Ministry of Higher Education and Scientific Research / Iraq

Northern Technical University

College of Oil & Gas Techniques Engineering / Kirkuk

Department of Renewable Energy Techniques Engineering

Electrical technology

First year

By: Dr. Naseer T. Alwan

2023-2024

MODULE DESCRIPTION FORM

Module Information					
Module Title	Electrical technology			**Module Delivery**	
Module Type	**Basic**			☒ **Theory** ☐ **Lecture** ☒ **Lab** ☒ **Tutorial** ☐ **Practical** ☐ **Seminar**	
Module Code	**COGTEK 101**				
ECTS Credits	**6**				
SWL (hr/sem)	**150**				
Module Level	UGx11 1	**Semester of Delivery**		1	
Administering Department	RETE	**College**	College of Oil & Gas Techniques Engineering/Kirkuk		
Module Leader	Naseer Tawfeeq Alwan	e-mail	naseer.t.alwan@ntu.edu.iq		
Module Leader's Acad. Title	Lecturer	Module Leader's Qualification		PhD	
Module Tutor	Naseer Tawfeeq Alwan	e-mail	naseer.t.alwan@ntu.edu.iq		
Peer Reviewer Name	Name	**e-mail**	E-mail		
Scientific Committee Approval Date	01/06/2023	**Version Number**	1.0		

Delivery Plan (Weekly Syllabus)

	Material Covered
Week 1	How to use measuring devices for the purpose of measuring (R, I, V)
Week 2	Ohm's law Connecting resistors to mixed parallel
Week 3	Kirchhoff's law for voltage and current
Week 4	Applications of Kirchhoff’s law
Week 5	Thevenin Theory
Week 6	Norton Theory
Week 7	Tractorism Theory
Week 8	Nodal theory
Week 9	Series circuits consisting of a coil
Week 10	Parallel circuits consisting of a coil
Week 11	Series circuits consisting of a capacitor
Week 12	Parallel circuits consisting of a capacitor
Week 13	Resonant circuit
Week 14	Applications of series circuits
Week 15	Applications of parallel circuits
Week 16	Preparatory week before the final Exam

Learning and Teaching Resources

	Text	Available in the Library
Required Texts	“Basic Electrical Engineering”, THERAJA.	Yes
Recommended Texts	“Electrical and Electronic Principles and Technology”, John Bird	Yes
Websites	Basic Electrical Circuits website tutorials	

Chapter One

1. Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e., electrons) through it.

1.2. The Unit of Resistance

The practical unit of resistance is ohm. The symbol for ohm is Ω.

Prefix	*Its meaning*	*Abbreviation*	*Equal to*
Mega-	One million	M Ω	10^6 Ω
Kilo-	One thousand	k Ω	10^3 Ω
Centi-	One hundredth	–	–
Milli-	One thousandth	m Ω	10^{-3} Ω
Micro-	One millionth	μ Ω	10^{-6} Ω

1.3 Laws of Resistance

The resistance R offered by a conductor depends on the following factors:

(i) It varies directly as its length, L.

(ii) It varies inversely as the cross-section A of the conductor.

(iii) It depends on the nature of the material.

(iv) It also depends on the temperature of the conductor.

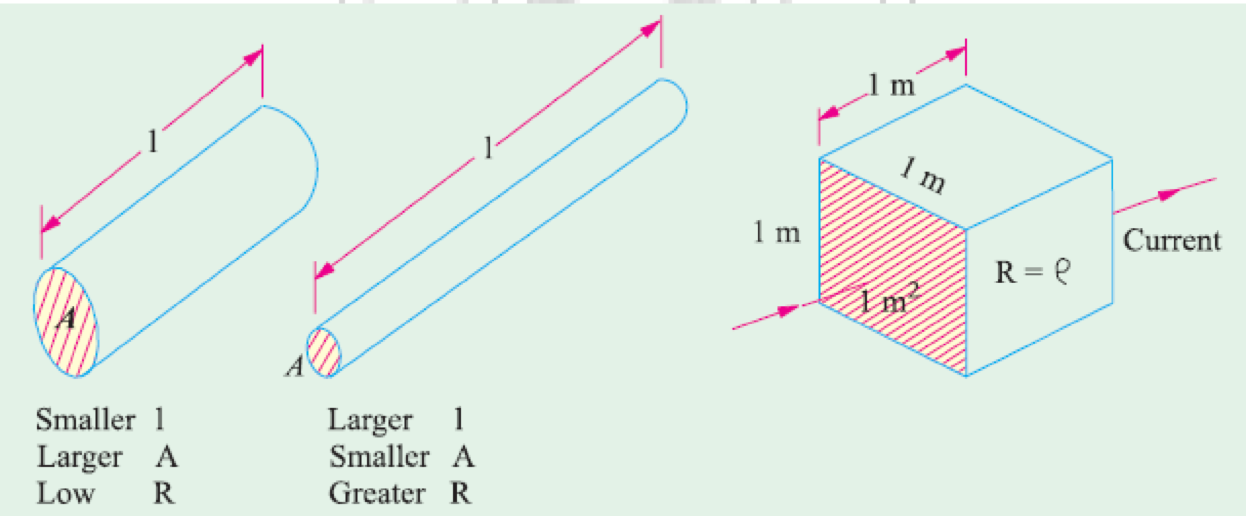


Fig. 1 Fig.2

Neglecting the last factor for the time being, we can say that:

$$R \propto \frac{l}{A} \quad or\ R = \rho \frac{l}{A} \quad \ldots (\boldsymbol{i})$$

Where ρ is a constant depending on the nature of the material of the conductor and is known as its ***specific resistance* or *resistivity***.
If in Eq. ***(i)***, we put
L = 1 meter and A = 1 metre^2, then $R = \rho$ (Fig. 2)
Hence, specific resistance of a material may be defined as ***the resistance between the opposite faces of a meter cube of that material***.

1.4 Units of Resistivity

From Eq. ***(i)***, we have $\rho = \frac{AR}{L}$

In the S.I. system of units,

$$\rho = \frac{A\ metre^2 \times R\ ohm}{1\ metre} = \frac{A\ R}{L}\ \ ohm - metre$$

Hence, the unit of resistivity is ohm-metre ($\Omega - m$).

Table 1.2. Resistivities and Temperature Coefficients

Material	*Resistivity in ohm-metre at 20°C ($\times 10^{-8}$)*	*Temperature coefficient at 20°C ($\times 10^{-4}$)*
Aluminium, commercial	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–5
Constantan or Eureka	49	+0.1 to -0.4
Copper (annealed)	1.72	39.3
German Silver	20.2	2.7
(84% Cu; 12% Ni; 4% Zn)		
Gold	2.44	36.5
Iron	9.8	65
Manganin	44 – 48	0.15
(84% Cu ; 12% Mn ; 4% Ni)		
Mercury	95.8	8.9
Nichrome	108.5	1.5
(60% Cu ; 25% Fe ; 15% Cr)		
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	5×10^{14}	
Bakelite	10^{10}	
Glass	$10^{10} - 10^{12}$	
Mica	10^{15}	
Rubber	10^{16}	
Shellac	10^{14}	
Sulphur	10^{15}	

Example: Most homes use solid copper wire having a diameter of 1.63 mm to provide electrical distribution to outlets and light sockets. Determine the resistance of 75 meters of a solid copper wire having the above diameter.

Solution:

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.63 \times 10^{-3}\ \text{m})^2}{4} = 2.09 \times 10^{-6}\ \text{m}^2$$

Now, using the Table above, the resistance of the length of wire is;

$$R = \frac{\rho \ell}{A} = \frac{(1.723 \times 10^{-8}\ \Omega\text{-m})(75\ \text{m})}{2.09 \times 10^{-6}\ \text{m}^2} = 0.619\ \Omega$$

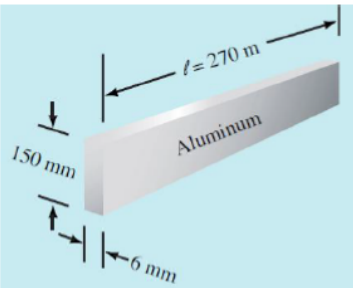
Example: Bus bars are bare solid conductors (usually rectangular) used to carry large currents within buildings such as power generating stations, telephone exchanges, and large factories. Given a piece of aluminum bus bar as shown in Figure, determine the resistance between the ends of this bar at a temperature of 20°C.

Solution The cross-sectional area is

$$A = (150\ \text{mm})(6\ \text{mm}) = (0.15\ \text{m})(0.006\ \text{m}) = 0.0009\ \text{m}^2 = 9.00 \times 10^{-4}\ \text{m}^2$$

The resistance between the ends of the bus bar is determined as

$$R = \frac{\rho \ell}{A} = \frac{(2.825 \times 10^{-8}\ \Omega\text{-m})(270\ \text{m})}{9.00 \times 10^{-4}\ \text{m}^2} = 8.48 \times 10^{-3}\ \Omega = 8.48\ \text{m}\Omega$$

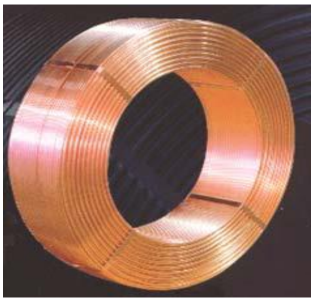


Example: A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm2. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ–m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.

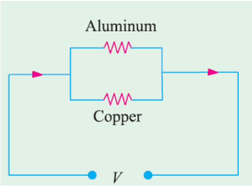
Solution. Length of the coil, $l = 0.8 \times 2000 = 1600$ m ; $A = 0.8\ \text{mm}^2 = 0.8 \times 10^{-6}\ \text{m}^2$.

$$R = \rho\frac{l}{A} = 0.02 \times 10^{-6} \times 1600/0.8 \times 10^{-6} = \mathbf{40\ \Omega}$$

$$\text{Power absorbed} = V^2/R = 110^2/40 = \mathbf{302.5\ W}$$



Example: An aluminium wire 7.5 m long is connected in a parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. The resistivity of copper is 0.017 μΩ- ; that of the aluminium is 0.028 μΩ-m.



Solution. Let the subscript 1 represent aluminium and subscript 2 represent copper.

$$R_1 = \rho\frac{l_1}{a_1} \text{ and } R_2 = \rho_2\frac{l_2}{a_2} \qquad \therefore \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1}\cdot\frac{l_2}{l_1}\cdot\frac{a_1}{a_2}$$

$$\therefore \quad a_2 = a_1\cdot\frac{R_1}{R_2}\cdot\frac{\rho_2}{\rho_1}\cdot\frac{l_2}{l_1} \qquad ...(i)$$

Now $I_1 = 3$ A ; $I_2 = 5 - 3 = 2$ A.

If V is the common voltage across the parallel combination of aluminium and copper wires, then

$$V = I_1 R_1 = I_2 R_2 \qquad \therefore R_1/R_2 = I_2/I_1 = 2/3$$

$$a_1 = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = \frac{\pi}{4}\ \text{mm}^2$$

Substituting the given values in Eq. (*i*), we get

$$a_2 = \frac{\pi}{4}\times\frac{2}{3}\times\frac{0.017}{0.028}\times\frac{6}{7.5} = 0.2544\ \text{m}^2$$

$$\therefore \quad \pi \times d_2^2/4 = 0.2544 \quad \text{or} \quad d_2 = \mathbf{0.569\ mm}$$

Tutorial Problems No. 1.1

1. Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m.
   If the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased? **[500 Ω; 9 times]**
2. A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length? **[0.064 Ω]**
3. A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 per cent more current than the latter and the latter is 47 per cent longer than the former. Determine the ratio of their cross-sectional area. **[2.5 : 1]**
4. A rectangular metal strip has the following dimensions :
   $$x = 10 \text{ cm}, y = 0.5 \text{ cm}, z = 0.2 \text{ cm}$$
   Determine the ratio of resistances R_x, R_y, and R_z between the respective pairs of opposite faces.
   [$R_x : R_y : R_z$: 10,000 : 25 : 4] *(Elect. Engg. A.M.Ae. S.I.)*
5. The resistance of a conductor 1 mm^2 in cross-section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material. **[1.73×10^{-8} Ω-m]** *(Elect. Circuits-1, Bangalore Univ. 1991)*
6. When a current of 2 A flows for 3 micro-seconds in a coper wire, estimate the number of electrons crossing the cross-section of the wire. *(Bombay University, 2000)*
   Hint : With 2 A for 3 μ Sec, charge transferred = 6 μ-coulombs
   Number of electrons crossed = $6 \times 10^{-6}/(1.6 \times 10^{-19}) = 3.75 \times 10^{+13}$

1.5 Conductance and Conductivity

المواصلة الكهربائية :هي معكوس المقاومة الكهربائية والمواصلة هي قدرة المادة الكهربائية على تمرير الشحنات وبالتالي التوصيل بين عناصر الدائرة الكهربائية وتتأثر بنفس العوامل المؤثرة على قيمة المقاومة الكهربائية , وتتناسب المواصلة الكهربائية طرديا مع مساحة المقطع الموصل وعكسيا مع طول الموصل ومقاومة المادة

Conductance (*G*) is reciprocal of resistance*. Whereas resistance of a conductor measures the ***opposition*** which it offers to the flow of current, the conductance measures the ***inducement*** which it offers to its flow.

From Eq. **(*i*)** of Art. 1.6, $R = \rho \frac{l}{A}$ or $G = \frac{1}{\rho} \cdot \frac{A}{L} = \frac{\sigma A}{l}$

where σ is called the ***conductivity*** **or** ***specific conductance*** of a conductor. The unit of conductance is siemens (S).

It is seen from the above equation that the conductivity of a material is given by

$$\sigma = G\frac{l}{A} = \frac{G \text{ siemens} \times l \text{ metre}}{A \text{ metre}^2} = G\frac{l}{A} \text{ siemens/metre}$$

1.6. Effect of Temperature on Resistance

The resistance of a conductor will not be constant at all temperatures. As temperature increases, more electrons will escape their orbits, causing additional collisions within the conductor. For most conducting materials, the increase in the number of collisions translates into a relatively linear increase in resistance.

1.7. Temperature Coefficient of Resistance

Let a metallic conductor having a resistance of R_0 at 0°C be heated of t°C and let its resistance at this temperature be R_t. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

(*i*) directly on its initial resistance

(*ii*) directly on the rise in temperature

(*iii*) on the nature of the material of the conductor.

or $$R_t - R_0 \propto R \times t \quad \text{or} \quad R_t - R_0 = \alpha R_0 t \qquad ...(i)$$

where α (alpha) is a constant and is known as the *temperature coefficient of resistance* of the conductor.

Rearranging Eq. (*i*), we get $\alpha = \dfrac{R_t - R_0}{R_0 \times t} = \dfrac{\Delta R}{R_0 \times t}$

If $R_0 = 1\ \Omega,\ t = 1°\text{C}$, then $\alpha = \Delta R = R_t - R_0$

Hence, the temperature-coefficient of a material may be defined as :

the increase in resistance per ohm original resistance per °C rise in temperature.

From Eq. (*i*), we find that $$R_t = R_0 (1 + \alpha t) \qquad ...(ii)$$

It should be remembered that the above equation holds good for both rise as well as fall in temperature. As temperature of a conductor is decreased, its resistance is also decreased. In Fig. 1.3 is shown the temperature/resistance graph for copper and is practically a straight line. If this line is extended backwards, it would cut the temperature axis at a point where temperature is −234.5°C (a number quite easy to remember). It means that theoretically, the resistance of copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 1.6 it is seen that :

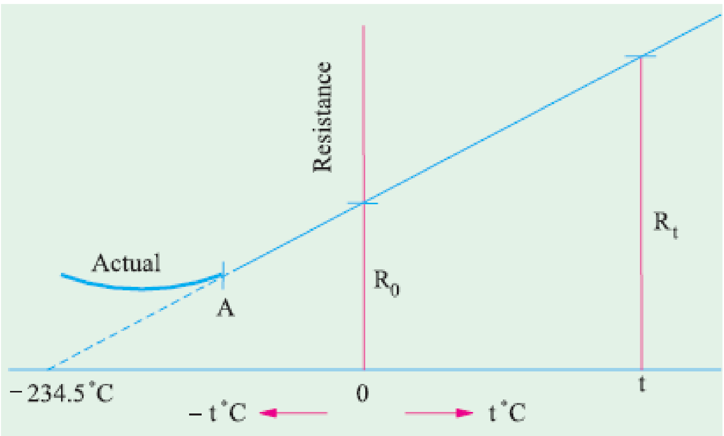
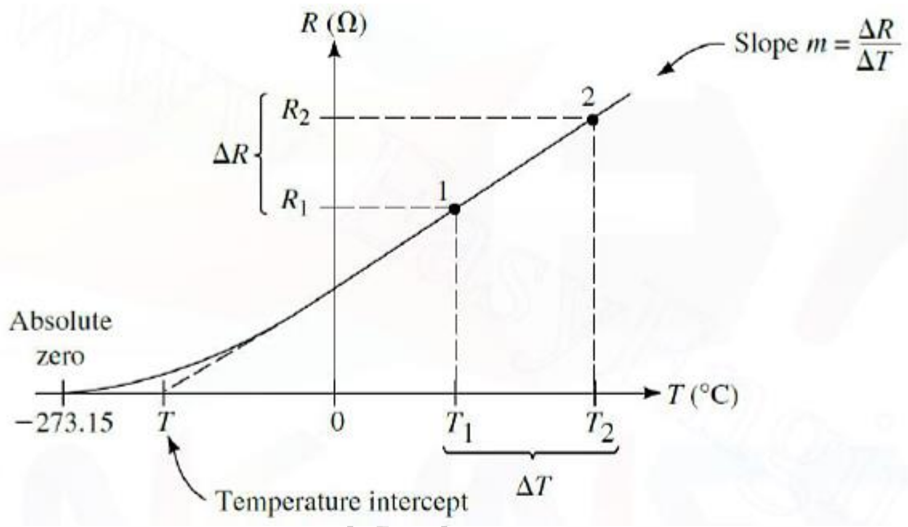


Fig. 1. 3

$$\frac{R_t}{R_0} = \frac{t + 234.5}{234.5} = \left(1 + \frac{t}{234.5}\right)$$

$$\therefore \quad R_t = R_0 \left(1 + \frac{t}{234.5}\right) \text{ or } R_t = R_0 (1 + \alpha t) \text{ where } \alpha = 1/234.5 \text{ for copper.}$$



Temperature Intercepts and Coefficients for Common Materials

	T (°C)	α (°C)$^{-1}$ at 20°C	α (°C)$^{-1}$ at 0°C
Silver	−243	0.003 8	0.004 12
Copper	−234.5	0.003 93	0.004 27
Aluminum	−236	0.003 91	0.004 24
Tungsten	−202	0.004 50	0.004 95
Iron	−162	0.005 5	0.006 18
Lead	−224	0.004 26	0.004 66
Nichrome	−2270	0.000 44	0.000 44
Brass	−480	0.002 00	0.002 08
Platinum	−310	0.003 03	0.003 23
Carbon		−0.000 5	
Germanium		−0.048	
Silicon		−0.075	

We observe an almost linear increase in resistance as the temperature increases. Further, we see that as the temperature is decreased to absolute zero (T=-273.15°C), the resistance approaches zero. In Figure, the point at which the linear portion of the line is extrapolated to cross the abscissa (temperature axis) is referred to as the temperature intercept or the inferred absolute temperature T of the material. By examining the straight-line portion of the graph, we see that we have two similar triangles, one with the apex at point 1 and the other with the apex at point 2. The following relationship applies for these similar triangles.

$$R_2 = \frac{T_2 - T}{T_1 - T} R_1$$

Example: An aluminum wire has a resistance of 20 Ω at room temperature (20°C). Calculate the resistance of the same wire at temperatures of -40°C, 100°C, and 200°C.

Solution:

At **T** = −40°C:

The resistance at −40°C is determined using Equation 3–6.

$$R_{-40°C} = \frac{-40°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{196°C}{256°C} 20\ \Omega = 15.3\ \Omega$$

At **T** = 100°C:

$$R_{100°C} = \frac{100°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{336°C}{256°C} 20\ \Omega = 26.3\ \Omega$$

At **T** = 200°C:

$$R_{200°C} = \frac{200°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{436°C}{256°C} 20\ \Omega = 34.1\ \Omega$$

1.8. Value of α at Different Temperatures

So far we did not make any distinction between values of α at different temperatures. But it is found that value of α itself is not constant but depends on the initial temperature on which the increment in resistance is based.

Suppose a conductor of resistance R_0 at 0°C (point *A* in Fig. 1.4) is heated to *t*°C (point *B*). Its resistance R_t after heating is given by

$$R_t = R_0 (1 + \alpha_0 t) \qquad ...(i)$$

where α_0 is the temperature-coefficient at 0°C.

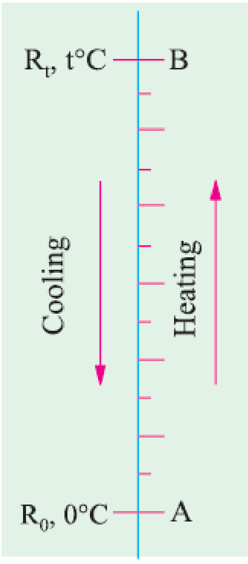


Fig. 1.4

Now, suppose that we have a conductor of resistance R_t at temperature *t*°C. Let this conductor be *cooled* from *t*°C to 0°C. Obviously, now the initial point is *B* and the final point is *A*. The final resistance R_0 is given in terms of the initial resistance by the following equation

$$R_0 = R_t [1 + \alpha_t (-t)] = R_t (1 - \alpha_t . t) \qquad ...(ii)$$

From Eq. (*ii*) above, we have $\alpha_t = \dfrac{R_t - R_0}{R_t \times t}$

Substituting the value of R_t from Eq. (*i*), we get

$$\alpha_t = \frac{R_0 (1 + \alpha_0 t) - R_0}{R_0 (1 + \alpha_0 t) \times t} = \frac{\alpha_0}{1 + \alpha_0 t} \quad \therefore \ \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t} \qquad ...(iii)$$

In general, let α_1 = tempt. coeff. at t_1°C ; α_2 = tempt. coeff. at t_2°C. Then from Eq. (*iii*) above, we get

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \text{ or } \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_1}{\alpha_0}$$

Similarly, $$\frac{1}{\alpha_2} = \frac{1+\alpha_0 t_2}{\alpha_0}$$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) \text{ or } \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) \text{ or } \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

Values of α for copper at different temperatures are given in Table No. 1.3.

Table 1.3. Different values of α for copper

Tempt. in °C	0	5	10	20	30	40	50
α	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

In case R_0 is not given, the relation between the known resistance R_1 at t_1°C and the unknown resistance R_2 at t_2°C can be found as follows:

$$R_2 = R_0(1+\alpha_0 t_2) \quad \text{and} \quad R_1 = R_0(1+\alpha_0 t_1)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1+\alpha_0 t_2}{1+\alpha_0 t_1} \qquad ...(iv)$$

The above expression can be simplified by a little approximation as follows :

$$\frac{R_2}{R_1} = (1+\alpha_0 t_2)(1+\alpha_0 t_1)^{-1}$$

$$= (1+\alpha_0 t_2)(1-\alpha_0 t_1)$$ [Using Binomial Theorem for expansion and

$$= 1+\alpha_0(t_2 - t_1)$$ neglecting squares and higher powers of $(\alpha_0 t_1)$]

$$\therefore \quad R_2 = R_1[1+\alpha_0(t_2 - t_1)]$$ [Neglecting product $(\alpha_0^2 t_1 t_2)$]

For more accurate calculations, Eq. (*iv*) should, however, be used.

Example 2. *A copper conductor has its specific resistance of 1.6×10^{-6} ohm-cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find (i) the specific resistance and (ii) the resistance - temperature coefficient at 60°C.* **(F.Y. Engg. Pune Univ. Nov.)**

Solution.

$$\alpha_{20} = \frac{\alpha_0}{1+\alpha_0 \times 20} \quad \text{or} \quad \frac{1}{254.5} = \frac{\alpha_0}{1+\alpha_0 \times 20} \quad \therefore \quad \alpha_0 = \frac{1}{234.5} \text{ per °C}$$

(*i*)
$$\rho_{60} = \rho_0 (1 + \alpha_0 \times 60) = 1.6 \times 10^{-6} (1 + 60/234.5) = \mathbf{2.01 \times 10^{-6}\ \Omega\text{-cm}}$$

(*ii*)
$$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60} = \frac{1/234.5}{1+(60/234.5)} = \frac{\mathbf{1}}{\mathbf{294.5}} \textbf{ per°C}$$

Example 3. *A platinum coil has a resistance of 3.146 Ω at 40°C and 3.767 Ω at 100°C. Find the resistance at 0°C and the temperature-coefficient of resistance at 40°C.*

(Electrical Science-II, Allahabad Univ.)

Solution.

$$R_{100} = R_0 (1 + 100\,\alpha_0) \qquad \ldots(i)$$

$$R_{40} = R_0 (1 + 40\,\alpha_0) \qquad \ldots(ii)$$

$$\therefore \quad \frac{3.767}{3.146} = \frac{1+100\,\alpha_0}{1+40\,\alpha_0} \quad \text{or} \quad \alpha_0 = 0.00379 \quad \text{or} \quad 1/264 \text{ per°C}$$

From (*i*), we have
$$3.767 = R_0 (1 + 100 \times 0.00379) \quad \therefore \quad R_0 = \mathbf{2.732\ \Omega}$$

Now,
$$\alpha_{40} = \frac{\alpha_0}{1+40\,\alpha_0} = \frac{0.00379}{1+40 \times 0.00379} = \frac{\mathbf{1}}{\mathbf{304}} \textbf{ per°C}$$

Tutorial Problems No. 1.2

1. It is found that the resistance of a coil of wire increases from 40 ohm at 15°C to 50 ohm at 60°C. Calculate the resistance temperature coefficient at 0°C of the conductor material.
   [1/165 per °C] (*Elect. Technology, Indore Univ.*)
2. A tungsten lamp filament has a temperature of 2,050°C and a resistance of 500 Ω when taking normal working current. Calculate the resistance of the filament when it has a temperature of 25°C. Temperature coefficient at 0°C is 0.005/°C. **[50 Ω] (*Elect. Technology, Indore Univ.*)**

Chapter two

2. Ohm's Law

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.

In other words, $\frac{V}{I} = constant\ or\ \frac{V}{I} = R$

Example 4: *A coil of copper wire has resistance of 90 Ω at 20°C and is connected to a 230-V supply. By how much must the voltage be increased in order to maintain the current consant if the temperature of the coil rises to 60°C ? Take the temperature coefficient of resistance of copper as 0.00428 from 0°C.*

Solution. As seen from Art. 1.10

$$\frac{R_{60}}{R_{20}} = \frac{1 + 60 \times 0.00428}{1 + 20 \times 0.00428} \qquad \therefore R_{60} = 90 \times 1.2568/1.0856 = 104.2\ \Omega$$

Now, current at 20°C = 230/90 = 23/9 A

Since the wire resistance has become 104.2 Ω at 60°C, the new voltage required for keeping the current constant at its previous value = 104.2 × 23/9 = 266.3 V

∴ increase in voltage required = 266.3 − 230 = **36.3 V**

2.1. Resistance in Series

When some conductors having resistances R1, R2 and R3 etc. are joined end-on-end as in Fig. 2.1, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that:

(i) Current is the same through all the three conductors.

(ii) voltage drop across each is different due to its different resistance and is given by Ohm's Law.

(iii) Sum of the three voltage drops is equal to the voltage applied across the three conductors.

There is a progressive fall in potential as we go from point A to D as shown in Fig. 2.2.

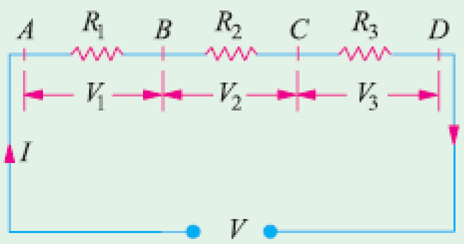


Fig 2.1

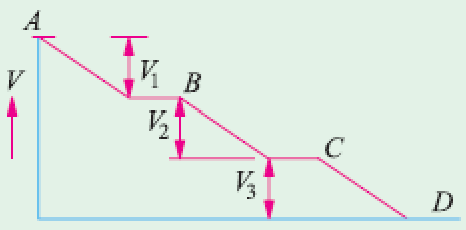


Fig 2.2

$\therefore \quad V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$ —Ohm's Law

But $\quad V = IR$

where R is the equivalent resistance of the series combination.

$\therefore \quad IR = IR_1 + IR_2 + IR_3 \quad$ or $\quad R = R_1 + R_2 + R_3$

Also $\quad \frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

2.3. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 2.3 is shown a 24-V battery connected across a series combination of three resistors.

Total resistance $\quad R = R_1 + R_2 + R_3 = 12\ \Omega$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V \cdot \frac{R_1}{R} = 24 \times \frac{2}{12} = 4\ \text{V}$$

$$V_2 = V \cdot \frac{R_2}{R} = 24 \times \frac{4}{12} = 8\ \text{V}$$

$$V_3 = V \cdot \frac{R_3}{R} = 24 \times \frac{6}{12} = 12\ \text{V}$$

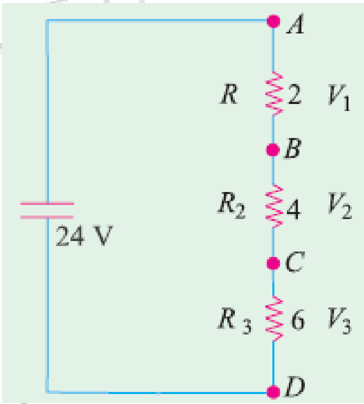
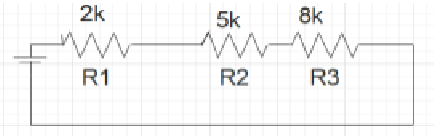


Fig.2.3

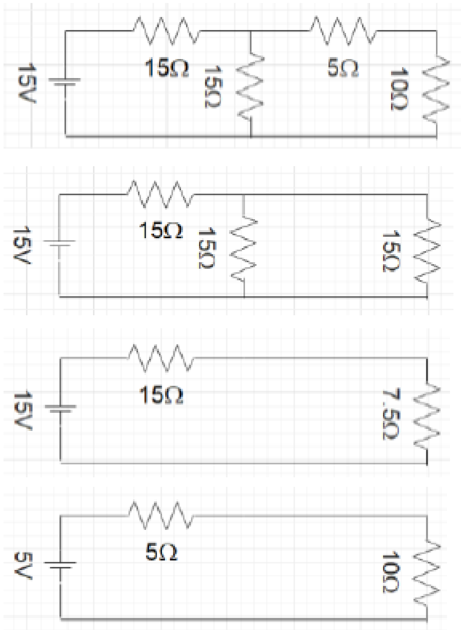
Example:-Using the voltage divider rule, determine the voltage across R_1&R_3 (if V_s = 45v).



Solution: $R_T = 15k\Omega$, $V_1 = \frac{R_1}{R_T} \cdot V_T = \frac{2}{15} \cdot 45 = 6v$

H.W.-: (Answer v_3 = 24v,v_2=15)

Example:- Determine the voltage across 5Ω in the following circuit.



Solution: ملاحظة: عندما تكون الدائرة الكهربائية مربوطة ربطا مختلطا فيجب تبسيطها وتحويلها الى دائرة توالي

$$V_{7.5\Omega} = \frac{7.5\Omega}{22.5\Omega} * 15 \rightarrow V_{7.5} = 5v$$

$$V_{5\Omega} = \frac{5\Omega}{(5+10)\Omega} * 5 \rightarrow V_5 = 1.667v$$

2.4. Resistances in Parallel

Three resistances, as joined in Fig. 2.4 are said to be connected in parallel. In this case:
(i) p.d. across all resistances is the same.
(ii) Current in each resistor is different and is given by Ohm's Law and
(iii) The total current is the sum of the three separate currents.

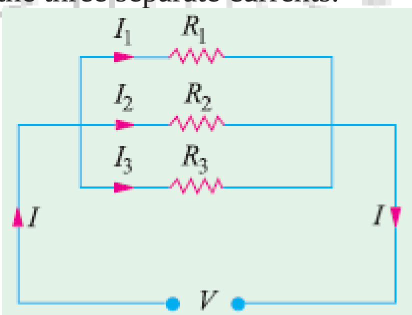


Fig.2.4

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now, $I = \frac{V}{R}$ where V is the applied voltage.

R = equivalent resistance of the parallel combination.

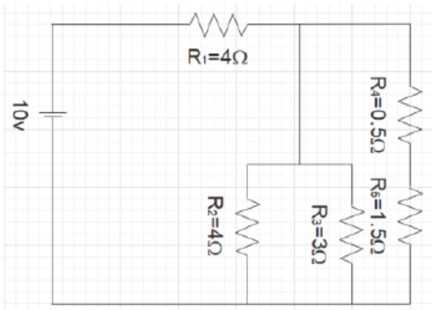
$$\therefore \quad \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also $G = G_1 + G_2 + G_3$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

Example:- Determine the voltage drop across each resistor and the current through each resistor using Ohm's law.



Solution: $R_a = 2\Omega$, $R_B = 2\Omega$, $R_c = 1\Omega$

$R_T = 5\Omega \rightarrow I_T = \frac{V_T}{R_T} = \frac{10v}{5\Omega} = 2A$

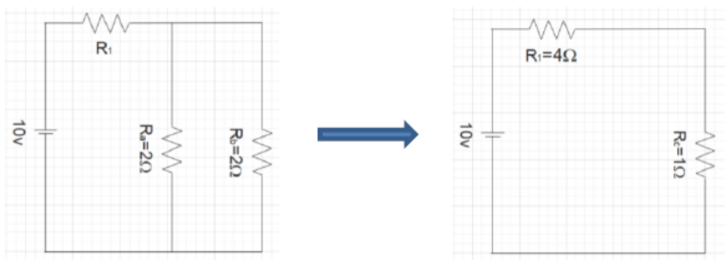
$I_1 = 2A \qquad V_1 = I_1 R_1 \rightarrow V_1 = 2*4 = 8v$

$R_A = 2v \qquad R_B = 2v$

$V_2 = 2v \qquad V_3 = 2v$

$I_2 = \frac{2v}{4\Omega} = 0.5A \qquad I_3 = \frac{2v}{4\Omega} = 0.5A \rightarrow R_A = 1A, \ R_B = 1A, \quad I_4 = I_5 = 1A$

$V_4 = I_4 R_4 = 0.5v, \quad V_5 = I_5 R_5 = 1.5V$



The Current Divider Rule:

في حالة وجود مقاومتان نستخدم القانون التالي:

$$I_1 = I_t\left(\frac{R_2}{R_1+R_2}\right),\quad I_2 = I_t\left(\frac{R_1}{R_1+R_2}\right)$$

هنالك حالات خاصة عندما يكون لدينا أكثر من مقاومتين فنستخدم:

$$I_x = \frac{R_T}{R_x}\, I_T$$

Example 6: *What is the value of the unknown resistor R in Fig. 1.16 if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.* **(Elect. Technology, Indore Univ.)**

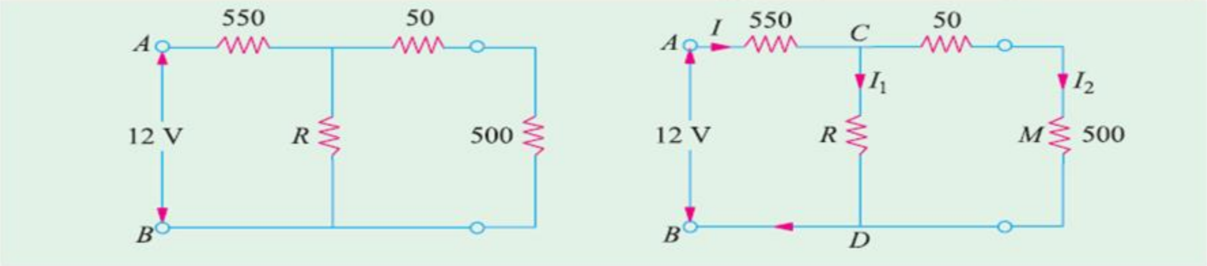


Fig. 1.16

Solution. By direct proportion, drop on 50 Ω resistance = 2.5 × 50/500 = 0.25 V

Drop across CMD or CD = 2.5 + 0.25 = 2.75 V

Drop across 550 Ω resistance = 12 − 2.75 = 9.25 V

I = 9.25/550 = 0.0168 A, I_2 = 2.5/500 = 0.005 A

I_1 = 0.0168 − 0.005 = 0.0118 A

∴ 0.0118 = 2.75/R; R = **233 Ω**

Example 1.26. *Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.*

Solution. Resistance between A and C (Fig. 1.17).

= 6 || 3 = 2 Ω

Resistance of branch ACD = 18 + 2 = 20 Ω

Now, there are two parallel paths between points A and D of resistances 20 Ω and 5 Ω

Hence, resistance between A and D = 20 || 5 = 4 Ω

∴Resistance between A and B = 4 + 8 = 12 Ω

Total circuit current = 60/12 = 5 A

Current through 5 Ω resistance = $5 \times \frac{20}{25} = 4$ A

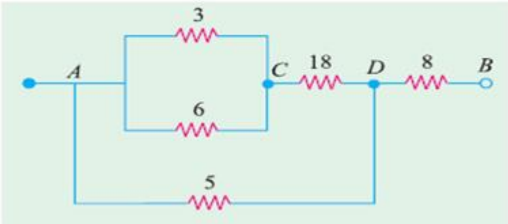
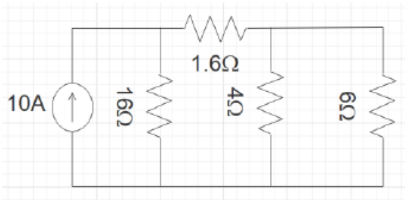


Fig. 1.17

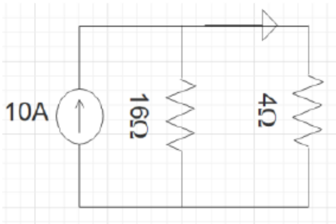
—**Art. 1.25**

Example:- Determine the power at 6Ω using the current divider rule for the circuit below.



Solution:

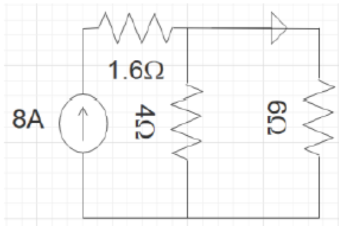
$$\frac{6*4}{6+4} = 2.4 + 1.6 = 4\Omega$$



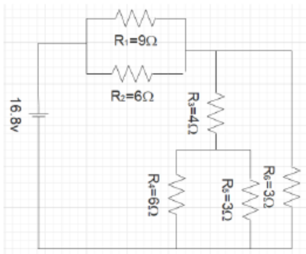
$$I_{4\Omega} = I_s\frac{16}{4+16} \rightarrow I_{4\Omega} = 10\frac{16}{20} = 8A$$

$$I_{6\Omega} = 8\frac{4}{4+6} = \frac{32}{10} = 3.2A$$

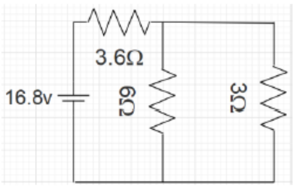
$$P = I^2R \rightarrow P = (3.2)^2 * 6 = 61.44W$$



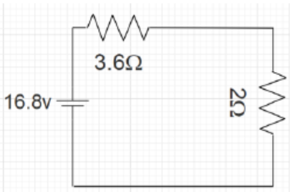
Example:- Find V_3 and I_5 for the circuit shown below.



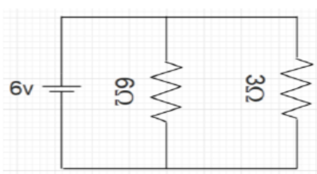
Solution:



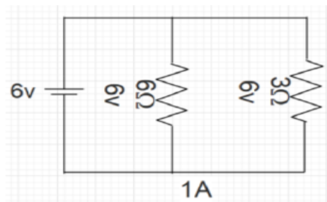
→

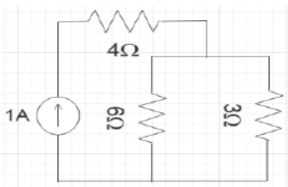


$$V_{2\Omega} = \frac{2}{5.6} * 16.8 = 6v$$

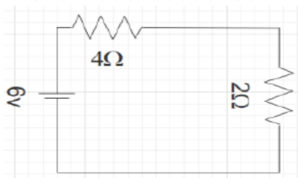


→



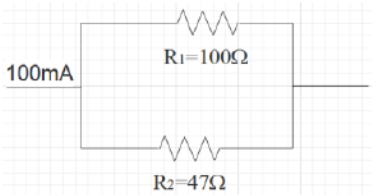


$$\rightarrow I_{3\Omega} = \frac{6}{9} * 1 = 0.667A$$



$$\rightarrow V_{4\Omega} = \frac{4}{6} * 6 = 4v$$

Example:- Find I_1 and I_2 in Figure below.



Kirchhoff's Laws: Kirchhoff's laws, two in number, are particularly useful ***(a)*** in determining the equivalent resistance of a complicated network of conductors and ***(b)*** for calculating the currents flowing in the various conductors. The two-laws are:

$$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$$

$$I_1 + I_4 - I_2 - I_3 - I_5 = 0 \quad \text{or} \quad I_1 + I_4 = I_2 + I_3 + I_5$$

incoming currents = outgoing currents

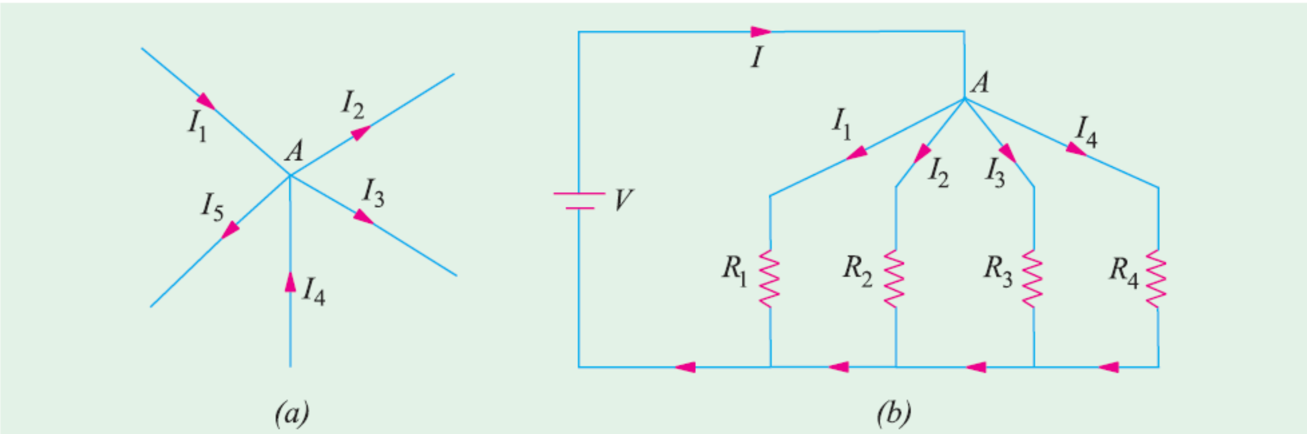
1. Kirchhoff's Point Law or Current Law (KCL)

It states as follows: in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.

Similarly, in Fig. below (*b*) for node *A*

$$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \quad \text{or} \quad I = I_1 + I_2 + I_3 + I_4$$

We can express the above conclusion thus : $\Sigma I = 0$ ....at a junction

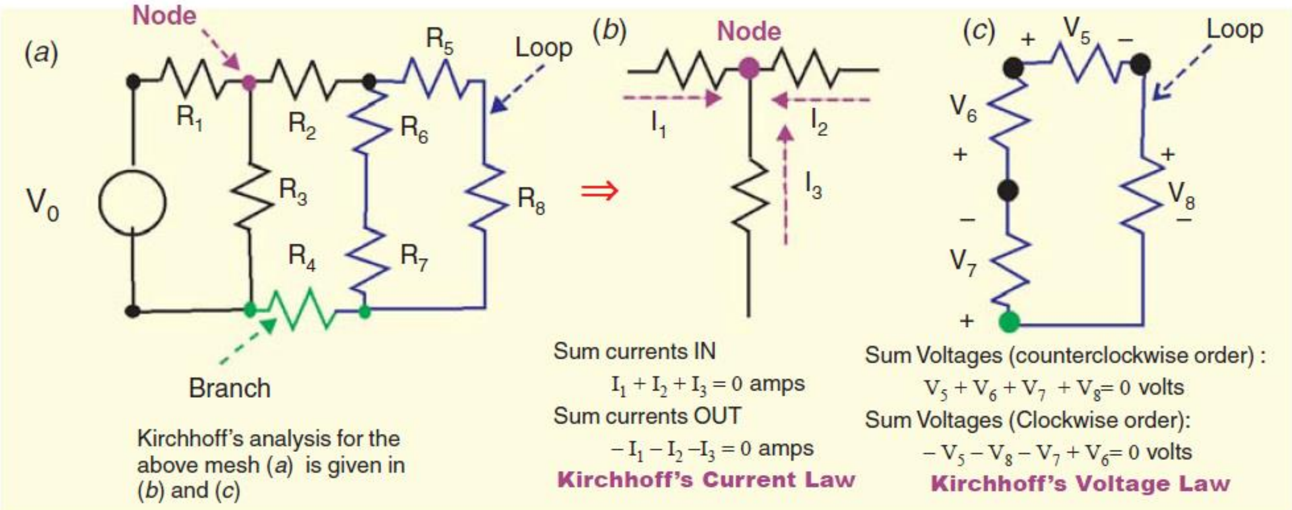


2. Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows: The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

It should be noted that algebraic sum is the sum that takes into account the polarities of the voltage drops.

In other words, $\Sigma IR + \Sigma e.m.f. = 0$ ...round a mesh



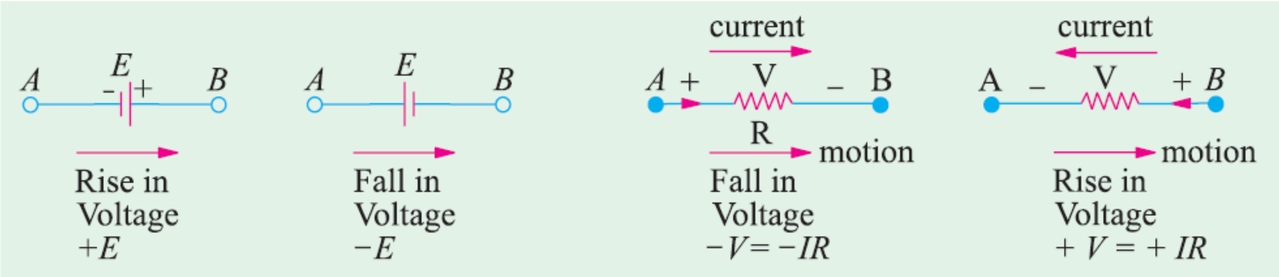
➢ Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. The following sign conventions are suggested:

a) Sign of Battery E.M.F.

A *rise* in voltage should be given a + ve sign and a ***fall*** in voltage a −ve sign. Keeping this in mind, it is clear that as we go from the −ve terminal of a battery to its +ve terminal (Fig. 2.3), there is a *rise* in potential, hence this voltage should be given a + ve sign. If, on the other hand, we go from +ve terminal to −ve terminal, then there is a *fall* in potential, hence this voltage should be preceded by a −ve sign. ***It is important to note that the sign of the***

battery e.m.f. is independent of the direction of the current through that branch.



b) Sign of IR Drop

Now, take the case of a resistor (Fig. below). If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken −ve. However, if we go in a direction opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Consider the closed path ABCDA in Fig. below. As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs:

I_1R_2 is −ve (fall in potential)

I_2R_2 is −ve (fall in potential)

I_3R_3 is +ve (rise in potential)

I_4R_4 is −ve (fall in potential)

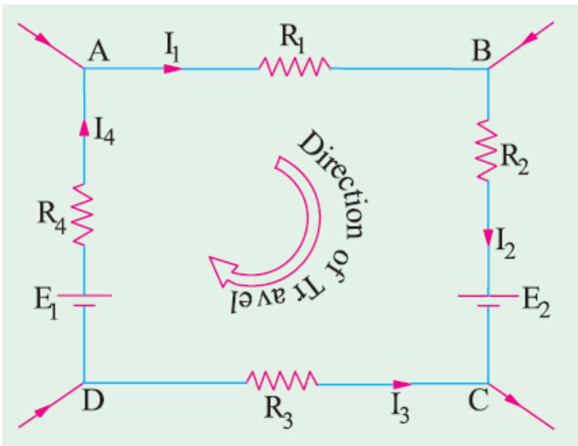
E_2 is −ve (fall in potential)

E_1 is +ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

or $$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$



Solving Simultaneous Equations

Electric circuit analysis with the help of Kirchhoff's laws usually involves the solution of two or three simultaneous equations. Determinants rule provide a simple and straight method for solving network equations.

➢ Determinants

The symbol $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called a determinant of the second order (or 2 × 2 determinant) because it contains two rows (*ab* and *cd*) and two columns (*ac* and *bd*). The numbers *a, b, c* and *d* are called the elements or constituents of the determinant. Their number in the present case is $2^2 = 4$.

The evaluation of such a determinant is accomplished by cross-multiplicaiton is illustrated below :

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

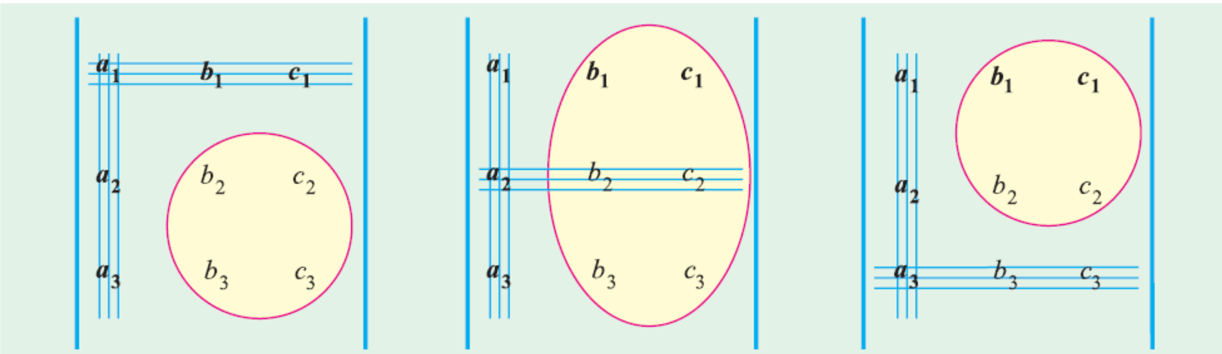
The above result for a second order determinant can be remembered as

upper left times lower right* minus *upper right times lower left

The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ represents a third-order determinant having $3^2 = 9$ elements. It may be evaluated (or expanded) as under :

1. Multiply each element of the first row (or alternatively, first column) by a determinant obtainedby omitting the row and column in which it occurs. (It is called minor determinant or just minor as shown in Fig. below.
2. Prefix + and − sing alternately to the terms so obtained.
3. Add up all these terms together to get the value of the given determinant.

Considering the first column, minors of various elements are as shown in Fig. below.



Minor of a_1 Minor of a_2 Minor of a_3

Expanding in terms of first column, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - a_2 (b_1c_3 - b_3c_1) + a_3 (b_1c_2 - b_2c_1) \qquad ...(i)$$

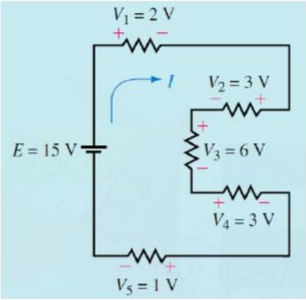
Expanding in terms of the first row, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

which will be found to be the same as above.

Example: Verify Kirchhoff's voltage law for the circuit of Figure



Solution If we follow the direction of the current, we write the loop equation as

$$15\text{ V} - 2\text{ V} - 3\text{ V} - 6\text{ V} - 3\text{ V} - 1\text{ V} = 0$$

Example. *Evaluate the determinan*

$$\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}$$

Solution. We will expand with the help of 1st column.

$$D = 7 \begin{vmatrix} 6 & -2 \\ -2 & 11 \end{vmatrix} - (-3) \begin{vmatrix} -3 & -4 \\ -2 & 11 \end{vmatrix} + (-4) \begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix}$$

$$= 7 [(6 \times 11) - (-2 \times -2)] + 3 [(-3 \times 11) - (-4 \times -2)] - 4 [(-3 \times -2) - (-4 \times 6)]$$

$$= 7 (66 - 4) + 3 (-33 - 8) - 4 (6 + 24) = \mathbf{191}$$

Solving Equations with Two Unknowns

Suppose the two given simultaneous equations are

$$ax + by = c$$
$$dx + ey = f$$

Here, the two unknown are x and y, a, b, d and e are coefficients of these unknowns whereas c and f are constants. The procedure for solving these equations by the method of determinants is as follows :

1. Write the two equations in the matrix form as $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$
2. The *common* determinant is given as $\Delta = \begin{bmatrix} a & b \\ d & e \end{bmatrix} = ae - bd$
3. For finding the determinant for x, replace the coefficients of x in the original matrix by the constants so that we get determinant Δ_1 given by $\Delta_1 = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = (ce - bf)$
4. For finding the determinant for y, replace coefficients of y by the constants so that we get $\Delta_2 = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = (af - cd)$
5. Apply Cramer's rule to get the value of x and y

$$x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd} \text{ and } y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$$

Example. Solve the following two simultaneous equations by the method of determinants:

$$4i_1 - 3i_2 = 1$$
$$3i_1 - 5i_2 = 2$$

Solution. The matrix form of the equations is $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = (4 \times -5) - (-3 \times 3) = -11$$

$$\Delta_1 = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix} = (1 \times -5) - (-3 \times 2) = 1$$

$$\Delta_2 = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (4 \times 2) - (1 \times 3) = 5$$

$$\therefore \quad i_1 = \frac{\Delta_1}{\Delta} = \frac{1}{-11} = -\frac{1}{11}; \quad i_2 = \frac{\Delta_2}{\Delta} = -\frac{5}{11}$$

Solving Equations with Three Unknowns

Let the three simultaneous equations be as under :

$$ax + by + cz = d$$
$$ex + fy + gz = h$$
$$jx + ky + lz = m$$

The above equations can be put in the matrix form as under :

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

The value of common determinant is given by

$$\Delta = \begin{vmatrix} a & b & c \\ e & f & g \\ j & k & l \end{vmatrix} = a\,(fl - gk) - e\,(bl - ck) + j(bg - cf)$$

The determinant for x can be found by replacing coefficients of x in the original matrix by the constants.

$$\therefore \quad \Delta_1 = \begin{vmatrix} d & b & c \\ h & f & g \\ m & k & l \end{vmatrix} = d\,(fl - gk) - h\,(bl - ck) + m(bg - cf)$$

Similarly, determinant for y is given by replacing coefficients of y with the three constants.

$$\Delta_2 = \begin{vmatrix} a & d & c \\ e & h & g \\ j & m & l \end{vmatrix} = a\,(hl - mg) - e\,(dl - mc) + j\,(dg - hc)$$

In the same way, determinant for z is given by

$$\Delta_3 = \begin{vmatrix} a & b & d \\ e & f & h \\ j & k & m \end{vmatrix} = a\,(fm - hk) - e\,(bm - dk) + j\,(bh - df)$$

As per Cramer's rule $\quad x = \dfrac{\Delta_1}{\Delta}, \; y = \dfrac{\Delta_2}{\Delta}, \; z = \dfrac{\Delta_3}{\Delta}$

Example. Solve the following three simultaneous equations by the use of determinants and Cramer's rule

$$i_1 + 3i_2 + 4i_3 = 14$$
$$i_1 + 2i_2 + i_3 = 7$$
$$2i_1 + i_2 + 2i_3 = 2$$

Solution. As explained earlier, the above equations can be written in the form

$$\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 1(4-1) - 1\,(6-4) + (3-8) = -9$$

$$\Delta_1 = \begin{bmatrix} 14 & 3 & 4 \\ 7 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 14\,(4-1) - 7\,(6-4) + 2(3-8) = 18$$

$$\Delta_2 = \begin{bmatrix} 1 & 14 & 4 \\ 1 & 7 & 1 \\ 2 & 2 & 2 \end{bmatrix} = 1\,(14-2) - 1\,(28-8) + 2\,(14-28) = -36$$

$$\Delta_3 = \begin{bmatrix} 1 & 3 & 14 \\ 1 & 2 & 7 \\ 2 & 1 & 2 \end{bmatrix} = 1\,(4-7) - 1\,(6-14) + 2\,(21-28) = -9$$

According to Cramer's rule,

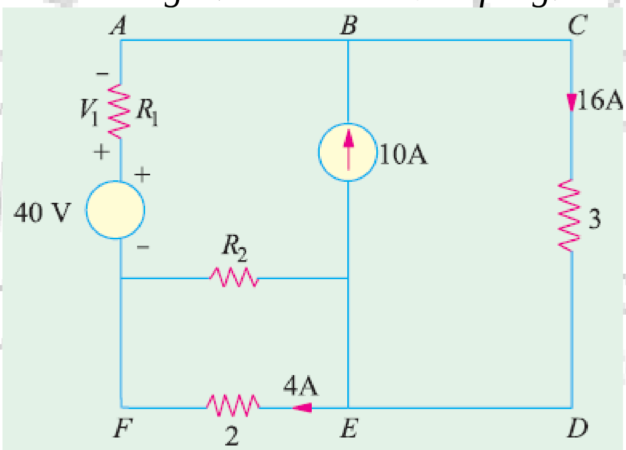
$$i_1 = \frac{\ }{1} \quad \frac{18}{9} \quad -2A \; ; \; i_2 \quad \frac{\ }{2} \quad \frac{36}{9} \quad -4A \; ; \; i_3 \quad \frac{\ }{3} \quad \frac{9}{9} \quad 1A$$

Example. *What is the voltage Vs across the open switch in the circuit of Fig. below?*

Solution. We will apply *KVL* to find *Vs*. Starting from point *A* in the clockwise direction and using the sign convention, we have

$$+V_s + 10 - 20 - 50 + 30 = 0 \quad \therefore \; V_s = 30 \text{ V}$$

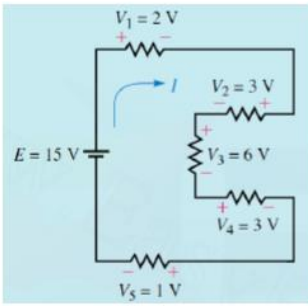
Example. *Find the unknown voltage V1 in the circuit of Fig. below.*



Solution. Taking the outer closed loop *ABCDEFA* and applying *KVL* to it, we get

$$-16 \times 3 - 4 \times 2 + 40 - V_1 = 0 \; ; \quad \therefore \; V_1 = -16 \text{ V}$$

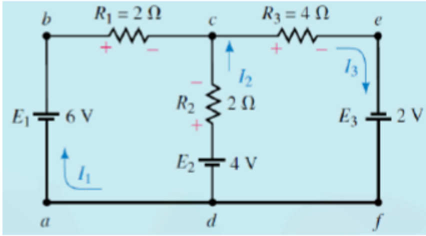
Example: Verify Kirchhoff's voltage law for the circuit of Figure



Solution If we follow the direction of the current, we write the loop equation as

$$15\text{ V} - 2\text{ V} - 3\text{ V} - 6\text{ V} - 3\text{ V} - 1\text{ V} = 0$$

Example: Find the current in each branch for the circuit shown.



Solution:

Step1) Assign the current as shown in figure,
Step 2) Indicate the polarities of the voltage drops on all resistors in the circuit using the assumed current directions.
Step 3) Write the Kirchhoff voltage law equations.
Loop *abcda*: $6\text{ V} - (2\ \Omega)I_1 + (2\ \Omega)I_2 - 4\text{ V} = 0\text{ V}$
Notice that the circuit still has one branch which has not been included in the KVL equations, namely the branch *cefd*. This branch would be included if a loop equation for *cefdc* or for *abcefda* were written. There is no reason for choosing one loop over another, since the overall result will remain unchanged even though the intermediate steps will not give the same results.
Loop *cefdc*: $4\text{ V} - (2\ \Omega)I_2 - (4\ \Omega)I_3 + 2\text{ V} = 0\text{ V}$
Now that all branches have been included in the loop equations, there is no need to write any more. Although more loops exist, writing more loop equations would needlessly complicate the calculations.
Step 4) Write the Kirchhoff current law equations. By applying KCL at node c, all branch currents in the network are included.
Node c: $I_3 = I_1 + I_2$
To simplify the solution of the simultaneous linear equations we write them as follows:

$2I_1 - 2I_2 - 0I_3 = 2$
$0I_1 - 2I_2 - 4I_3 = -6$
$1I_1 + 1I_2 - 1I_3 = 0$

The principles of linear algebra allow us to solve for the determinant of the denominator as follows:

$$D = \begin{vmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= 2\begin{vmatrix} -2 & -4 \\ 1 & -1 \end{vmatrix} - 0\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix}$$
$$= 2(2 + 4) - 0 + 1(8) = 20$$

Now, solving for the currents, we have the following:

$$I_1 = \frac{\begin{vmatrix} 2 & -2 & 0 \\ -6 & -2 & -4 \\ 0 & 1 & -1 \end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix}-2 & -4\\ 1 & -1\end{vmatrix} - (-6)\begin{vmatrix}-2 & 0\\ 1 & -1\end{vmatrix} + 0\begin{vmatrix}-2 & 0\\ -2 & -4\end{vmatrix}}{20}$$

$$= \frac{2(2+4)+6(2)+0}{20} = \frac{24}{20} = 1.200 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix}2 & 2 & 0\\ 0 & -6 & -4\\ 1 & 0 & -1\end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix}-6 & -4\\ 0 & -1\end{vmatrix} - 0\begin{vmatrix}2 & 0\\ 0 & -1\end{vmatrix} + 1\begin{vmatrix}2 & 0\\ -6 & -4\end{vmatrix}}{20}$$

$$= \frac{2(6)+0+1(-8)}{20} = \frac{4}{20} = 0.200 \text{ A}$$

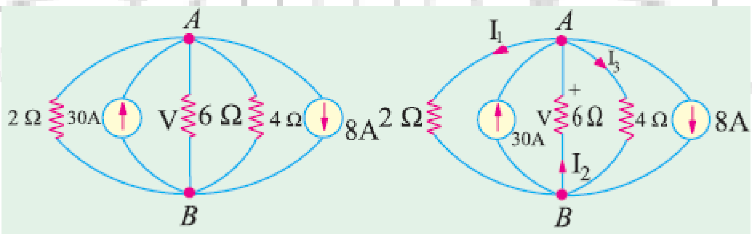
$$I_3 = \frac{\begin{vmatrix}2 & -2 & 2\\ 0 & -2 & -6\\ 1 & 1 & 0\end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix}-2 & -6\\ 1 & 0\end{vmatrix} - 0\begin{vmatrix}-2 & 2\\ 1 & 0\end{vmatrix} + 1\begin{vmatrix}-2 & 2\\ -2 & -6\end{vmatrix}}{20}$$

$$= \frac{2(6)-0+1(12+4)}{20} = \frac{28}{20} = 1.400 \text{ A}$$

Example 2.6. *Using Kirchhoff's Current Law and Ohm's Law, find the magnitude and polarity of voltge V in Fig. below (a). Directions of the two current sources are as shown.*

Solution. *Let us arbitrarily choose the directions of I_1, I_2 and I_3 and polarity of V as shown in Fig below. (b). We will use the sign convention for currents. Applying KCL to node A, we have*



(a) (b)

$$-I_1 + 30 + I_2 - I_3 - 8 = 0$$

or $$I_1 - I_2 + I_3 = 22 \quad ...(i)$$

Applying Ohm's law to the three resistive branches in Fig. 2.9 (*b*), we have

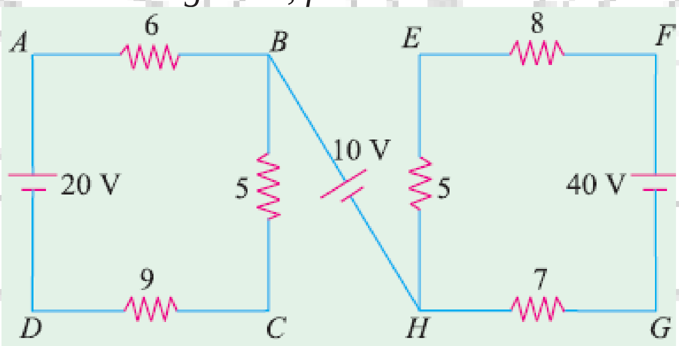
$$I_1 = \frac{V}{2}, I_3 = \frac{V}{4}, I_2 = -\frac{V}{6}$$ (Please note the –ve sign.)

Substituting these values in (*i*) above, we get

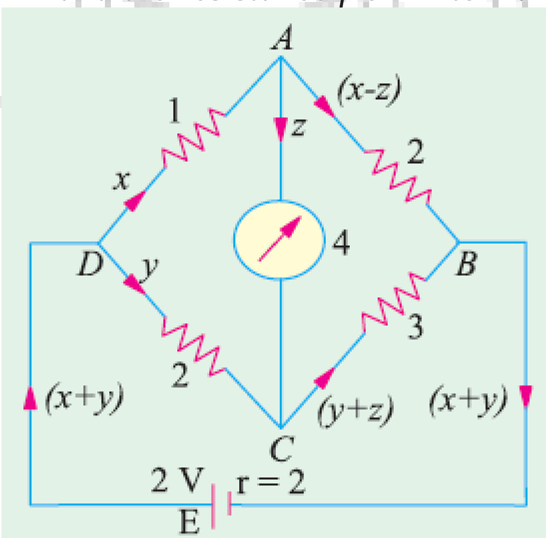
$$\frac{V}{2} - \left(\frac{-V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = 24 \text{ V}$$

The negative sign of I2 indicates that actual direction of its flow is opposite to that shown in Fig. (b). Actually, I2, flows from A to B and not from B to A as shown. Incidentally, it may be noted that all currents are outgoing except 30A which is an incoming current.

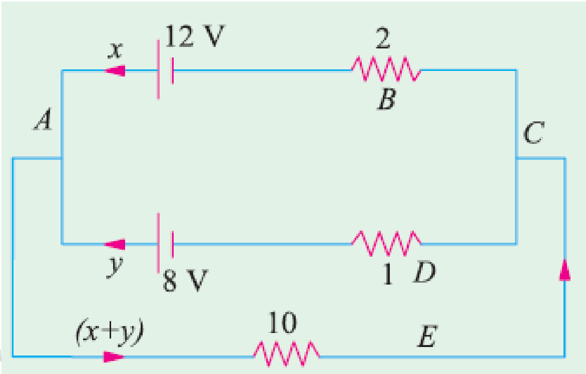
H.W. *For the circuit shown in Fig. 2.10, find VCE and VAG.*



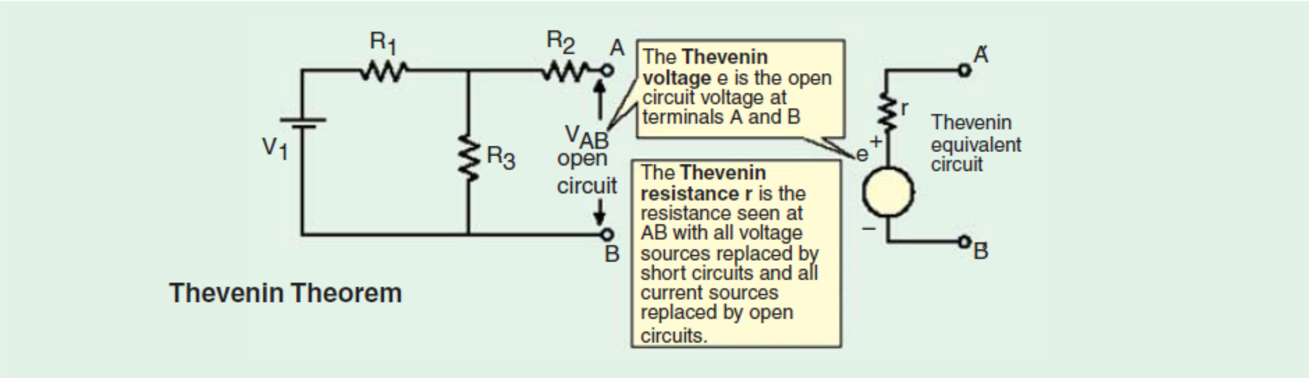
H.W. *Determine the currents in the unbalanced bridge circuit of Fig. below. Also, determine the p.d. across BD and the resistance from B to D.*



H.W. *Two batteries A and B are connected in parallel and load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω; B has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.*



Thevenin Theorem:



It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by ***a single voltage source with a series resistance***. It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.

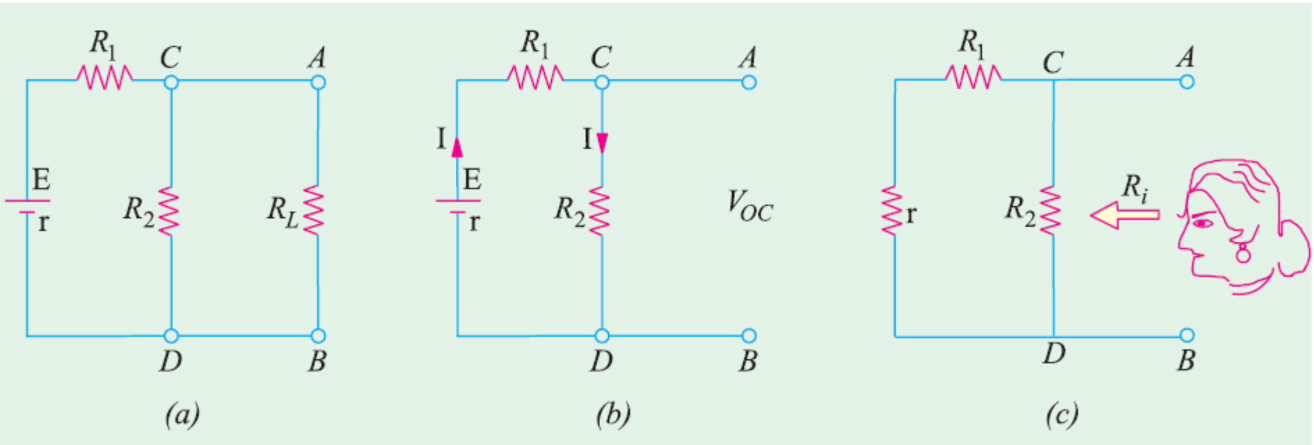


Fig. 4.1

Suppose it is required to find current flowing through load resistance R_L, as shown in Fig. 4.1 (a). We will proceed as under:

1. Remove R_L from the circuit terminals *A* and *B* and redraw the circuit as shown in Fig. 4.1 (*b*). Obviously, the terminals have become open-circuited.

2. Calculate the open-circuit voltage V_{oc} which appears across terminals *A* and *B* when they are open *i.e.* when *RL* is removed. As seen, V_{oc} = drops across $R_2 = IR_2$ where *I* is the circuit current when *A* and *B* are open.

$I = \frac{E}{R_1 + R_2 + r}$ $\therefore$ $V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r}$ [r is the internal resistance of battery]

It is also called 'Thevenin voltage' V_{th}.

3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance r behind, and redrawing the circuit, as shown in Fig. 4.1 (c). When viewed inwards from terminals A and B, the circuit consists of two parallel paths : one containing R_2 and the other containing (R_1 + r). The equivalent resistance of the network, as viewed from these terminals is given as

$$R = R_2 \| (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called,* Thevenin resistance R_{sh} (though, it is also sometimes written as R_i or R_0). Consequently, as viewed from terminals A and B, the whole network (excluding R1) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals Voc (or Vsh) and whose internal resistance equals R_{sh} (or R_i) as shown in Fig. 4.2.

4. RL is now connected back across terminals A and B from where it was temporarily removed earlier. Current flowing through RL is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by Voc || (R_i + R_L) where V_{oc} is the open-circuit voltage (i.e. voltage across the two terminals when RL is removed) and R_i is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.

2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.

3. Compute the resistance of whose network as investigated from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.

4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.

5. Connect *RL* back to its terminals from where it was previously removed.

6. Finally, calculate the current flowing through R_L by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

Example. *State Thevenin's theorem and give a proof. Apply this theorem to calculate the current through the 4 Ω resistor of the circuit of Fig. 4.2 (a).*

Solution. As shown in Fig. 4.1 (*b*), 4 Ω resistance has been removed thereby open circuiting the terminals *A* and *B*. We will now find V_{AB} and R_{AB} which will give us *Vth* and R_{th} respectively. The potential drop across 5 Ω resistor can be found with the help of voltage-divider rule. Its value is = 15 × 5/(5 + 10) = 5 V.

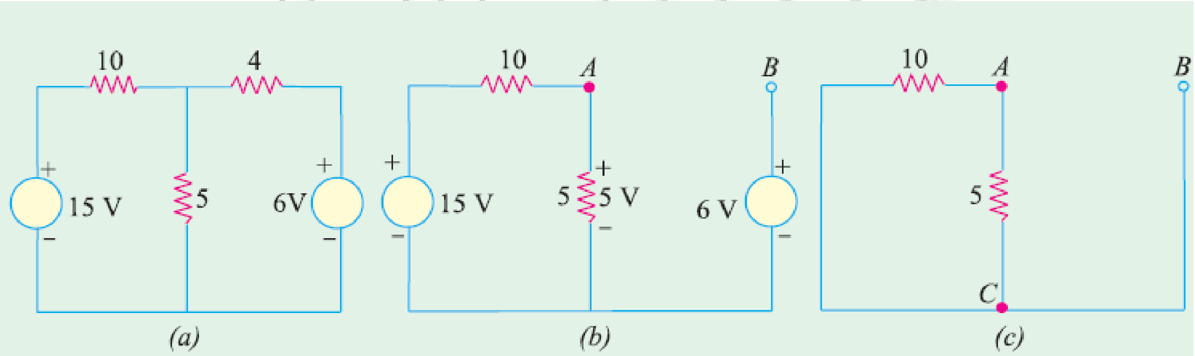


Fig. 4.1

For finding V_{AB}, we will go from point *B* to point *A* in the clockwise direction and find the algebraic sum of the voltages met on the way.

$\therefore V_{AB} = -6 + 5 = -1$ V.

It means that point *A* is negative with respect to point *E*, or point *B* is at a higher potential than point *A* by one volt. In Fig. 2.130 (*c*), the two-voltage source has been short-circuited.

The resistance of the network as viewed from points *A* and *B* is the same as viewed from points *A* and *C*.

$\therefore R_{AB} = R_{AC} = 5 \parallel 10 = 10/3\ \Omega$

Thevenin's equivalent source is shown in Fig. 4.3 in which 4 Ω resistors have been joined back across terminals *A* and *B*. Polarity of the voltage source is worth nothing.

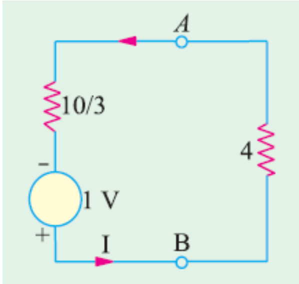


Fig. 4.3

$$I = \frac{1}{(10/3)+4} = \frac{3}{22} = 0.136 \text{ A} \qquad \text{From } E \text{ to } A$$

Example . *With reference to the network of Fig. 2.132 (a), by applying Thevenin's theorem find the following :*

(*i*) *the equivalent e.m.f. of the network when viewed from terminals A and B.*

(*ii*) *the equivalent resistance of the network when looked into from terminals A and B.*

(*iii*) *current in the load resistance RL of 15 Ω.*

Solution. (*i*) Current in the network before load resistance is connected [Fig. 4.4 (*a*)] = 24/(12 + 3 + 1) = 1.5 A

∴ voltage across terminals *AB* = *Voc* = *Vth* = 12 × 1.5 = **18 V**

Hence, so far as terminals *A* and *B* are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(*ii*) There are two parallel paths between points *A* and *B*. Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point *A* and *B* is [Fig. 4.4 (*c*)] $R_i = R_{th} = 12 \times 4/(12 + 4) = 3\ \Omega$

(*iii*) When load resistance of 15 Ω is connected across the terminals, the network is reduced to the structure shown in Fig. 4.4 (*d*).

I = *Vth*/(*Rth* + *RL*) = 18/(15 + 3) = **1 A**

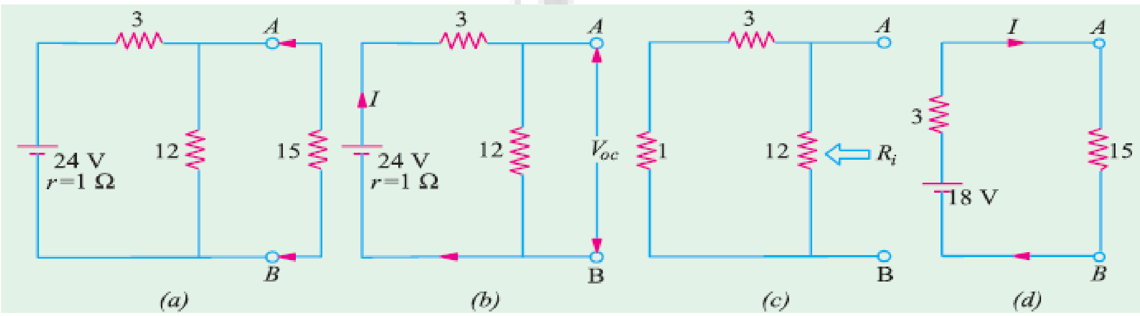


Fig. 4.4

Example. *Using Thevenin theorem, calculate the current flowing through the 4 Ω resistor of Fig. 4.5* (*a*).

Solution. (*i*) Finding V_{th}

If we remove the 4-Ω resistor, the circuit becomes as shown in Fig. 4.5 (*b*). Since full 10 A current passes through 2 Ω resistors, drop across it is 10 × 2 = 20 V. Hence, *VB* = 20 V with respect to the common ground. The two resistors of 3 Ω and 6 Ω are connected in series across the 12 V battery.

Hence, drop across 6 Ω resistor = 12 × 6/(3 + 6) = **8 V**.

∴ *VA* = 8 V with respect to the common ground*

∴ V_{th} = *VBA* = *VB* − *VA* = 20 − 8 = 12 V—with *B* at a higher potential

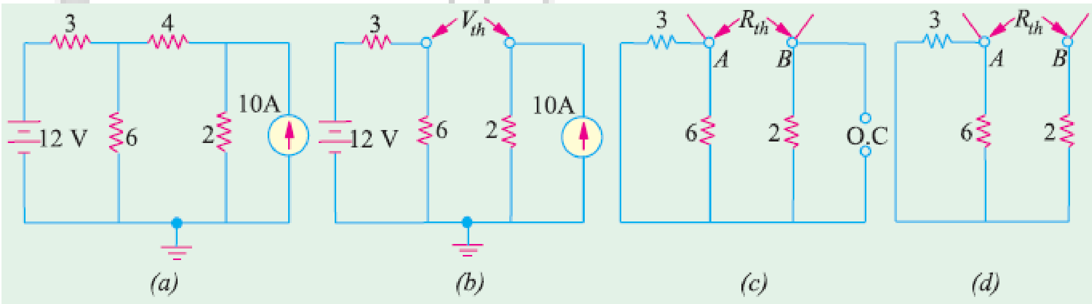


Fig. 4.5

(*ii*) Finding *Rth*

Now, we will find R_{th} *i.e.* equivalent resistance of the network as looked back into the open-circuited terminals *A* and *B*. For this purpose, we will replace both the voltage and current sources. Since voltage source has no internal resistance, it would be replaced by a short circuit *i.e.* zero resistance. However,

current source would be removed and replaced by an 'open' *i.e.* infinite resistance (Art. 1.18). In that case, the circuit becomes as shown in Fig. 2.5 (*c*). As seen from Fig. 2.5 (*d*), F_{th} = 6 || 3 + 2 = 4 Ω. Hence, Thevenin's equivalent circuit consists of a voltage source of 12 V and a series resistance of 4 Ω as shown in Fig. 2.6 (*a*). When 4 Ω resistors are connected across terminals *A* and *B*, as shown in Fig. 2.6 (*b*).

I = 12/(4 + 4) = **1.5 A—from *B* to *A***

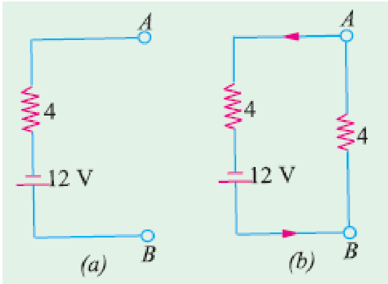


Fig. 2.6

Example 2.63. *For the circuit shown in Fig. 2.7 (a), calculate the current in the 10-ohm resistance. Use Thevenin's theorem only.*

Solution. *When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.7 (b).*

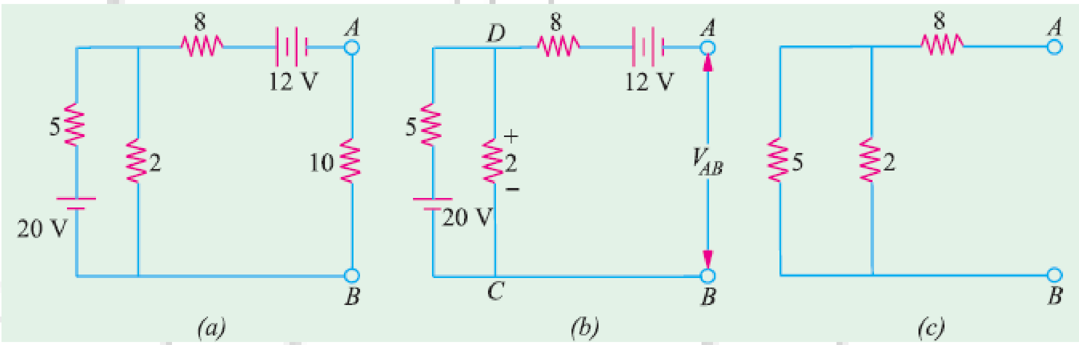


Fig. 2.7

Now, we will find the open-circuit voltage $V_{AB} = V_{th}$. For this purpose, we will go from point B to point A and find the algebraic sum of the voltages met on the way. It should be noted that with terminals A and B open, there is no voltage drop on the 8 Ω resistance.

However, the two resistances of 5 Ω and 2 Ω are connected in series across the 20-V battery. As per voltage- divider rule, drop on 2 Ω resistance = 20 × 2/(2 + 5) = 5.71 V with the polarity as shown in figure. As per the sign convention of Art.

$V_{AB} = Vth = + 5.71 - 12 = - 6.29\ V$

The negative sign shows that point A is negative with respect to point B, or which is the same thing, point B is positive with respect to point A.

For finding $R_{AB} = R_{th}$, we replace the batteries by short-circuits .

$\therefore RAB = Rth = 8 + 2 \,||\, 5 = 9.43\ \Omega$

Hence, the equivalent Thevenin's source with respect to terminals A and B is as shown in Fig. 2.8. When 10 Ω resistance is reconnected across A and B, current through it is I = 6.24/(9.43 + 10) = ***0.32 A.***

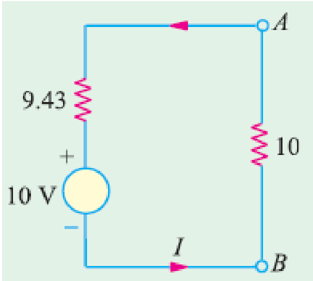


Fig. 2.8

H.W-1. *Using Thevenin's theorem, calculate the p.d. across terminals A and B in Fig. 2.9.*

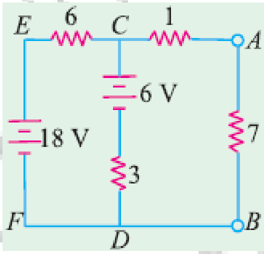


Fig. 2.9.

H.W-2. *Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of the network shown in Fig. 2.10 if the load is (a) 2 Ω (b) 1 Ω.*

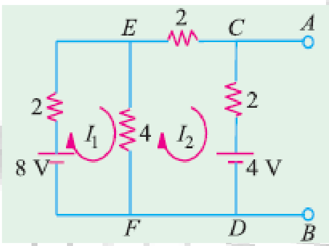


Fig. 2.10

H.w-3. *The four arms of a Wheatstone bridge have the following resistances : AB = 100, BC = 10, CD = 4, DA = 50 Ω. A galvanometer of 20 Ω resistance is connected across BD. Use Thevenin's theorem to compute the current through the galvanometer when a p.d. of 10 V is maintained across AC.*

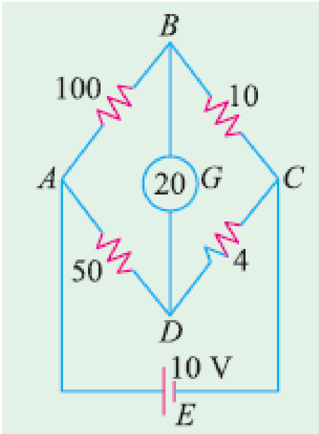


Fig. 1.11.