: The temperature gradients in the three materials are shown, and the heat flow may be written





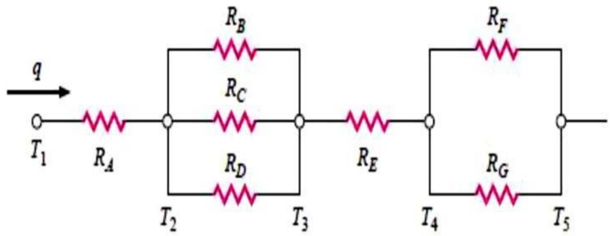
$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}$$

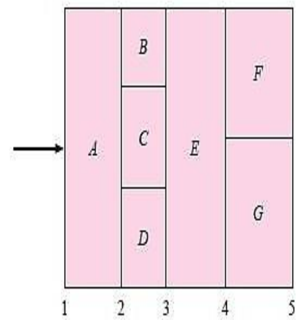
The heat flow must be the same through all sections, therefore Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A}$$

The one-dimensional heat-flow equation for this type problem may be written

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}}$$





$$\frac{1}{R_1}=\frac{1}{R_A};\quad \frac{1}{R_2}=\frac{1}{R_B}+\frac{1}{R_C}+\frac{1}{R_D};\quad \frac{1}{R_3}=\frac{1}{R_E};\quad \frac{1}{R_4}=\frac{1}{R_F}+\frac{1}{R_G}$$

$$\therefore\ \frac{1}{R_{th}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}$$

(R_{th} is the thermal resistances)

Example (1)

An outside wall of a building consists of 0.1m layer of common brick [k=0.69W/m.K] and 25mm layer of fiber glass [k=o.o5W/m.K]. Calculate the heat flow with through the wall for a 45°C temperature differences.

Solution:

$$q=\frac{\Delta T}{\sum R_{th}}=\frac{\Delta T_{overall}}{\frac{\Delta x_b}{k_b A}+\frac{\Delta x_f}{k_f A}}$$

$$\Rightarrow\quad q=\frac{45}{\frac{0.1}{0.69}+\frac{0.025}{0.05}}=69.78\,W/m^2$$

Example (2)

Two large aluminum plates (k = 240 W/m K), each 1 cm thick, with 10 µm surface roughness the contact resistance Ri = 2.75 x 10^{-4} m^2 K/W. The temperatures at the outside surfaces are 395°C and 405°C. Calculate the heat flux

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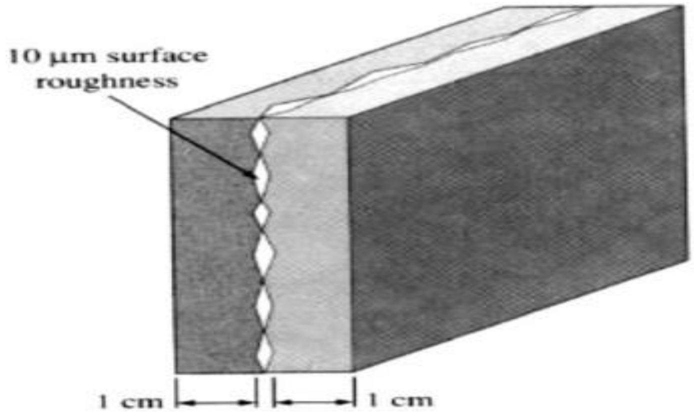


T

h

e

The rate of heat flow per unit area, q" through the sandwich wall is

S

O

L

$$q'' = \frac{T_{s1} - T_{s3}}{R_1 + R_2 + R_3} = \frac{\Delta T}{(L/k)_1 + R_i + (L/k)_2}$$

U

U

(

(L/k) = (0.01 m)/(240 W/m.K) = 4.17 x 10^{-5} m^2 K/W

Hence, the heat flux is

$$q'' = \frac{(405 - 395)^\circ C}{(4.17 \times 10^{-5} + 2.75 \times 10^{-4} + 4.17 \times 10^{-5})\ m^2\ K/W}$$

o

$$= 2.79 \times 10^4\ W/m^2\ K$$

f

h

e

a

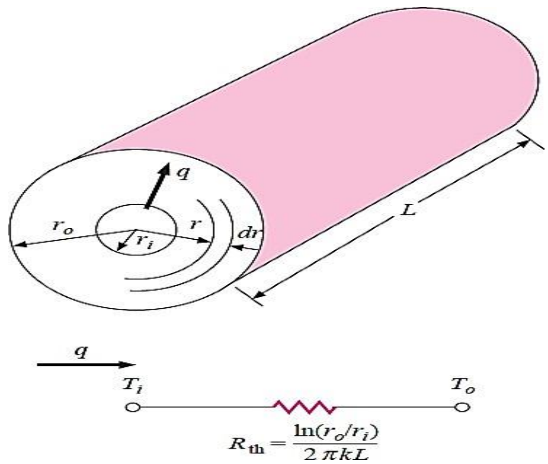
t

2- Radial systems

A) Cylindrical

i- One material

Consider a long cylinder of inside radius ri, outside radius ro, and length L. The inner side temperature is Ti, The outer side is T0, when the heat flows only in a radial direction. The area for heat flow in the cylindrical system is



$$A_r = 2\pi rL$$

So that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi\, krL \frac{dT}{dr}$$

$$\frac{q}{2\pi kL}\int_{r_i}^{r_0}\frac{dr}{r}=-\int_{T_i}^{T_0}dT$$

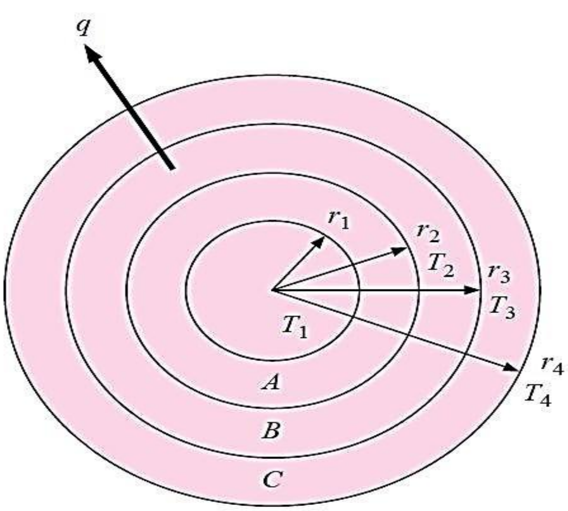
The solution is

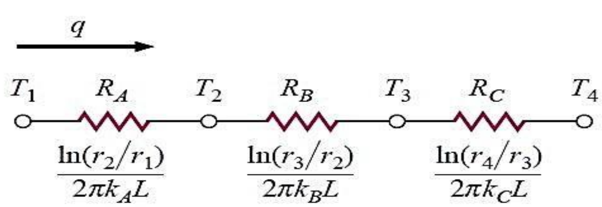
$$q=\frac{2\pi kL\,(T_i-T_o)}{\ln(r_o/r_i)}$$

and the thermal resistance in this case is

$$R_{th}=\frac{\ln(r_o/r_i)}{2\pi kL}$$

ii- Multi-Layer cylindrical wall





T =Ti

at r = ri

T =To

at r = ro

The solution to Equation

$$q = \frac{2\pi kL\,(T_i - T_o)}{\ln\,(r_o/r_i)}$$

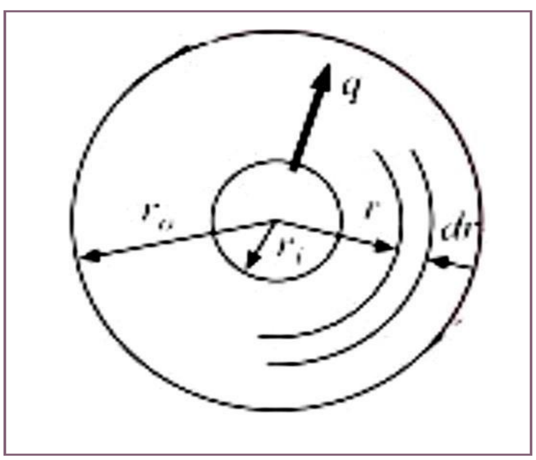
and the thermal resistance in this case is

$$R_{\text{th}} = \frac{\ln\,(r_o/r_i)}{2\pi kL}$$

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln\,(r_2/r_1)/k_A + \ln\,(r_3/r_2)/k_B + \ln\,(r_4/r_3)/k_C}$$

B) Spherical

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then



$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -4k\pi\, r^2 \frac{dT}{dr}$$

$$\frac{}{4\pi\, k} \cdot \int_{r_i}^{r_0} \frac{dr}{r^2} = -\int_{T_i}^{T_0} dT$$

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o}$$

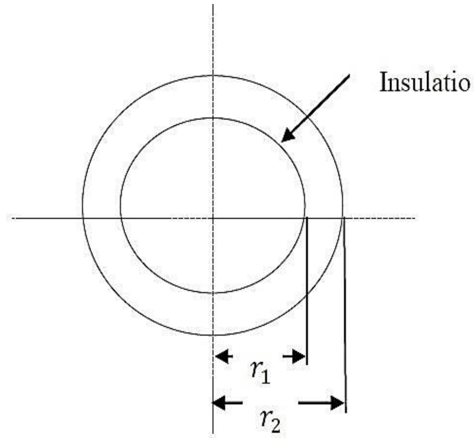
The thermal resistance in spherical system is:

$$R_{th} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right)$$

$$q_r = \frac{4\pi k(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Example :

A spherical container having outer diameter (500 mm) is insulated by (100 mm) thick layer of material with thermal conductivity (k=0.03(1+0.006T)) W/m. °C, where T in °C. If the surface temperature of sphere is (-200 °C) and temperature of outer surface is (30 °C) determine the heat flow.



$$q = \frac{4\pi k(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$r_1 = \frac{D}{2} = \frac{500}{2} = 250\, mm$$

$$r_2 = r_1 + 100 = 350\, mm$$

$$k = 0.3(1 + 0.006T) = 0.3(1 + 0.006\left(\frac{-200 + 30}{2}\right))$$

$$k = 0.147\, W$$

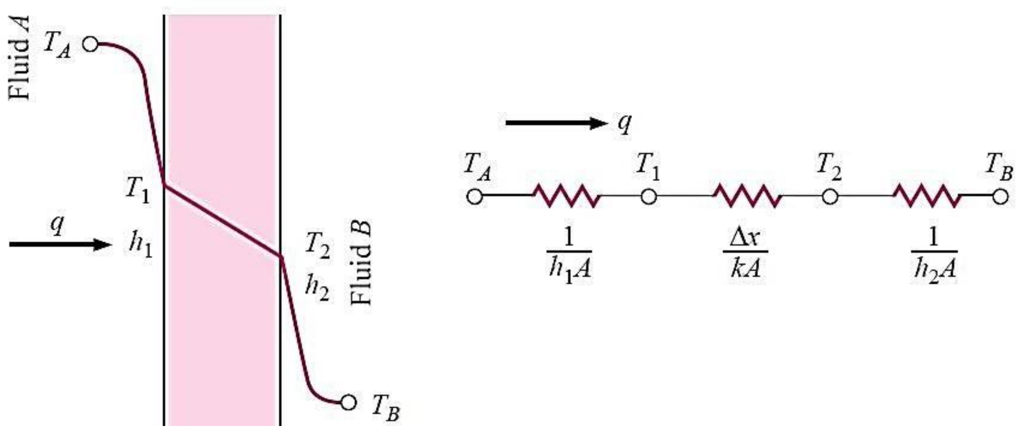
$$q = \frac{4\pi * 0.147(-200 - 30)}{\left(\frac{1}{0.025} - \frac{1}{0.035}\right)} = -37.14\, W$$

THE OVERALL HEAT TRANSFER COEFFICIENT

We noted previously that a common heat transfer problem is to determine the rate of heat flow between two fluids, gaseous or liquid, separated by a wall. If the wall is plane and heat is transferred only by convection on both sides, the rate of heat transfer in terms of the two fluid temperatures is given by

$$q = h_1 A\,(T_A - T_1) = \frac{kA}{\Delta x}\,(T_1 - T_2) = h_2 A\,(T_2 - T_B)$$

The heat-transfer process may be represented by the resistance network in Figure



and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A}$$

And the figure below show Resistance analogy for hollow cylinder with convection boundaries

Observe that the value 1/*hA* is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat- transfer coefficient *U*, defined by the relation

$q = UA\Delta T_{overall}$

where *A* is some suitable area for the heat flow. In accordance with Equation the overall heat-transfer coefficient would be

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

The overall heat-transfer coefficient is also related to the *R* value of Equation through

$$U = \frac{1}{R \text{ value}}$$

For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electric-resistance analogy would appear as in Figure where, again, *TA* and *TB* are the two fluid temperatures. Note that the area for convection is not the same for both fluids in this case, these areas depending on the inside tube diameter and wall thickness. The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}}$$

in accordance with the thermal network shown in Figure. The terms *Ai* and *Ao* represent the inside and outside surface areas of the inner tube. The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i}\dfrac{1}{h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o}}$$

The general notion, for either the plane wall or cylindrical coordinate system, is that

$$UA = 1/\sum R_{th} = 1/R_{th,\text{overall}}$$

Example(1)

Water flows at 50◦C inside a 2.5-cm-inside-diameter tube such that *hi* =3500 W/m2 · ◦C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m . ◦C. The outside of the tube loses heat by free convection with *ho* =7.6W/m2 . ◦C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20◦ C.

Solution

There are three resistances in series for this problem L=1.0 m, di =0.025 m, and do =0.025+(2)(0.0008)=0.0266 m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ ^\circ\mathrm{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L}$$

$$= \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ ^\circ\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ ^\circ\mathrm{C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so*. This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad [a]$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)}$$

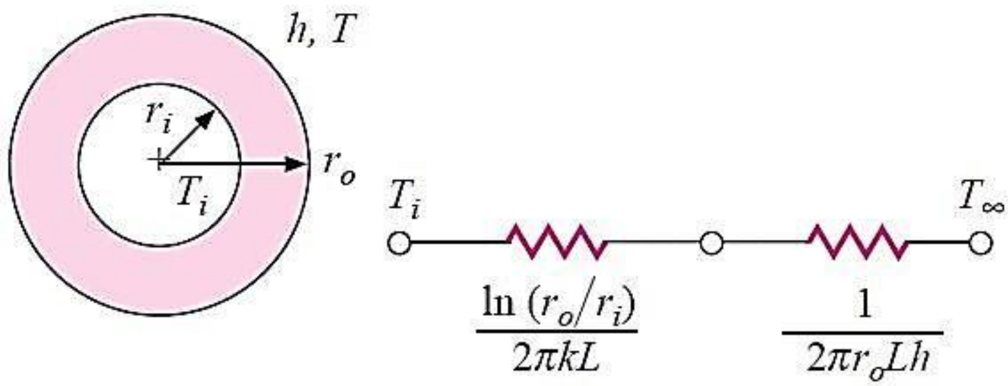
$$= 7.577\ \mathrm{W/m^2 \cdot {}^\circ C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (a), with

$$q = U A_o\, \Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19\ \mathrm{W}\ \text{(for 1.0 m length)}$$

CRITICAL THICKNESS OF INSULATION

Let us consider a layer of insulation which might be installed around a circular pipe. The inner temperature of the insulation is fixed at *Ti*, and the outer



surface is exposed to a convection environment at T_∞. From the thermal network the heat transfer is

$$q = \frac{2\pi L\,(T_i - T_\infty)}{\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}}$$

$$r_o = \frac{k}{h}$$

Example(1)

Calculate the critical radius of insulation for asbestos [*k* =0.17 W/^{0}C] surrounding a pipe and exposed to room air at 20◦C with *h*=3.0 W/m2 · ◦C. Calculate the heat loss from a 200◦C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567\ \text{m} = 5.67\ \text{cm}$$

The inside radius of the insulation is 5.0/2 = 2.5 cm, so the heat transfer is calculated from Equation

$$\frac{q}{L} = \frac{2\pi\,(200-20)}{\dfrac{\ln(5.67/2.5)}{0.17} + \dfrac{1}{(0.0567)(3.0)}} = 105.7\ \text{W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200-20) = 84.8\ \text{W/m}$$

The general equation for heat transfer by conduction

The general equation for heat transfer by conduction can be derived by making energy balance on a solid system. For the element of thickness dx, the following energy balance may be made:

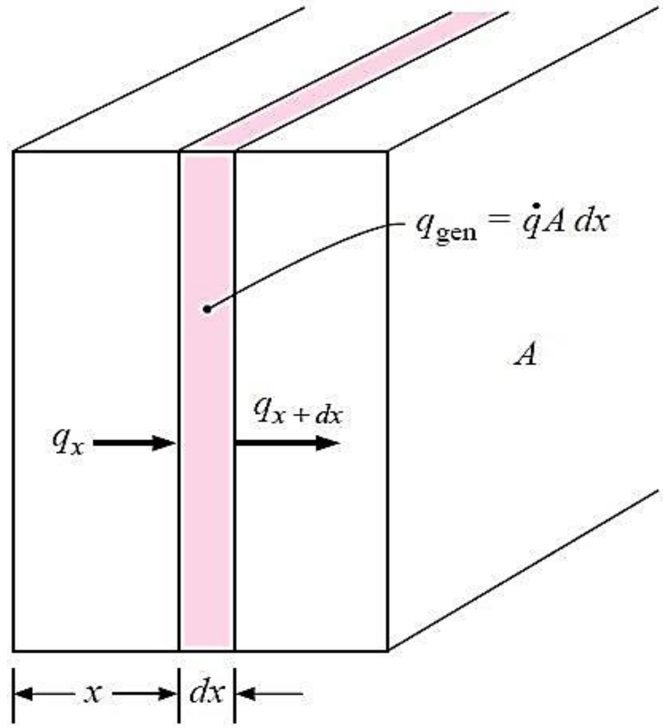
Energy conducted in left face + heat generated within element

=change in internal energy + energy conducted out right face

These energy quantities are given as follows:

$$\text{Energy in left face} = q_x = -kA\,\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$



$$\text{Change in internal energy } = \rho c A \frac{\partial T}{\partial \tau} dx$$

$$\text{Energy out right face } = q_{x+dx} = -kA \left.\frac{\partial T}{\partial x}\right]_{x+dx}$$

$$= -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

where

$\dot{q}$ = energy generated per unit volume, W/m^3
c = specific heat of material, J/kg · °C
ρ = density, kg/m^3

Combining the relations above gives

$$-kA\frac{\partial T}{\partial x} + \dot{q}A\,dx = \rho c A\frac{\partial T}{\partial \tau}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

or

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

This is the one-dimensional heat-conduction equation.

General three dimension heat transfer by conduction

(A) Cartesian coordinates

so that the general three-dimensional heat-conduction equation is

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\,\frac{\partial T}{\partial \tau}$$

For constant thermal conductivity,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

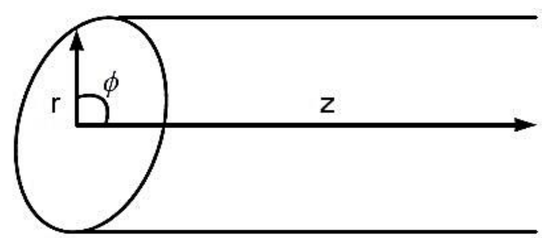
where the quantity $\alpha = k/\rho c$ is called the *thermal diffusivity* of the material (m^2/s).

B) Cylindrical Coordinate

$x = r\,Cos\phi$

$y = r\,Sin\phi$

$z = z$



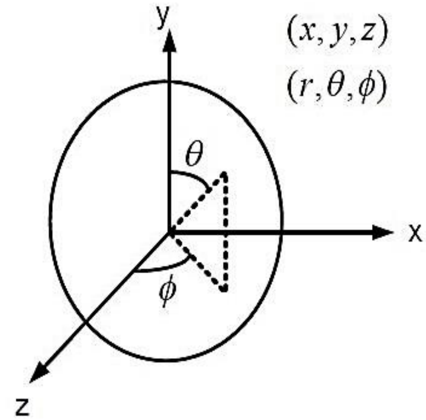
$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

C) Spherical Coordinate

$x = r\,Cos\theta + Sin\phi$

$y = r\,Sin\theta + Cos\phi$

$z = r\,Cos\theta$



The general equation is written:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial\tau}$$

1-Steady-state one-dimensional heat flow (no heat generation):

$$\frac{d^2T}{dX^2} = 0$$

2- Steady-state one-dimensional heat flow in cylindrical coordinates (no heat generation):

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

3- Steady-state one-dimensional heat flow with heat sources:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

4-Two-dimensional steady-state conduction without heat sources:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

HEAT-SOURCE SYSTEMS

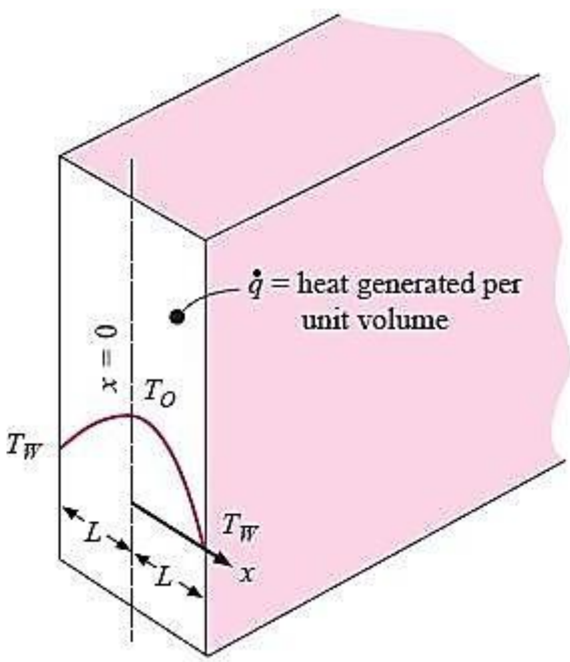
A number of interesting applications of the principles of heat transfer are concerned with systems in which heat may be generated internally.

1. Nuclear reactors are one example
2. electrical conductors
3. chemically reacting systems

At this point we shall confine our discussion to one-dimensional systems, or, more specifically, systems where the temperature is a function of only one space coordinate.

1- Plane Wall with Heat Sources

Consider the plane wall shown with uniformly distributed heat sources as shown in the figure. The heat generated per unit volume is The general equation is ***q·***



$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

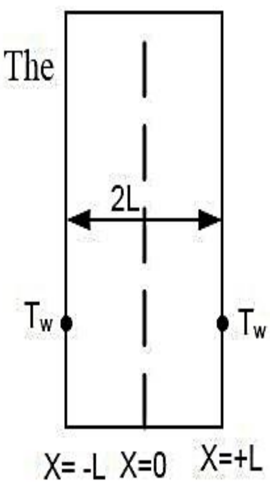
For one-dimensional, steady state with heat generation

$$\frac{d^2T}{dx^2}+\frac{q^\bullet}{k}=0 \Rightarrow \frac{d^2T}{dx^2}=-\frac{q^\bullet}{k} \Rightarrow by\int \Rightarrow \frac{dT}{dx}=-\frac{q^\bullet x}{k}+C_1 \Rightarrow by\int \Rightarrow$$

$$T=-\frac{q^\bullet x^2}{2k}+C_1X+C_2 \quad \textit{(general solution)}$$

The both sides of the plane wall are subjected to a constant temperature Tw. The Boundary conditions will be

$$T=T_w \qquad \text{at } x=\pm L$$



By applying the boundary conditions above,

$$C_1=0$$

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$$T_W=-\frac{q^\bullet L^2}{2k}+C_2 \Rightarrow C_2=T_W+\frac{q^\bullet L^2}{2k}$$

$$\therefore\ T=-\frac{q^\bullet x^2}{2k}+T_W+\frac{q^\bullet L^2}{2k}$$

$$T_{\max} = T_W + \frac{q^{\bullet} L^2}{2k}$$

$$T = -\frac{q^{\bullet} x^2}{2k} + C_1 X + C_2$$

B.C.1 $x = 0, \ T = T_1$

B.C. 2 $x = L, \ T = T_2$

From B.C.1, $T_1 = C_2$

From B.C.2, $T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + C_2$

By sub. C_2, we get

$$T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + T_1 \Rightarrow$$

$$C_1 = \frac{(T_2 - T_1)}{L} + \frac{q^{\bullet} L}{2k}$$

Application

Consider the plane wall with uniformly distributed heat sources shown in Figure below. The thickness of the wall in the x direction is 2L, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one dimensional. The heat generated per unit volume is $\dot{q}$, and assume that the thermal conductivity does not vary with temperature. Derive an expression of the temperature distribution

Solution:

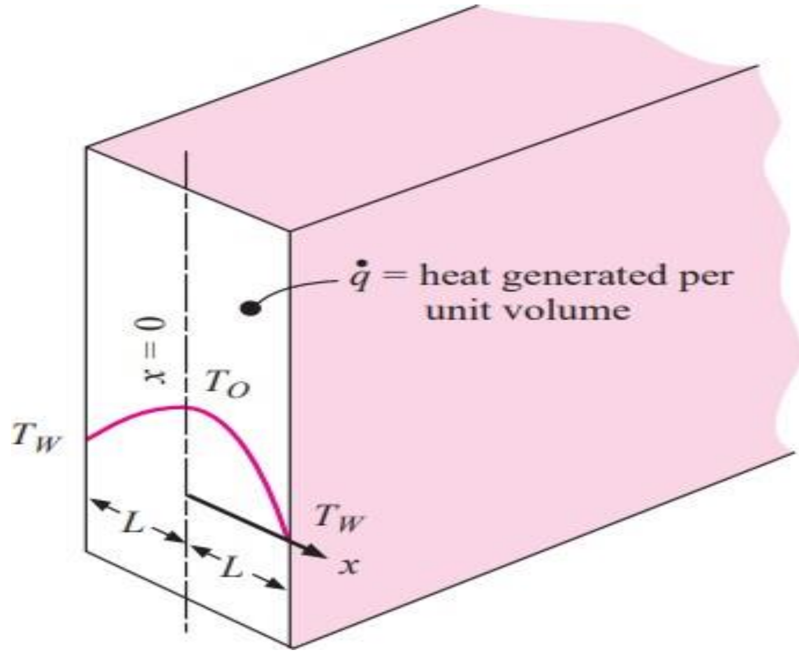
Assumption:

1- One-Dimension ($\partial/\partial y$=0,$\partial/\partial z$=0).

2- Steady state (∂/∂t).

3- Uniform heat generation($\dot{q}$).

4- Homogeneous(k=constant).



$$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{q}}{k}=0 \qquad integrate$$

$$\frac{\partial T}{\partial x}+\frac{\dot{q}}{k}x=C_1 \qquad (1) \quad integrate\ again$$

$$T+\frac{\dot{q}}{k}x^2=C_1x+C_2$$

$$T=-\frac{\dot{q}}{2k}x^2+C_1x+C_2 \qquad (2)$$

B.C1: $at\ x=0$ $\qquad T=T_0$ $\quad$ Sub. in Eq. (2)

$$T_0=-\frac{\dot{q}}{2k}(0)^2+C_1*0+C_2$$

$C_2 = T_0$ Sub. in Eq. (2)

B.C2: $at\ x = \pm L$ $T = T_w$ Sub. in Eq. (2)

$$T_w = -\frac{\dot{q}}{2k}L^2 + C_1 L + T_0 \qquad (3)$$

$$T_w = -\frac{\dot{q}}{2k}L^2 - C_1 L + T_0 \qquad (4)$$

———————— Subtract

$$0 = 0 + 2LC_1 + 0$$

$C_1 = 0$ Sub. in Eq. (2)

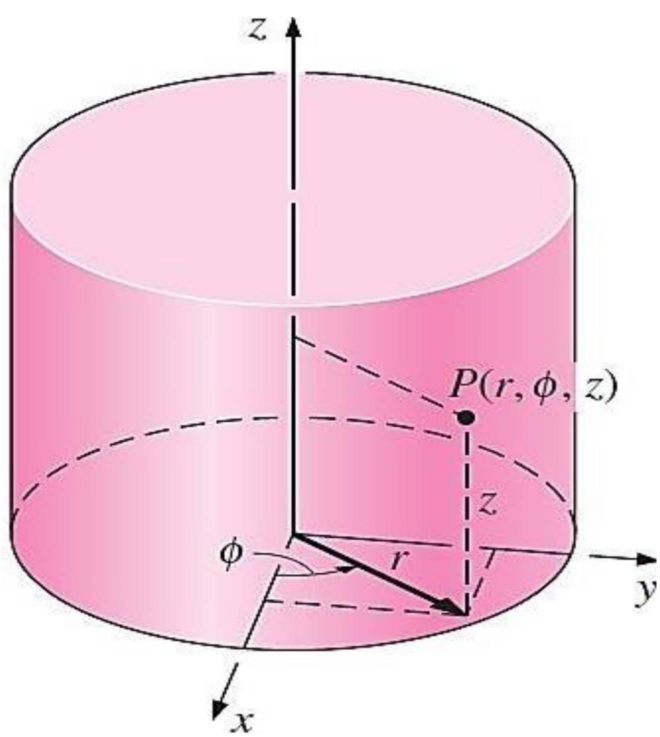
$$T = -\frac{\dot{q}}{2k}x^2 + T_0$$

$$T - T_0 = -\frac{\dot{q}}{2k}x^2 \qquad (5)$$

The Conduction Equation of Cylindrical Coordinate

A common example is the hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures. For a general transient three dimensional in the cylindrical coordinates *T=T(r,* φ *,z, t)*, the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



For a general transient three-dimensional in the cylindrical coordinates *T= T(r,* φ *,z, t)*,the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If the heat flow in a cylindrical shape is only in the radial direction and for steady-state conditions with no heat generation, the conduction equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=0$$

Integrating once with respect to radius gives

$$r\frac{\partial T}{\partial r}=C_1 \quad \text{and} \quad \frac{\partial T}{\partial r}=\frac{C_1}{r}$$

A second integration gives $T = C_1 \ln r + C_2$.

To obtain the constants (C_1 and C_2), we introduce the following boundary conditions

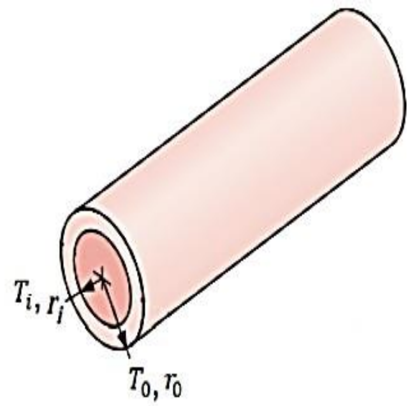
B.C.1 $T=T_i$ at $r=r_i$ $T_i = C_1 \ln r_i + C_2$.

B.C.2 $T=T_o$ at $r=r_o$ $T_o = C_1 \ln r_o + C_2$.

Example

consider a steam pipe of length (L), inner radius (ri), outer radius ($r0$) and thermal conductivity (k). The inner and outer surface of pipe are maintained at average temperature of (Ti) and ($T0$) respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions and determine the rate of heat loss from the steam through the pipe

Solution:



Assumption:

1- Steady state ($\partial/\partial t = 0$).

2- Homogenous material (isotropic material).

3- With heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad \textit{multiply by } r$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad \textit{integrate}$$

$$r\frac{\partial T}{\partial r} = C_1 \quad \rightarrow \quad \frac{\partial T}{\partial r} = \frac{C_1}{r} \qquad \textit{integrate again}$$

$$T = C_1 \ln r + C_2 \qquad (1)$$

B.C1: $at\ r = r_i \qquad T = T_i \qquad sub. in\ Eq. (1)$

$$T_i = C_1 \ln r_i + C_2 \qquad (3)$$

B.C2: $at\ r = r_0 \qquad T = T_0 \qquad sub. in\ Eq. (1)$

$$T_0 = C_1 \ln r_0 + C_2 \qquad (4)$$

Subtract Eq. (3) and Eq. (4)

$$T_i - T_0 = C_1 \ln\frac{r_i}{r_0}$$

$$C_1 = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \qquad sub. in\ Eq. (3)$$

$$T_i = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i + C_2$$

$$C_2 = T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

Sub. C_1 and C_2 in Eq. (1)

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r + T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln\frac{r}{r_i} + T_i$$

$$q = -kA\frac{\partial T}{\partial r} = -k(2\pi rL)\frac{C_1}{r} = -\frac{(2\pi rLk)}{r}\frac{T_i - T_0}{\ln\frac{r_i}{r_0}}$$

$$q = -2\pi Lk\frac{(T_i - T_0)}{\ln\frac{r_i}{r_0}}$$